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SUPERSYMMETRIC CP VIOLATION IN B DECAYS AT LHCB EXPERIMENT

By
Dris Boubaa

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Table of Contents

List of Figures	viii
List of Tables	x
Acknowledgements	xii
Abstract	xiii
Introduction	1
1 Supersymmetry	5
1.1 Standard Model	6
1.2 What is Supersymmetry?	12
1.3 Supersymmetry Algebra	15
1.4 Supersymmetric Lagrangians	21
1.4.1 Wess-Zumino Lagrangian	21
1.4.2 Chiral Superfield Lagrangian	25
1.4.3 Lagrangians for Gauge Supermultiplets	27
1.4.3.1 Free Supersymmetric Abelian Gauge theory	28
1.4.3.2 Supersymmetric Gauge Interactions	31
1.4.4 Soft Supersymmetry Breaking Lagrangian	34
1.4.5 The Complete Lagrangian	35
1.5 Minimal Supersymmetric Standard Model	36
1.5.1 Electroweak Symmetry Breaking	39
1.5.2 Masses of the Gauge Bosons	42
1.5.3 MSSM Mass Spectrum	44
1.5.3.1 Charginos	44

1.5.3.2	Neutralinos	47
1.5.3.3	Sfermions	51
1.5.3.4	Higgs Sector	56
1.6	SUSY searches at the LHC	57
2	CP Violation in Supersymmetric Models	61
2.1	CP Transformation	61
2.2	CP Violation in Meson Decay	64
2.3	Classification of CP violation	67
2.4	Recent Measurements of CP Violating Observables	72
2.5	CP Violation in the Standard Model	74
2.6	CP Violation in $K^0 - \bar{K}^0$ Mixing	80
2.7	CP Violation in the MSSM	86
2.8	Flavour Changing Neutral Currents	88
2.9	Electric Dipole Moments and SUSY CP Problem	90
2.9.1	SM contributions to EDMs	93
2.9.2	SUSY contribution to EDMs	94
3	Prediction for $B \rightarrow \tau \bar{\mu} + \mu \bar{\tau}$ in the MSSM and 2HDM	100
3.1	Looking for New Physics in $B_{s,d} \rightarrow \ell_i \ell_j$	100
3.2	Weak Effective Hamiltonian	102
3.3	Effective Hamiltonian for $b \rightarrow q(=d,s) \ell_i^- \ell_j^+$	111
3.4	$B_{s,d} \rightarrow \ell_i^- \ell_j^+$ in 2HDM	113
3.5	$B_{s,d} \rightarrow \ell_i^- \ell_j^+$ in the MSSM	117
3.5.1	Chargino Contribution	119
3.5.2	Neutralino Contribution	122
3.5.3	Result and Discussion	125
4	$R(D)$ and $R(D^*)$ in the MSSM	128
4.1	Recent Experimental Result for $\mathcal{R}(D)$ and $\mathcal{R}(D^*)$	128
4.2	Helicity Amplitudes and Related Observables	130
4.3	New Physics Wilson Coefficients for $b \rightarrow c \tau \nu_\tau$	137
4.4	Experimental Constraints	142
4.5	$\mathcal{R}(D)$ and $\mathcal{R}(D^*)$ Result in the MSSM with CP Conservation	144
4.6	CP Violation Effect on $\mathcal{R}(D)$ and $\mathcal{R}(D^*)$	149
5	Conclusion	154
	APPENDICES	156

A	Notations and Definitions	156
A.1	Spinors	156
A.1.1	Weyl Spinors	156
A.1.2	Dirac Spinors	158
A.1.3	Majorana Spinors	159
A.2	Superspace	159
A.3	Superfields	161
A.4	Constrained Superfields	165
A.4.1	Chiral Superfield	165
A.4.2	Vector Superfield	167
B	Amplitudes and Form Factors	169
	Bibliography	174

List of Figures

1.1	ATLAS summary of SUSY particles search and limits [60]	59
1.2	CMS summary of SUSY particles search and limits 2017 [61]	60
2.1	C, P, and CP transformation of the decay $\pi^+ \rightarrow \mu^+ \nu_\mu$.	63
2.2	Experimental constraints on the $\rho - \eta$ plane as obtained by the UTfit collaboration [69].	77
2.3	The unitarity triangle of the CKM matrix.	79
2.4	Box diagrams contributing to $K^0 - \bar{K}^0$ mixing in the SM.	81
2.5	Possible two-loop SM contributions to the EDMs.	94
2.6	Leading SUSY contributions to the EDMs. The photon and gluon lines are to be attached to the loop in all possible ways.	95
2.7	Two-loop of gluino-quark-squark contribution to the EDMs.	95
3.1	One-loop current-current diagrams in the full theory.	105
3.2	One loop current-current diagrams in the effective theory.	105
3.3	Typical diagrams in the full theory, tree, penguin and box in the SM.	107
3.4	$b \longrightarrow ql_+l_-$ through the scalar X mediation.	113
3.5	The variation of $BR(B_s \rightarrow \mu^- \tau^+ + \mu^+ \tau^-)$ with the couplings $ Y_{sb} - Y_{bs}^* $ and $ Y_{\mu\tau} - Y_{\tau\mu}^* $ for $m_X = 125 \text{ GeV}$ and $f_{B_s} = .229 \pm 0.006 \text{ GeV}$. Scatter points are allowed by ΔM_s^{exp} and the experimental bounds in Eq. (3.4.12).	116

3.6	Figure shows the $(Y_{\mu\tau} ^2 - Y_{\tau\mu}^* ^2)$ dependence of $P_L(B_s \rightarrow \mu^- \tau^+)$. <i>Scatter points are allowed by ΔM_s^{exp} and the experimental bounds in Eq. (3.4.12).</i>	116
3.7	One loop SUSY contributions to $B_s \rightarrow \tau^- \mu^+$.	118
3.8	The chargino contribution for $BR(B_s \rightarrow \tau \bar{\mu} + \mu \bar{\tau})$ as function of $\tan \beta$ for stop masses of order 200 and 800 GeV and $M_2 \in [100, 1000]$ GeV and $\mu \in [250, 1000]$ GeV.	126
4.1	Comparison of the BaBar results (light band, blue) with predictions that include a charged Higgs boson of type II 2HDM (red band, dark). The SM corresponds to $\tan \beta / m_H^+ = 0$ [26, 132].	130
4.2	The allowed regions in the (g_{SL}, g_{SR}) (left) and (g_{VL}, g_{VR}) (right) planes by the 1σ and 2σ experimental results on $\mathcal{R}(D)$ (magenta) and $\mathcal{R}(D^*)$ (blue) of Belle.	136
4.3	Diagrams contributing to $b \rightarrow c \tau \nu_\tau$ transitions	137
4.4	Triangle diagrams (penguins) contributing to, <i>e.g.</i> , $b \rightarrow c(\tau, e)\nu_{(\tau, e)}$ affecting the leptonic vertex.	138
4.5	Behaviour of the first term in Eq (4.3.2), $m_{\tilde{\chi}^0} m_{\tilde{\chi}^-} C_0(m_{\tilde{\chi}^0}^2, m_{\tilde{\chi}^-}^2, m_{\tilde{\tau}}^2)$ (left) and $C_0(m_{\tilde{\chi}^0}^2, m_{\tilde{\chi}^-}^2, m_{\tilde{\tau}}^2)$ (right), respectively, with $m_{\tilde{\chi}^-}$ and $m_{\tilde{\tau}}$ for degenerate chargino/neutralino masses.	141
4.6	The allowed regions in the (g_{SL}, g_{SR}) plane by the 1σ and 2σ experimental results on $\mathcal{R}(D)$ (magenta) and $\mathcal{R}(D^*)$ (blue) of BaBar (left) and Belle (middle). SM and complete SUSY predictions (tree, penguin and box contributions, where tree-level effect is the dominant for the scalar interactions) are also included and they coincide with the black point. The correlation between the SUSY corrected values of g_{SL} and g_{SR} is displayed in the right panel.	145

4.7	The allowed regions in the (g_{VL}, g_{VR}) plane by the 1σ and 2σ experimental results on $\mathcal{R}(D)$ (magenta) and $\mathcal{R}(D^*)$ (blue) of BaBar (left) and Belle (middle). SM (black point) and complete SUSY (red points) predictions (penguin and box contributions, where the penguin is the dominant one) are also included. The correlation between the SUSY corrected values of g_{VL} and g_{VR} is displayed in the right panel, where yellow points represent the configurations that yield $\Gamma(\tau \rightarrow \mu\nu_\tau\nu_\mu)/\Gamma(\tau \rightarrow e\nu_\tau\nu_e)$ within experimental limits while the red ones are the complete sample.	145
4.8	The correlation between $\mathcal{R}(D)$ and $\mathcal{R}(D^*)$ at tree level (top-left) and after the one-loop SUSY contributions through the lepton penguins (top-right) where the blue points show the constrained ones by $\Gamma(\tau \rightarrow \mu\nu_\tau\nu_\mu)/\Gamma(\tau \rightarrow e\nu_\tau\nu_e)$. The correlation between sneutrino and stau masses is on the bottom-left (same colour scheme as in the top-right frame with the additional yellow points representing the region with $\mathcal{R}(D) > 0.33$). (Bottom-right) This represents the correlation between chargino and neutralino masses where the colours are the same meaning as in the top and bottom-left panels.	146
4.9	$\mathcal{R}(D)$ and $\mathcal{R}(D^*)$ as functions of the complex phase $\phi_{VL} \in [-\pi, \pi]$ (top) and $ g_{VL} $ (bottom) for $ g_{VL} = 0.01, 0.05, 0.1$ and $ \phi_{VL} = 0, \pi/2, \pi$, respectively.	151
4.10	$\mathcal{R}(D)$ and $\mathcal{R}(D^*)$ as functions of eEDM (top) and complex phase ϕ_{VL} (bottom) in the MSSM.	152

List of Tables

4.1	Loop functions values which correspond to the benchmark given in Tab. 4.2.	144
4.2	Illustrative benchmark point yielding large $\mathcal{R}(D)$ and $\mathcal{R}(D^*)$ values. .	148

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Abstract

One of the most promising signals of new physics at colliders is the rare decay $B \rightarrow \tau\bar{\mu} + \mu\bar{\tau}$. If the observation of $B \rightarrow \tau\bar{\mu} + \mu\bar{\tau}$ occurs at present experiments, it would be a clear sign of new physics. In this regards, we calculate this process in an extended Higgs sector framework where the decay is mediated by the exchange of spin zero particle with flavour changing neutral current couplings. If we identify the scalar with the newly discovered state at LHC with a mass ~ 125 GeV then we find that, after imposing all experimental constraints, the $BR(B_s \rightarrow \tau\bar{\mu} + \mu\bar{\tau})$ can be as high $\sim 10^{-6}$ and $BR(B_d \rightarrow \tau\bar{\mu} + \mu\bar{\tau})$ can be as high as $\sim 10^{-7}$. We also calculate this process in the minimal supersymmetric standard model and find the $BR(B_s \rightarrow \tau\bar{\mu} + \mu\bar{\tau})$ is typically of the order $\sim 10^{-8}$.

Also, we study the processes $\bar{B} \rightarrow D\tau\bar{\nu}_\tau$ and $\bar{B} \rightarrow D^*\tau\bar{\nu}_\tau$ on the light of a deviation from the standard model expectations which has been recently reported by BaBar, Belle and LHCb. Since these processes are all tree-level decays in the SM, it is difficult to explain this deviation with models of new physics. Thus, we show that supersymmetric effects driven by penguin contributions to the $b \rightarrow c\tau\nu_\tau$ transition are able to account simultaneously for a significant increase of both branching ratios of $\bar{B} \rightarrow D\tau\bar{\nu}_\tau$ and $\bar{B} \rightarrow D^*\tau\bar{\nu}_\tau$ with respect to the Standard Model predictions, thereby approaching their experimentally measured values. We emphasis that a degeneracy between the lightest chargino and neutralino, with masses less than 300 GeV, in addition to a large stau/sneutrino mass and a large $\tan\beta$ are essential for enhancing the effect of the lepton penguin $\tau\nu_\tau W^\pm$, which is responsible for the improved theoretical predictions with respect to current data.

Introduction

One of the most important tasks for new physics (NP) searches is to identify experimental observables where one can find large deviations from the standard model (SM) expectation and are also experimentally accessible in the future. After discovering the only scalar boson of the SM at Large Hadron Collider (LHC) by CMS [1] and ATLAS [2], the SM has been confirmed as being extremely successful in describing most aspects of Nature with remarkable precision at the 100 GeV scale. However, even after this success, the hierarchy of masses in the SM still very poorly understood. In fact, the second and third generation of the quarks and leptons are quite special because they are comparatively heavier and are expected to be relatively more sensitive to NP.

Supersymmetry (SUSY) is one of the most attractive candidates for physics beyond the Standard model (BSM), such as the Minimal Supersymmetric Standard Model (MSSM). LHCb collaboration has been reported data of the branching ratios of rare B decays and the CP violation of B mesons [3–5], since the B meson decays is a good place for indirect search for NP. For a long years the CP violation in the K and B^0 mesons has been successfully understood within the framework of the SM, where the source of the CP violation is the Kobayashi-Maskawa (KM) phase in the quark sector with three families [6]. In supersymmetric extensions of the SM there are possible sources for CP violation arising from complexity of the soft SUSY

breaking terms. The soft squark and slepton mass matrices contain the CP-violating phases, which contribute to the flavour changing neutral current (FCNC) with the CP violation. Therefore, new phases have significant implications and can modify the SM predictions in CP violating phenomena. However, there is no deviation from the prediction of the SM has been observed yet in the LHCb experiment.

FCNC events are rare in the SM both in the quark and the lepton sectors. These processes can be affected by NP. FCNC involving the third family quark and leptons are interesting as their larger masses make them more susceptible to NP effects in various extensions of the SM. As an example, in certain versions of the two Higgs doublet models (2HDM) the couplings of the new Higgs bosons are proportional to the masses and so new physics effects are more pronounced for the heavier generations. Moreover, the constraints on NP involving, specially the third generation leptons and quarks, are somewhat weaker allowing for larger NP effects. Hence, in the down quark sector there is intense interest in the search for NP in FCNC $b \rightarrow s$ and $b \rightarrow d$ transitions in $B_{d,s}$ mixing, $b \rightarrow s(d)q\bar{q}$ and $b \rightarrow s(d)\ell^+\ell^-$ decays. Many measurements in the B sector have constrained NP effects though slight discrepancies from SM predictions still remain in several decays [7–13]. In the up quark sector one can look for NP is the FCNC $t \rightarrow c(u)$ transitions [14, 15].

In the lepton sector FCNC decays are severely suppressed and the lepton number for each family is conserved in the SM. Therefore, FCNC effects and lepton flavor violation in this sector are sensitive probes of NP. The presence of neutrino masses and mixing suggest NP in the lepton sector which in turn could enhance FCNC effects and flavor violation. It is interesting that measurements involving the tau lepton show some discrepancies from the SM expectations. The branching ratio of $B \rightarrow \tau\nu_\tau$ shows some tension with the SM predictions [16] though more recent measurement are consistent with the SM [17]. This could indicate NP [18], possibly coming from an extended scalar or gauge sector. There is also a seeming violation of universality in

the tau lepton coupling to the W suggested by the LEP II data which could indicate new physics associated with the third generation leptons [19].

If there is NP involving the third generation leptons one can search for it in Bdecays such as $B \rightarrow D\tau\nu$ and $B \rightarrow D^*\tau\nu$. The semileptonic of the B meson to the τ lepton is mediated by W boson in the SM and it is very well understood theoretically. In many models this decay gets contributions from additional states like new vectors, new scalar or new fermions.

Recently, the BaBar, Belle and LHCb collaborations have reported for the following observables

$$\mathcal{R}(D) = \frac{BR(\bar{B} \rightarrow D\tau^-\bar{\nu}_\tau)}{BR(\bar{B} \rightarrow D\ell^-\bar{\nu}_\ell)}, \quad \mathcal{R}(D^*) = \frac{BR(\bar{B} \rightarrow D^*\tau^-\bar{\nu}_\tau)}{BR(\bar{B} \rightarrow D^*\ell^-\bar{\nu}_\ell)}, \quad (0.0.1)$$

where, $l = e$ or μ , the following measurements [20–27]:

$$\mathcal{R}(D) = 0.375 \pm 0.064, \quad \mathcal{R}(D^*)_{ave} = 0.288 \pm 0.019, \quad \text{Belle} \quad (0.0.2)$$

$$\mathcal{R}(D) = 0.440 \pm 0.0724, \quad \mathcal{R}(D^*) = 0.332 \pm 0.030, \quad \text{BaBar} \quad (0.0.3)$$

$$\mathcal{R}(D^*) = 0.332 \pm 0.030. \quad \text{LHCb} \quad (0.0.4)$$

The SM predictions for $\mathcal{R}(D)$ and $\mathcal{R}(D^*)$ are [21, 28, 29]:

$$\mathcal{R}(D) = 0.305 \pm 0.012, \quad \mathcal{R}(D^*) = 0.252 \pm 0.004, \quad (0.0.5)$$

which deviate from the experimental measurements considerably. This deviation could be a sign of NP. For this reason we will perform a complete analysis in the MSSM in order to find an explanation for these deviations, in both cases of CP-violating and CP-conserving interactions.

The thesis is organized as follows:

In Chapter 1 We give a brief introduction to the SM of particle physics displaying the problems that facing particle physics. We also give various SUSY Lagrangians construction. In addition we give review to the MSSM with electroweak symmetry

breaking where we present the mass spectra. Finally, we wrap up the chapter by giving the SUSY searches limits at LHC.

In chapter 2 we start with some definitions which are important for the study of CP violation. We give in discussion the CP violation source in the SM through the K^0 and $\bar{K}^0$ system. We also discuss the new sources of the CP violation in SUSY. In the rest of the chapter, we give the SM and MSSM contributions to electric dipole moments.

In chapter 3 we begin with promising vision for looking for NP in B decays. We review the basic approach for studying B decays namely the effective Hamiltonian. We also provide the calculation of the process $B_s \rightarrow \tau \bar{\mu} + \mu \bar{\tau}$ in an extended Higgs sector framework where the decay is mediated by the exchange of spin zero particle with flavour changing neutral current couplings. Also, we calculate this process in the MSSM through the chargino and neutralino contributions in the box graph. Finally, we present the results with discussion.

In chapter 4 we present the recent experimental results for the processes $B \rightarrow D\tau\nu$ and $B \rightarrow D^*\tau\nu$ and related experimental limits. We calculate the helicity amplitudes for all possible interactions, scalar, vector and tensor. We also calculate the Wilson coefficients in the MSSM through penguin topologies since they give the dominant contribution for vector interactions. In the other hand, we give the explicit formulae for the observables $\mathcal{R}(D)$ and $\mathcal{R}(D^*)$ as functions of the new physics Wilson coefficients which are the natural place for CP violation. Finally, we provide the results in both cases of CP-conserving and CP-violating.

1

Supersymmetry

Although the SM [30, 31] Higgs boson discovered in 2012 by the ATLAS [2] and CMS [1, 32] experiments at the CERN Large Hadron Collider (LHC) [1, 2, 32, 33], which becomes extremely successful phenomenologically to describe the most aspects of the Nature, it remains unable to explain some outstanding issues in the particle physics world at 100 GeV scale. Therefore, the SM must be extended to be valid at higher energies. Various candidate theories have been proposed to go beyond the SM (BSM). Among these theories, SUSY is the most renowned one. In this chapter we provide the main aspects of the SUSY theory. We start with a brief review on the SM. Also, we proceed through the rest of the chapter the construction of SUSY Lagrangian, the supersymmetric spectrum structure and SUSY signature at LHC.

1.1 Standard Model

To go beyond the SM, we first present something about the SM itself. The SM is based on the gauge symmetry group

$$G_{SM} = SU(3)_C \otimes SU(2)_L \otimes U(1)_Y, \quad (1.1.1)$$

where C represents the color, L denotes left-handed chirality and Y stands for hypercharge. This group contains three symmetries as known Color, Weak Isospin and Hypercharge respectively. It provides a unified framework to describe these three forces of Nature (the SM). The specific gauge bosons associated with the generators of the algebra of the group are:

$$SU(3)_C \rightarrow 8G_\mu^\alpha (\alpha = 1, \dots, 8), \quad (1.1.2)$$

$$SU(2)_L \rightarrow 3W_\mu^a (a = 1, 2, 3), \quad (1.1.3)$$

$$U(1)_Y \rightarrow B_\mu. \quad (1.1.4)$$

The eight spin-one particles, G_μ^α , are called gluons which are massless, because this theory describes only massless particles. Since gluons themselves carry color charge, they participate in strong interactions between quarks. The four spin-one bosons associated with $SU(2)_L \otimes U(1)_Y$ are related to the physical gauge bosons that mediate the weak interactions, $W^\pm$ and Z^0 , and the familiar photon from QED. The gauge boson mass terms are not invariant under their respective gauge transformations, they violate local $SU(2) \otimes U(1)$ gauge invariance. In the other hand, the strength of interactions are determined by three coupling constants g_s , g and g' associated with the three groups of Eq. (1.1.1), respectively.

The matter fields are of two types, quarks and leptons which are ordered in left-handed doublets and right-handed singlets. Each doublet defines a *flavour* family:

$$L_L \in \left\{ \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L, \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L \right\}, \quad Q_L \in \left\{ \begin{pmatrix} u' \\ d' \end{pmatrix}_L, \begin{pmatrix} c' \\ s' \end{pmatrix}_L, \begin{pmatrix} t' \\ b' \end{pmatrix}_L \right\}, \quad (1.1.5)$$

1.1 Standard Model

$$e_R \in \{e_R, \mu_R, \tau_R\}, \quad u'_R \in \{u'_R, c'_R, t'_R\}, \quad d'_R \in \{d'_R, s'_R, b'_R\}. \quad (1.1.6)$$

Here, we indicate the quarks with a primed symbol since the electroweak eigenstates do not necessarily coincide with the mass eigenstates.

The scalar field (called Higgs field) corresponds to the complex scalar $SU(2)_L$ doublet, Φ ,

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \quad (1.1.7)$$

which breaks the symmetry

$$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \rightarrow SU(3)_C \otimes U(1)_{EM}, \quad (1.1.8)$$

so that the eight gluons of Quantum Chromodynamics (QCD) and the photon of Quantum Electrodynamics (QED) remain massless. Furthermore, the Higgs doublet is also able to generate masses for the fermions, which acquire a non-zero vacuum expectation value (VEV),

$$\phi = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}. \quad (1.1.9)$$

The Higgs potential, which describes the scalar self-interactions, is given by

$$V(\Phi) = \mu^2 \Phi^\dagger \Phi - \lambda (\Phi^\dagger \Phi)^2. \quad (1.1.10)$$

The parameter λ must be non-negative, $\lambda > 0$, in order for the potential to be bounded from below. On the other hand, the pattern of spontaneous symmetry breaking require $\mu^2 < 0$. Let us now write the SM Lagrangian that describes all fields which is given by

$$\begin{aligned} \mathcal{L}_{SM} = & -\frac{1}{4} \sum_{a=1}^8 G_{\mu\nu}^a G^{a\mu\nu} - \frac{1}{4} \sum_{b=1}^3 W_{\mu\nu}^b W^{b\mu\nu} - \frac{1}{4} \sum B_{\mu\nu} B^{\mu\nu} \\ & + (D^\mu \Phi)^\dagger D_\mu \Phi - V(\Phi^\dagger \Phi) \end{aligned}$$

$$\begin{aligned}
 & + \sum_{i=1}^3 i \bar{Q}_L^i \gamma^\mu D_\mu Q_L^i + \sum_{i=1}^3 i \bar{u}_R^i \gamma^\mu D_\mu u_R^i + \sum_{i=1}^3 i \bar{d}_R^i \gamma^\mu D_\mu d_R^i \\
 & + \sum_{i=1}^3 i \bar{L}_L^i \gamma^\mu D_\mu L_L^i + \sum_{i=1}^3 i \bar{e}_R^i \gamma^\mu D_\mu e_R^i \\
 & - \sum_{i,j=1}^3 Y_{ij}^u \bar{Q}_{Li} \Phi u_{Rj} - \sum_{i,j=1}^3 Y_{ij}^d \bar{Q}_{Li} \tilde{\Phi} d_{Rj} - \sum_{i,j=1}^3 Y_{ij}^e \bar{L}_{Li} \Phi e_{Rj} + h.c., \quad (1.1.11)
 \end{aligned}$$

where $\tilde{\Phi} = i\tau_2 \Phi^\dagger$. $G_{\mu\nu}^a$, $W_{\mu\nu}^b$ and $B_{\mu\nu}$ are the strength tensors for the gauge fields of $SU(3)$, $SU(2)$ and $U(1)$ respectively, which are given by

$$G_{\mu\nu}^a = \partial_\mu G_\nu^a - \partial_\nu G_\mu^a - g_s f^{abc} G_\mu^b G_\nu^c, \quad (1.1.12)$$

$$W_{\mu\nu}^a = \partial_\mu W_\nu^a - \partial_\nu W_\mu^a - g \epsilon^{abc} W_\mu^b W_\nu^c, \quad (1.1.13)$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu, \quad (1.1.14)$$

and the covariant derivative or the weak doublet is

$$D_\mu = \partial_\mu - i \frac{g}{2} \tau^a W_\mu^a - i \frac{g'}{2} B_\mu Y. \quad (1.1.15)$$

We shall now use the Higgs mechanism to generate mass terms for the gauge bosons of the electroweak theory. We need Φ to be a complex doublet of scalar fields and we develop the theory around the minimum;

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \eta_1 + i\eta_2 \\ v + h + i\eta_3 \end{pmatrix}. \quad (1.1.16)$$

Φ has the hypercharge $Y_\Phi = 1$, and zero electromagnetic charge, which means that the electromagnetic $U(1)$ symmetry is preserved. The electric charge of the particles is given by the Gell-Mann-Nishijima formula,

$$Q = I^3 + \frac{Y}{2}. \quad (1.1.17)$$

The three fields η_i are three Goldstone bosons which give mass to the three weak gauge fields. The spontaneous symmetry breaking also leads to a massive boson from

the field h , the Higgs boson. After the Higgs acquires a VEV, the scalar kinetic term, of Eq. (1.1.10), leads to gauge bosons masses,

$$M_{gauge\ bosons}^2 \sim \frac{1}{2} \begin{pmatrix} 0 & v \end{pmatrix} \left(\frac{1}{2} g \tau^a W_\mu^a + \frac{1}{2} g' B_\mu \right)^2 \begin{pmatrix} 0 \\ v \end{pmatrix}. \quad (1.1.18)$$

Three massive vector bosons appear,

$$W_\mu^\pm = \frac{1}{\sqrt{2}} (W_\mu^1 \mp i W_\mu^2), \quad Z^\mu = \frac{-g' B_\mu + g W_\mu^3}{\sqrt{g'^2 + g^2}}, \quad (1.1.19)$$

and the fourth vector field remains massless,

$$B^\mu = \frac{g B_\mu + g' W_\mu^3}{\sqrt{g'^2 + g^2}}, \quad (1.1.20)$$

with the weak mixing angle θ_W ,

$$\sin \theta_W = \frac{g}{\sqrt{g'^2 + g^2}}, \quad \cos \theta_W = \frac{g'}{\sqrt{g'^2 + g^2}}. \quad (1.1.21)$$

Then the gauge bosons, $W^\pm$, Z^0 and B^0 masses appear to be:

$$M_W^2 = \frac{1}{4} g^2 v^2, \quad M_Z^2 = \frac{1}{4} (g^2 + g'^2) v^2, \quad M_\gamma = 0. \quad (1.1.22)$$

The fermion-fermion-boson interactions can be written as:

$$\begin{aligned} \mathcal{L}_{int} = & \frac{g}{\sqrt{2}} \left[\bar{u}'_i \gamma^\mu \frac{1 - \gamma^5}{2} d'_i + \bar{\nu}_{f_i} \gamma^\mu \frac{1 - \gamma^5}{2} f_i \right] W_\mu^+ + h.c \\ & - e J_\mu^{em} B^\mu - \frac{g}{2 \cos \theta_W} J_\mu^Z Z^\mu, \end{aligned} \quad (1.1.23)$$

where the neutral currents are defined as

$$J_\mu^{em} = \sum_f q_f \bar{f} \gamma_\mu f, \quad (1.1.24)$$

$$J_\mu^Z = \sum_f q_f \bar{f} \gamma_\mu (g_V^f - g_A^f) f, \quad (1.1.25)$$

with

$$g_V^f = I_3^f - 2q_f \sin^2 \theta_W, \quad (1.1.26)$$

$$g_A^f = I_3^f. \quad (1.1.27)$$

In the above equations, $e_i \in \{e, \mu, \tau\}$, $\nu_i \in \{\nu_e, \nu_\mu, \nu_\tau\}$, $u'_i \in \{u', c', t'\}$ (up sector), $d'_i \in \{d', s', b'\}$ (down sector), f is a generic fermion with electric charge q_f and I_f^3 is the third component of the weak isospin ($\frac{1}{2}$ for u_i and ν_i , $-\frac{1}{2}$ for d_i and e_i), θ_W is the Weinberg angle defining the mixing that produces the Z and the photon.

From Eq.(1.1.11) we split the Yukawa part into two, the leptonic and quark parts. At the renormalizable level the Yukawa interactions are given by

$$-\mathcal{L}_{Yukawa}^{quarks} = Y_{ij}^u \bar{Q}_{Li} \Phi u'_{Rj} + Y_{ij}^d \bar{Q}_{Li} \tilde{\Phi} d'_{Rj} + h.c. \quad (1.1.28)$$

$$-\mathcal{L}_{Yukawa}^{leptons} = Y_{ij}^e \bar{L}_{Li} \Phi e'_{Rj} + h.c. \quad (1.1.29)$$

After symmetry breaking the leptons and quarks become massive and flavor arises in . Note that the SM predicts massless neutrinos. Therefore, These coupling can mix the flavour families, so that, the flavour and mass eigenstates can be different. The Yukawa couplings, for quarks, turns into

$$\mathcal{L}_{mass} = \bar{u}_L^i M_u^{ij} u_R'^j + \bar{d}_L^i M_d^{ij} d_R'^j + h.c., \quad (1.1.30)$$

where the mass matrices $M_u^{ij} = \frac{1}{\sqrt{2}} Y_u^{ij} v$ and $M_d^{ij} = \frac{1}{\sqrt{2}} Y_d^{ij} v$ are, in general, non-diagonal 3×3 matrices. In fact that the eigenstates in the original Lagrangian may not coincide with the physically useful ones. To obtain the particle masses, we need to diagonalize the mass matrices via unitary fields transformations that can be written as:

$$u'_{Li} \rightarrow V_{ij}^u u_{Lj}, \quad u'_{Ri} \rightarrow U_{ij}^u u_{Rj}, \quad (1.1.31)$$

$$d'_{Li} \rightarrow V_{ij}^d d_{Lj}, \quad d'_{Ri} \rightarrow U_{ij}^d d_{Rj}, \quad (1.1.32)$$

and likewise for the leptons case. In term of the matrix notations, the transformations (1.1.31) and (1.1.32) turn the quark mass matrices M^u and M^d into

$$M_u \rightarrow V_u^* M_u U_u^\dagger, \quad M_d \rightarrow V_d^* M_d U_d^\dagger. \quad (1.1.33)$$

With $m_{(u,d)}^i = v Y_{(u,d)}^{ii} / \sqrt{2}$ are the masses of the quarks.

When introducing the above transformations in the interaction between fermions and charged gauge boson in Eq. (1.1.23), the interaction terms can be written, in the mass basis, as

$$\mathcal{L}_W^{\text{int}} = \frac{g}{2\sqrt{2}} \bar{u}^i \gamma^\mu (1 - \gamma^5) V_{CKM}^{ij} d^j W_\mu^+ + h.c., \quad (1.1.34)$$

where the unitary 3×3 matrix $V_{CKM} = V_u V_d^\dagger$ ($V_{CKM} V_{CKM}^\dagger = 1$) in general is not diagonal and is known as the Cabibbo-Kobayashi-Maskawa (CKM) matrix (see chapter 2).

Although the SM provides a correct description for particle physics, there are a number of theoretical and phenomenological issues that the SM fails to address adequately, therefore, the SM is not the complete fundamental theory:

- *Hierarchy problem:* It is one of the naturalness problems of the Higgs sector in the SM. This model predicted that the Higgs mass must be in the range of $\mathcal{O}(100 \text{ GeV})$ which is so much lighter than the Planck mass ($\mathcal{O}(10^{19} \text{ GeV})$). Also, this model is associated with the absence of a symmetry protecting the Higgs mass at electroweak scale. It would be expected that the large quantum contributions to the square of the Higgs boson mass, through the one loop radiative corrections to the Higgs mass, would inevitably make the mass huge when the natural cutoff scale is at or above the grand unification theory (GUT) scale $M_G \approx 10^{16} \text{ GeV}$.
- *Massive neutrinos:* The SM predicts that the neutrinos are massless, but the discovery of the neutrino oscillations in the nineties of the last century indicated that this is not true and neutrinos have tiny masses.

- *Gauge coupling unification:* The values of running gauge couplings do not unify if we evolve these to high energies using the renormalization group equations of the Standard Model. If instead, we extrapolate the gauge couplings from $Q = M_Z$ to $Q = M_{GUT}$ using supersymmetric evolution equations, they unify remarkably well provided superpartner masses are in the range 100 GeV – 10 TeV.
- *Family structure and fermion masses:* The SM has three families of particles although that the universe is made up entirely of first generation. The SM describes all three generations, but it cannot explain why more than one exists and hierarchical values of the fermion masses, it also does not account for the neutrino masses (neutrinos are strictly left-handed and massless).
- *Cosmological challenges:* In fact the early universe began as a huge burst of energy, it should have evolved into equal parts matter and anti-matter (CP symmetry). But the visible world is made of protons, neutrons and electrons and not their antiparticles (their antimatter equivalents). This matter asymmetry of the universe cannot be explained by the Standard Model. The SM also does not have a viable candidate for the cold dark matter of the universe, nor a viable inflation.

Therefore, the SM must be extended to be valid at higher energies.

1.2 What is Supersymmetry?

As we seen above that the SM is widely considered to be an incomplete theory, even at energies as low as the weak scale (about 100 GeV). It becomes clear that requiring certain specific properties and symmetries helps to build new physical theories. One such requirement is that the theory must be Lorentz invariant. Another symmetry

that has proved unexpectedly powerful in the construction of the SM of particle physics is gauge invariance. Therefore, it is useful to have some symmetry principle to guide us in designing those theories. SUSY is such a principle.

SUSY is a spacetime symmetry that relates two types of elementary particles: bosons have an integer-valued spin, and fermions have a half-integer spin. Each particle belongs to the same superfield. In quantum theory, a symmetry that links a boson with a fermion is generated by the operator Q that carries spin-1/2 (implies that SUSY must be a spacetime symmetry), with

$$Q |\text{Boson}\rangle = |\text{Fermion}\rangle, \quad Q |\text{Fermion}\rangle = |\text{Boson}\rangle. \quad (1.2.1)$$

Since Q is a complex spinor, so the hermitian conjugate of Q ($Q^\dagger$) is also a symmetry generator. In this respect, SUSY is an extension of the spacetime symmetry reflected in the Poincaré group.

On the other hand, symmetries are an important aspect in quantum field theory (QFT) which describe features of elementary particles. The known symmetries can be classified into two categories, spacetime symmetries and internal symmetries. Since the particle states should belong to representations of the Poincaré group, it is natural to also group particles into multiplets of some additional internal symmetry group, and this viewpoint has been very successful in the construction of gauge theories. An obvious question to ask, is there a bigger group could combine internal symmetry group with the Poincaré group, and this implies is whether the generators of the internal symmetry group could have nonzero commutators with the generators of the Poincaré group (the operator Q does not commute with the generators of the Poincaré group $M_{\mu\nu}$ and P_μ , this means that Q should carry some Lorentz index). This is an interesting issue because such a theory then would combine spacetime transformations with internal transformations and would lead to a highly nontrivial extension of gauge theories. In 1967 Sidney Coleman and Jeffrey Mandula [34] have

shown that it is impossible to achieve a non-trivial combination of internal and external symmetries.

Let us illustrate the theorem in the simple case of a symmetric tensor charge, $\Sigma_{\mu\nu} = \Sigma_{\nu\mu}$. So we can show that for a scattering process $a+b \rightarrow c+d$, the conservation of the eigenvalue of $\Sigma_{\mu\nu}$ together with the conservation of total momentum implies that either there is no scattering at all. The most general symmetric two-index tensor is given by

$$\Sigma_{\mu\nu} = ag_{\mu\nu} + bP_\mu P_\nu, \quad (1.2.2)$$

where a and b are constants. Then, if we apply this to a two body scattering state, we simply get

$$\langle pk | \Sigma_{\mu\nu} | pk \rangle = 2ag_{\mu\nu} + b(p_\mu p_\nu + k_\mu k_\nu). \quad (1.2.3)$$

Therefore, conservation of $\Sigma_{\mu\nu}$ leads to

$$2ag_{\mu\nu} + b(p_\mu^a p_\nu^a + p_\mu^b p_\nu^b) = 2ag_{\mu\nu} + b(p_\mu^c p_\nu^c + p_\mu^d p_\nu^d), \quad (1.2.4)$$

equivalently,

$$p_\mu^a p_\nu^a + p_\mu^b p_\nu^b = p_\mu^c p_\nu^c + p_\mu^d p_\nu^d. \quad (1.2.5)$$

In other hand, conservation of four-momentum implies

$$p_\mu^a + p_\mu^b = p_\mu^c + p_\mu^d. \quad (1.2.6)$$

Therefore, there are only two possible solutions are: either $p_\mu^a = p_\mu^c$ and $p_\mu^b = p_\mu^d$ (there is no scattering at all) or $p_\mu^a = p_\mu^d$ and $p_\mu^b = p_\mu^c$ (four-momenta exchange).

So, the Coleman-Mandula theorem can not be applied. Latter, Haag, Lopuszanski and Sohnius [35] generalized the Coleman-Mandula theorem, and showed that the possible symmetries of a consistent 4-dimensional quantum field theory do not only consist of internal symmetries and Poincaré symmetry (direct product of the internal symmetry group and the Poincaré group), but can also include supersymmetry

algebra as a nontrivial extension of the Poincaré algebra. This theorem implies that the supersymmetry algebra must be satisfied by anticommutation and commutation relations of the generators Q and $Q^\dagger$.

1.3 Supersymmetry Algebra

The simplest and most useful supersymmetry algebra in four dimension Minkowski space is a particularly simple of a graded Lie algebra. It is obtained by adding to Poincaré algebra a Majorana spinor charge, with the components $Q_{\dot{\alpha}}, \dot{\alpha} = 1, 2$, satisfying the properties ¹

$$\{Q_\alpha, Q_\beta\} = \{\bar{Q}_{\dot{\alpha}}, \bar{Q}_{\dot{\beta}}\} = 0, \quad (1.3.1)$$

$$\{Q_\alpha, \bar{Q}_{\dot{\alpha}}\} = 2\sigma_{\alpha\dot{\alpha}}^\mu P_\mu, \quad (1.3.2)$$

$$[P^\mu, Q_\alpha] = [P^\mu, \bar{Q}_{\dot{\alpha}}] = 0, \quad (1.3.3)$$

$$[M^{\mu\nu}, Q_\alpha] = -i(\sigma^{\mu\nu})_\alpha^\beta Q_\beta, \quad (1.3.4)$$

$$[M^{\mu\nu}, \bar{Q}_{\dot{\alpha}}] = -i(\sigma^{\mu\nu})_{\dot{\alpha}}^{\dot{\beta}} \bar{Q}_{\dot{\beta}}, \quad (1.3.5)$$

where $\sigma^{\mu\nu}$ is defined as

$$(\sigma^{\mu\nu})_\alpha^\beta = \frac{1}{4}(\sigma_{\alpha\rho}^\mu \bar{\sigma}^{\nu\rho\beta} - \sigma_{\alpha\rho}^\nu \bar{\sigma}^{\mu\rho\beta}), \quad (1.3.6)$$

$$(\bar{\sigma}^{\mu\nu})_{\dot{\alpha}}^{\dot{\beta}} = \frac{1}{4}(\bar{\sigma}^{\mu\dot{\alpha}\rho} \sigma_{\rho\dot{\beta}}^\nu - \bar{\sigma}^{\nu\dot{\alpha}\rho} \sigma_{\rho\dot{\beta}}^\mu), \quad (1.3.7)$$

and $P^\mu, M^{\mu\nu}$ are the generators of displacement and homogeneous Lorentz transformations of space time. Since P^μ has the dimension L^{-1} , the SUSY generators Q_α must have dimension $L^{-\frac{1}{2}}$ while $M^{\mu\nu}$ are dimensionless. Furthermore the square mass operator $P^\mu P_\mu$ is a Casimir operator while the spin operator $W^\mu W_\mu$ (W_μ being the Pauli-Lubansky-Bargman vector) is not. This implies

$$[Q_\alpha, P^2] = 0, \quad [Q_\alpha, W^2] \neq 0, \quad (1.3.8)$$

¹For more details about the SUSY, the references [36–44] can be consulted.

where $W^\mu = -\frac{i}{2}\varepsilon^{\mu\nu\rho\sigma}M^{\nu\rho}P^\sigma$. Therefore, all particles belonging to irreducible representation of SUSY, *i.e.* within one supermultiplet, have the same mass. This is obvious since P^2 commutes with all generators of the SUSY, *i.e.* it is still a Casimir operator. Thus, if $|F\rangle$ is a fermion state, with m_F , obtained from bosonic state $|B\rangle$, with mass m_B , through SUSY generator Q_α : $|F\rangle = Q_\alpha|B\rangle$. One can have particles of different spin within one supermultiplet.

To get a new Casimir C^2 (corresponding to superspin), we define

$$B_\mu = W_\mu + \frac{1}{8}\bar{Q}_M\gamma_\mu\gamma_5Q_M, \quad (1.3.9)$$

$$C_{\mu\nu} = B_\mu P_\nu - B_\nu P_\mu, \quad (1.3.10)$$

$$C^2 = C_{\mu\nu}C^{\mu\nu}, \quad (1.3.11)$$

where

$$[C^2, P_\mu] = [C^2, M_{\mu\nu}] = [C^2, Q_\alpha] = [C^2, \bar{Q}^{\dot{\alpha}}] = 0. \quad (1.3.12)$$

Note that in the definition of B_μ the 4-component Majorana spinor charge Q_M is used, where

$$Q_M = \begin{pmatrix} Q_\alpha \\ \bar{Q}^{\dot{\alpha}} \end{pmatrix}. \quad (1.3.13)$$

Therefore, the eigenvalues of Casimir operator C^2 label the irreducible representation. It is interesting to note that C^2 can be written as

$$C^2 = 2m^4 J_\mu J^\mu, \quad (1.3.14)$$

where

$$J_\mu = S_\mu - \frac{1}{4m}\bar{Q}_{\dot{\alpha}}(\bar{\sigma}_\mu)^{\dot{\alpha}\beta}Q_\beta, \quad (1.3.15)$$

and

$$[J_\mu, J_\nu] = i\varepsilon_{\mu\nu\rho}J_\rho. \quad (1.3.16)$$

Recall that the rotation group $\{J_i, i = 1, 2, 3\}$ satisfies

$$[J_i, J_j] = i\varepsilon_{ijk}J_k. \quad (1.3.17)$$

The Casimir operator J^2 commutes with all the J_i and labels irreducible representations by eigenvalues $j(j+1)$ of J^2 as in case of ordinary angular momentum. Accordingly, the irreducible representations of SUSY algebra are specified by the values of m^2 and $j(j+1)$, the eigenvalues of the Casimir operators. These states are not in general eigenstates of spin-squared operator, S^2 .

Since the generators Q_α and $\bar{Q}^{\dot{\alpha}}$ commute with the all internal symmetry generators, e, g , the gauge transformation, the particle contents of the supermultiplets must have the same quantum numbers like electric charge, weak isospin, etc. Whereas in any supersymmetric multiplet, the number n_B of bosons equals to the number n_F of fermions, $n_B = n_F$. To prove this, consider the fermion number operator $(-1)^F = (-)^F$, defined as

$$(-)^F |B\rangle = |B\rangle, \quad (-)^F |F\rangle = -|F\rangle. \quad (1.3.18)$$

This new operator $(-)^F$ anticommutes with Q_α since

$$(-)^F Q_\alpha |F\rangle = (-)^F |B\rangle = |B\rangle = Q_\alpha |F\rangle = -Q_\alpha (-)^F |F\rangle. \quad (1.3.19)$$

Therefore

$$\{(-)^F, Q_\alpha\} = 0. \quad (1.3.20)$$

On the other hand

$$\begin{aligned} Tr\{(-)^F \{Q_\alpha, \bar{Q}_{\dot{\beta}}\}\} &= Tr\{(-)^F Q_\alpha \bar{Q}_{\dot{\beta}} + (-)^F \bar{Q}_{\dot{\beta}} Q_\alpha\} \\ &= Tr\{-Q_\alpha (-)^F \bar{Q}_{\dot{\beta}} + Q_\alpha (-)^F \bar{Q}_{\dot{\beta}}\} \\ &= 0. \end{aligned} \quad (1.3.21)$$

Using $\{Q_\alpha, \bar{Q}_{\dot{\beta}}\} = 2(\sigma^\mu)_{\alpha\dot{\beta}}P_\mu$, one finds

$$Tr\{(-)^F 2(\sigma^\mu)_{\alpha\dot{\beta}}P_\mu\} = 0, \quad (1.3.22)$$

and thus

$$Tr(-)^F = 0. \quad (1.3.23)$$

Then

$$\begin{aligned} Tr(-)^F &= \sum_{bosons} \langle B|(-)^F|B\rangle + \sum_{fermions} \langle F|(-)^F|F\rangle \\ &= \sum_{bosons} \langle B|B\rangle - \sum_{fermions} \langle F|F\rangle = 0. \end{aligned} \quad (1.3.24)$$

Thus, one concludes that the number of bosonic and fermionic states are equal in the supermultiplet.

Let us now provide the representation of SUSY algebra. We present separately the massless and massive cases.

Massless Supermultiplet:

For states of massless particles we choose a reference frame where $P_\mu = (E, 0, 0, E)$. The Casimirs $P^2 = P_\mu P^\mu$ and $C^2 = C_{\mu\nu} C^{\mu\nu}$ are zero. In this respect one can write the equation of eigenstates as

$$P^0 |p, \lambda\rangle = P^2 |p, \lambda\rangle = 0, \quad (1.3.25)$$

$$P^3 |p, \lambda\rangle = P^0 |p, \lambda\rangle = E |p, \lambda\rangle. \quad (1.3.26)$$

Therefore, the SUSY algebra is

$$\begin{aligned} \{Q_\alpha, \bar{Q}_{\dot{\beta}}\} &= 2(\sigma^\mu)_{\alpha\dot{\beta}} P_\mu = 2E(\sigma^0 + \sigma^3)_{\alpha\dot{\beta}} \\ &= 4E \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}_{\alpha\dot{\beta}}. \end{aligned} \quad (1.3.27)$$

This implies that

$$\{Q_1, \bar{Q}_1\} = 4E, \quad (1.3.28)$$

$$\{Q_2, \bar{Q}_2\} = 0. \quad (1.3.29)$$

Using the last above relation, we deduce that

$$\{Q_2, \bar{Q}_2\} |p, \lambda\rangle = 0. \quad (1.3.30)$$

This equation is particularly useful, its expectation value in a state $|p, \lambda\rangle$ is

$$\begin{aligned} \langle p, \lambda | \{Q_2, \bar{Q}_2\} |p, \lambda\rangle &= \langle p, \lambda | Q_2 \bar{Q}_2 |p, \lambda\rangle + \langle p, \lambda | \bar{Q}_2 Q_2 |p, \lambda\rangle \\ &= \|Q_2 |p, \lambda\rangle\|^2 + \|\bar{Q}_2 |p, \lambda\rangle\|^2 \\ &= 0. \end{aligned} \quad (1.3.31)$$

Thus $Q_2 = \bar{Q}_2 = 0$, we therefore only need Q_1 and $\bar{Q}_1$. It is clear that each supermultiplet has a state of minimum helicity because Q_1 is the only operator at our disposal that lowers the helicity, and $Q_1^2 = 0$. Let us then start with the state in a supermultiplet with the lowest helicity, which we will denote by $|p, \lambda_{\min}\rangle$.

By definition, acting with Q_1 on this state gives zero, $Q_1 |p, \lambda_{\min}\rangle = 0$. On the other hand, acting with $\bar{Q}_1$ gives

$$\bar{Q}_1 |p, \lambda_{\min}\rangle = \left| p, \lambda_{\min} + \frac{1}{2} \right\rangle. \quad (1.3.32)$$

We cannot generate states of higher helicity because $(Q_1^\dagger)^2 = 0$. So a massless supermultiplet contains only two states of the same momentum but with helicities differing by half a unit can be denoted by $(\lambda, \lambda + \frac{1}{2})$.

To build $N = 1$ SUSY theory we need only two states. But it would not be physical because it would not be CPT invariant. Therefore, to satisfy CPT invariant, a physical supersymmetric theory must contain a supermultiplet plus its CPT conjugate with opposite quantum numbers, for a total of four states. Finally, one finds the following massless $N = 1$ supermultiplets:

- **Chiral multiplet:** Consists of $(0, \frac{1}{2})$ and its CPT conjugate $(-\frac{1}{2}, 0)$, corresponding to Weyl fermion and a complex scalar.

- **Vector multiplet:** Consists of $(\frac{1}{2}, 1)$ and $(-1, -\frac{1}{2})$, corresponding to gauge boson (massless vector) and Weyl fermion.

• **Gravitino multiplet:** Consists of $(1, \frac{3}{2})$ and $(-\frac{3}{2}, -1)$, corresponding to gravitino and gauge boson.

• **Graviton multiplet:** Consists of $(\frac{3}{2}, 2)$ and $(-2, -\frac{3}{2})$, corresponding to graviton and gravitino.

Massive Supermultiplet:

In case of $m \neq 0$, the rest frame can be chosen as $P^\mu = (m, 0, 0, 0)$ and Casimirs

$$P^2 = P_\mu P^\mu = m^2, \quad C^2 = C_{\mu\nu} C^{\mu\nu} = 2mJ^2. \quad (1.3.33)$$

Therefore, the eigenvalues of J^2 are $j(j+1)$, so the irreducible representations are labelled by $|m, j\rangle$. Again, from the anticommutation relation of Q and $\bar{Q}$, one gets

$$\begin{aligned} \{Q_\alpha, \bar{Q}_{\dot{\beta}}\} &= 2(\sigma^\mu)_{\alpha\dot{\beta}} P_\mu = 2m(\sigma^0)_{\alpha\dot{\beta}} \\ &= 2m \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}_{\alpha\dot{\beta}} \\ &= 2m\delta_{\alpha\dot{\beta}}. \end{aligned} \quad (1.3.34)$$

Now we define two sets of operators

$$a_{1,2} = \frac{Q_{1,2}}{\sqrt{2m}}, \quad a_{1,2}^\dagger = \frac{\bar{Q}_{\dot{1},\dot{2}}}{\sqrt{2m}}, \quad (1.3.35)$$

with anticommutation relations

$$\{a_i, a_j^\dagger\} = \delta_{ij}, \quad \{a_i, a_j\} = \{a_i^\dagger, a_j^\dagger\} = 0. \quad (1.3.36)$$

Let $|\Omega\rangle = |m, j, j_3\rangle$ be the vacuum state, where j_3 is the third spin component, such that

$$Q |\Omega\rangle = 0 \Rightarrow a_1 |\Omega\rangle = a_2 |\Omega\rangle = 0. \quad (1.3.37)$$

In this case, the remaining states are

$$|\Omega\rangle = |m, 0, 0\rangle, \quad (1.3.38)$$

$$a_{1,2}^\dagger |\Omega\rangle = \left| m, \frac{1}{2}, \pm \frac{1}{2} \right\rangle, \quad (1.3.39)$$

$$a_2^\dagger a_1^\dagger |\Omega\rangle = |m, 0, 0\rangle. \quad (1.3.40)$$

if $J_3 |\Omega\rangle = j_3 |\Omega\rangle$, then one can find that the eigenvalues of J_3 for the above four states are given by: $0, \frac{1}{2}, -\frac{1}{2}, 0$ respectively. It implies that there are two bosons and two fermions. Therefore, a massive chiral supermultiplet ($j = 0$) consists of a Weyl spinor and complex scalar field, while a massive vector supermultiplet ($j = \frac{1}{2}$) consists of a scalar field, two Weyl spinors and a gauge boson.

1.4 Supersymmetric Lagrangians

In this section we aim to present the construction of supersymmetric Lagrangians. In this context, we will provide a description of theories with and without gauge, called Wess Zumino and Yang Mills models respectively. Thus we have the recipe that allow us to write down the most terms of general supersymmetric field theory, interaction and mass terms.

1.4.1 Wess-Zumino Lagrangian

The simplest non-interacting Lagrangian of on-shell (two degree of freedom) Weyl fermion has 2 degrees of freedom and a complex scalar (two degrees of freedom) is given by

$$\mathcal{L} = \partial_\mu \phi^\dagger \partial^\mu \phi + \psi^\dagger i \bar{\sigma}^\mu \partial_\mu \psi. \quad (1.4.1)$$

However, under supersymmetric transformation given in Eq. (A.3), the scalar boson field ϕ turns into some thing the fermion field ψ as shown in Eqs. (A.4.13), (A.4.14) and (A.4.15). This implies that the corresponding action is not SUSY invariant, $\delta \mathcal{S} \neq 0$. In case of an off-shell Weyl-fermion has four degrees of freedom while a complex scalar still has two degrees of freedom, we need to add an additional auxiliary

complex bosonic field, F , which has two degrees of freedom off-shell and no on-shell degrees of freedom to make the numbers of bosonic and fermionic degrees of freedom match off-shell as well as on-shell [43]. Thus the free Lagrangian with all required degrees of freedom is given by

$$\mathcal{L}_{\text{WZ}}^{\text{free}} = \partial_\mu \phi^\dagger \partial^\mu \phi + \psi^\dagger i \bar{\sigma}^\mu \partial_\mu \psi + F^\dagger F, \quad (1.4.2)$$

where the F equation of motion is $F = F^\dagger = 0$. This theory is known as free Wess Zumino model [45]. Now it is clear that the above Lagrangian is invariant under SUSY transformation up to total derivative, hence the corresponding action is SUSY invariant, $\delta \mathcal{S} = 0$.

Since the free Lagrangian given in Eq. (1.4.2) contains only massless and non-interacting fields, we will now find the most general set of renormalizable interactions for these fields that is consistent with supersymmetry. Then we now generalize the free Wess-Zumino Lagrangian to a set of n copies of the fields. Such generalization will be useful when we consider the MSSM. The most generalization of the Lagrangian is given by

$$\begin{aligned} \mathcal{L}_{\text{WZ}} = \mathcal{L}_{\text{WZ}}^{\text{free}} + \mathcal{L}_{\text{int}} = & \partial_\mu \phi^\dagger \partial^\mu \phi + \psi^\dagger i \bar{\sigma}^\mu \partial_\mu \psi + F^\dagger F + W_i(\phi, \phi^\dagger) F_i \\ & - \frac{1}{2} W_{ij}(\phi, \phi^\dagger) \psi_i \psi_j + h.c., \end{aligned} \quad (1.4.3)$$

where W_i and W_{ij} are arbitrary functions of the scalar ϕ_i and $\phi_i^\dagger$ with dimensions 1 and 2 respectively. Thus W_i and W_{ij} cannot depend on ψ_i nor on F_i . The indices i and j are summed from 1 to n .

Since $\mathcal{L}_{\text{WZ}}^{\text{free}}$ was already SUSY invariant, the interaction Lagrangian, $\mathcal{L}_{\text{int}}$, must be also invariant. From $\delta \mathcal{L}_{\text{int}}$, which can be divided into several parts, one gets

$$\begin{aligned} \delta W_i(\phi, \phi^\dagger) &= \frac{\partial W_i}{\partial \phi_k} \delta \phi_k + \frac{\partial W_i}{\partial \phi_k^\dagger} \delta \phi_k^\dagger \\ &= \frac{\partial W_i}{\partial \phi_k} \xi \psi_k + \frac{\partial W_i}{\partial \phi_k^\dagger} \bar{\xi} \bar{\psi}_k, \end{aligned} \quad (1.4.4)$$

$$\begin{aligned}
 \delta(W_i(\phi, \phi^\dagger)F_i + h.c) &= \frac{\partial W_i}{\partial \phi_k} \xi \psi_k F_i + \frac{\partial W_i}{\partial \phi_k^\dagger} \bar{\xi} \bar{\psi}_k F_i + W_i \delta F_i + h.c \\
 &= \frac{\partial W_i}{\partial \phi_k} \xi \psi_k F_i + \frac{\partial W_i}{\partial \phi_k^\dagger} \bar{\xi} \bar{\psi}_k F_i + i\sqrt{2}W_i \partial_\mu \psi \sigma^\mu \bar{\xi} + h.c, \quad (1.4.5)
 \end{aligned}$$

and similarly

$$\delta\left(-\frac{1}{2}W_{ij}(\phi, \phi^\dagger)\psi_i\psi_j\right) = -\frac{1}{2}\frac{\partial W_{ij}}{\partial \phi_k} \xi \psi_k \psi_i \psi_j - \frac{\partial W_{ij}}{\partial \phi_k^\dagger} \bar{\xi} \bar{\psi}_k \psi_i \psi_j - \frac{1}{2}W_{ij}\delta(\psi_i\psi_j) + h.c. \quad (1.4.6)$$

We are looking for conditions on W_i and W_{ij} such that the sum of Eqs. (1.4.5) and (1.4.6) vanishes, modulo total derivatives. Let us start with the first term of Eq. (1.4.6):

$$-\frac{1}{2}\frac{\partial W_{ij}}{\partial \phi_k} \xi \psi_k \psi_i \psi_j, \quad (1.4.7)$$

in case of $i = j = k$, it is identically zero because it contains the square of spinor components. While in case for $i \neq j \neq k$, the Eq. (1.4.7) can be written as

$$-\frac{1}{2}\left(\frac{\partial W_{12}}{\partial \phi_3} \xi \psi_3 \psi_1 \psi_3 + \frac{\partial W_{31}}{\partial \phi_2} \xi \psi_2 \psi_3 \psi_1 + \frac{\partial W_{23}}{\partial \phi_1} \xi \psi_1 \psi_2 \psi_3\right). \quad (1.4.8)$$

Using the Fierz identity

$$\xi \psi_3 \psi_1 \psi_3 + \xi \psi_2 \psi_3 \psi_1 + \xi \psi_1 \psi_2 \psi_3 = 0. \quad (1.4.9)$$

If $\frac{\partial W_{ij}}{\partial \phi_k}$ is symmetric under interchange of i, j and k , we will have cancelation of Eq. (1.4.7). The second term of Eq. (1.4.6) vanishes only if $\frac{\partial W_{ij}}{\partial \phi_k^\dagger}$ cannot depend on $\phi_k^\dagger$, $\frac{\partial W_{ij}}{\partial \phi_k^\dagger} = 0$. This implies that W_{ij} must be a holomorphic function of the complex field ϕ (analytic function of the complex field ϕ). The most general holomorphic W_{ij} is

$$W_{ij} = M_{ij} + y_{ijk}\phi_k, \quad (1.4.10)$$

where M_{ij} is a symmetric mass matrix, and y_{ijk} is Yukawa coupling that must be symmetric under interchange of i, j, k . Also we can write W_{ij} as the second derivative of another function $\mathcal{W}$, of the scalar fields, where $\mathcal{W}$ is given by

$$\mathcal{W} = \frac{1}{2}M_{ij}\phi_i\phi_j + \frac{1}{6}y_{ijk}\phi_i\phi_j\phi_k, \quad (1.4.11)$$

$$W_{ij} = \frac{\partial^2 \mathcal{W}}{\partial_i \phi \partial \phi_j}, \quad W_i = \frac{\partial \mathcal{W}}{\partial_i \phi}, \quad (1.4.12)$$

where $\mathcal{W}$ is the superpotential. The remaining terms are

$$\begin{aligned} & i\sqrt{2}W_{ij}\psi_i(\sigma^\mu \bar{\xi} \partial_\mu \phi_j) - \sqrt{2}W_{ij}F_i(\xi\psi) + \frac{\partial W_i}{\partial \phi_k} \xi \psi_k F_i + \frac{\partial W_i}{\partial \phi_k^\dagger} \bar{\xi} \bar{\psi}_k F_i \\ & + i\sqrt{2}W_i \partial_\mu \psi_i \sigma^\mu \bar{\xi}, \end{aligned} \quad (1.4.13)$$

where

$$-\frac{1}{2}W_{ij}\delta(\psi_i\psi_j) = -i\sqrt{2}W_{ij}\psi_i(\sigma^\mu \bar{\xi} \partial_\mu \phi_j) + W_{ij}F_i(\xi\psi_j). \quad (1.4.14)$$

Using the fact that $W_{ij} = \frac{\partial^2 \mathcal{W}}{\partial \phi_i \partial \phi_j}$ to write

$$W_{ij}\partial_\mu \phi_j = \frac{\partial^2 \mathcal{W}}{\partial \phi_i \partial \phi_j} \partial_\mu \phi_j = \partial_\mu \left(\frac{\partial \mathcal{W}}{\partial \phi_i} \right), \quad (1.4.15)$$

so the sum of the first and last terms of the Eq. (1.4.13) is

$$i\sqrt{2}\partial_\mu \left(\frac{\partial \mathcal{W}}{\partial \phi_i} \right) \psi_i \sigma^\mu \bar{\xi} + i\sqrt{2}W_i \partial_\mu \psi_i \sigma^\mu \bar{\xi}. \quad (1.4.16)$$

This equation is a total derivative at the condition $W_i = \frac{\partial \mathcal{W}}{\partial \phi_i}$. Now we have only terms in Eq. (1.4.13):

$$-\sqrt{2}W_{ij}F_i(\xi\psi_j) + \frac{\partial W_i}{\partial \phi_k} \xi \psi_k F_i + \frac{\partial W_i}{\partial \phi_k^\dagger} \bar{\xi} \bar{\psi}_k F_i, \quad (1.4.17)$$

we have $W_{ij} = \frac{\partial W_i}{\partial \phi_j}$, then this equation becomes

$$-\sqrt{2}\frac{\partial W_i}{\partial \phi_j} F_i(\xi\psi_j) + \frac{\partial W_i}{\partial \phi_k} \xi \psi_k F_i + \frac{\partial W_i}{\partial \phi_k^\dagger} \bar{\xi} \bar{\psi}_k F_i. \quad (1.4.18)$$

Therefore, the first two terms cancel each other. Since we can not cancel the last term with anything, W_i must be independent on $\phi^\dagger$, *i.e.*, $\frac{\partial W_i}{\partial \phi_j^\dagger} = 0$. Finally, to eliminate the auxiliary fields we use the equation of motion of F_i , we get $F_i = -W_i$. And thus the full Wess-Zumino Lagrangian can be written as

$$\mathcal{L}_{WZ} = \partial_\mu \phi^\dagger \partial^\mu \phi + \psi^\dagger i \bar{\sigma}^\mu \partial_\mu \psi - \left| \frac{\partial \mathcal{W}}{\partial \phi_i} \right|^2 - \frac{1}{2} \left(\frac{\partial \mathcal{W}}{\partial \phi_i \partial \phi_j} \psi_i \psi_j + h.c. \right) \quad (1.4.19)$$

Using the general superpotential $\mathcal{W}$, one finds

$$\begin{aligned}\mathcal{L}_{WZ} = & \partial_\mu \phi^\dagger \partial^\mu \psi + \psi^\dagger i \bar{\sigma}^\mu \partial_\mu \psi - \left| M_{ij} \phi_j + \frac{1}{2} Y_{ijk} \phi_j \phi_k \right|^2 \\ & - \frac{1}{2} (M_{ij} \psi_i \psi_j + Y_{ijk} \phi_i \psi_j \psi_k + h.c.).\end{aligned}\quad (1.4.20)$$

It is clear now that the potential of this model is given by

$$\begin{aligned}V(\phi, \phi^\dagger) = & |F_i|^2 = \left| \frac{\partial \mathcal{W}}{\partial \phi_i} \right|^2 = M_{jk}^* M^{kl} \phi_j \phi_l + \left(\frac{1}{2} M^{jl} Y_{klm}^* \phi_j^* \phi^k \phi^m + h.c. \right) \\ & + \frac{1}{4} Y^{jkl} Y_{mnl}^* \phi_j \phi_k \phi^{*m} \phi^{*n}.\end{aligned}\quad (1.4.21)$$

1.4.2 Chiral Superfield Lagrangian

To construct a supersymmetric Lagrangian in superspace from chiral superfields which include spin-zero and spin-1/2 particles, one should select the highest order components of the composite superfield $\Phi_i^\dagger \Phi^i$, where all fields are evaluated as finite of x^μ , which can be obtained by the integration over the Grassmann parameters θ and $\bar{\theta}$. The most general SUSY and renormalizable Lagrangian for chiral superfield has the form

$$\mathcal{L} = \Phi_i^\dagger \Phi^i|_D + \mathcal{W}(\Phi_i)|_F + \bar{\mathcal{W}}(\Phi_i^\dagger)|_F. \quad (1.4.22)$$

Furthermore, this Lagrangian can be written as

$$\mathcal{L} = K(\Phi, \Phi^\dagger)|_D + (\mathcal{W}(\Phi)|_F + h.c.), \quad (1.4.23)$$

where $K(\Phi, \Phi^\dagger)$ is the Kähler potential, it is a real function of Φ and $\Phi^\dagger$, $|_D$ refers to D term of the corresponding superfield and similar for F-terms. A possible term for K is

$$K(\Phi, \Phi^\dagger) = \Phi^\dagger \Phi, \quad (1.4.24)$$

and a general expression for $\mathcal{W}$ is given by

$$\mathcal{W}(\Phi) = \frac{1}{2} M_{ij} \Phi^i \Phi^j + \frac{1}{6} Y_{ijk} \Phi^i \Phi^j \Phi^k, \quad (1.4.25)$$

where

$$\Phi(x, \theta, \bar{\theta}) = \phi(x) + \sqrt{2}\theta\psi(x) + \theta\theta F(x) + i\theta\sigma^\mu\bar{\theta}\partial_\mu\phi(x) - \frac{i}{\sqrt{2}}\theta\theta\sigma^\mu\bar{\theta}\partial_\mu\psi(x) + \frac{1}{4}\theta\theta\bar{\theta}\bar{\theta}\partial_\mu\partial^\mu\phi(x). \quad (1.4.26)$$

To derive the $\theta\theta$ terms for the general superpotential in terms of the components of a chiral superfield, we just perform a Taylor - expansion around $\Phi = \phi$:

$$\mathcal{W}(\Phi) = \mathcal{W}(\phi) + \frac{\partial\mathcal{W}}{\partial\phi_i}(\Phi - \phi) + \frac{1}{2}\frac{\partial^2\mathcal{W}}{\partial\phi^2}(\Phi - \phi)^2 + \dots \quad (1.4.27)$$

where

$$\frac{\partial\mathcal{W}}{\partial\phi}(\Phi - \phi) = \dots + \theta\theta F(x) + \dots, \quad \frac{1}{2}\frac{\partial^2\mathcal{W}}{\partial\phi^2}(\Phi_i - \phi_i)^2 = \dots + (\theta\psi(x))(\theta\psi(x)) + \dots \quad (1.4.28)$$

Therefore, the Lagrangian of chiral superfield is obtained by taking the D and F terms components:

$$K|_D = \Phi^\dagger\Phi|_D = \int d^4\theta\Phi^\dagger\Phi = \partial_\mu\phi_i^\dagger\partial^\mu\phi^i + i\bar{\psi}\bar{\sigma}^\mu\partial_\mu\psi_i + F_i^*F^i + \dots \quad (1.4.29)$$

$$\mathcal{W}(\Phi)|_F = \int d^2\theta\mathcal{W}(\Phi) = \frac{\partial\mathcal{W}}{\partial\phi_i}F^i - \frac{1}{2}\frac{\partial^2\mathcal{W}}{\partial\phi_i\partial\phi_j}\psi_i\psi_j + h.c. \quad (1.4.30)$$

After eliminating the auxiliary fields F_i and F^{*i} by their equations of motion as

$$F_i = -M_{ij}^*\phi_j^* - Y_{ijk}^*\phi_j^*\phi_k^* = -\frac{\partial\mathcal{W}^*}{\partial\phi^{*i}}, \quad (1.4.31)$$

$$F_i^* = -M_{ij}\phi_j - Y_{ijk}\phi_j\phi_k = -\frac{\partial\mathcal{W}}{\partial\phi^i}, \quad (1.4.32)$$

one finds

$$\begin{aligned} \mathcal{L}_{\text{chiral}} &= \partial_\mu\phi_i^\dagger\partial^\mu\phi^i + i\bar{\psi}\bar{\sigma}^\mu\partial_\mu\psi_i + F_i^*F^i + \left(\frac{\partial\mathcal{W}}{\partial\phi_i}F^i + h.c.\right) \\ &\quad - \frac{1}{2}\frac{\partial^2\mathcal{W}}{\partial\phi_i\partial\phi_j}\psi_i\psi_j + h.c. \\ &= \partial_\mu\phi_i^\dagger\partial^\mu\phi^i + i\bar{\psi}\bar{\sigma}^\mu\partial_\mu\psi_i - |M_{ij}\phi_j + Y_{ijk}\phi_j\phi_k|^2 \\ &\quad - \left(\frac{1}{2}M_{ij}\psi_i\psi_j + Y_{ijk}\psi_i\psi_j\phi_k + h.c.\right). \end{aligned} \quad (1.4.33)$$

From this Lagrangian we can conclude that the bosonic and fermionic masses are equal as expected, derived from the tree level effective potential, V , such that

$$V = \sum_i |F_i|^2 = \sum_i \left| \frac{\partial \mathcal{W}}{\partial \phi_i} \right|^2. \quad (1.4.34)$$

Similar for the interaction terms. While the Kinematic terms are derived from the Kähler potential.

1.4.3 Lagrangians for Gauge Supermultiplets

Besides the chiral supermultiplets there also exist gauge supermultiplets that contain gauge bosons and their fermionic superpartners. Before spontaneous symmetry breaking, the gauge bosons are described by a massless gauge boson field v_μ^a and a two component Weyl fermion gaugino λ^a . The index $a = 1, \dots, 8$ for $SU(3)_C$ color gluons and gluinos; $a = 1, 2, 3$ for $SU(2)_L$ weak isospin; $a = 1$ for $U(1)_Y$ weak hypercharge. The gauge transformations of the vector supermultiplet fields are

$$v_\mu^a \rightarrow v_\mu^a + \partial_\mu \Lambda^a + g f^{abc} v_\mu^b \Lambda^c, \quad (1.4.35)$$

$$\lambda^a \rightarrow \lambda^a + g f^{abc} \lambda^b \Lambda^c, \quad (1.4.36)$$

where f^{abc} are the totally antisymmetric structure constants of the gauge group where for an Abelian group $f^{abc} = 0$, g is the gauge coupling, Λ^a is an infinitesimal gauge transformation parameter.

If the fermionic and bosonic degrees of freedom in our theory are On-shell, v_μ^a has two degrees of freedom (the two transverse polarization states), and so does a left-chiral Weyl fermion λ^a (for each a), as required by supersymmetry. However, off-shell, a vector field v_μ has three degrees of freedom; one degree of freedom is removed by the inhomogeneous gauge transformation in Eq. (1.4.35), whereas λ^a has four (consists of two complex, or four real, fermionic degrees of freedom). For this reason we will need one extra real bosonic auxiliary field that is conventionally denoted D which does not have a kinetic term, in order for supersymmetry to be consistent off-shell.

In this section we would like to see how to construct the Lagrangian of free supersymmetric abelian gauge theory and gauge invariant interactions of chiral and vector superfield, the supersymmetric non-abelian gauge theory.

1.4.3.1 Free Supersymmetric Abelian Gauge theory

To introduce a field strength $\mathcal{F}$ that would contain the gauge invariant field strength $F_{\mu\nu}$, we should start from the superfield $V(x, \theta, \bar{\theta})$ and applying the SUSY derivative D_α and $\bar{D}^{\dot{\alpha}}$. We need to apply at least three SUSY derivatives to $V(x, \theta, \bar{\theta})$ because if we apply fewer, the photon field A_μ will remain present, and our result therefore will not be gauge invariant. In this respect, the field strength superfield $\mathcal{F}_a$ can be defined as

$$\mathcal{F}_\alpha = -\frac{1}{4}\bar{D}\bar{D}D_\alpha V(x, \theta, \bar{\theta}), \quad (1.4.37)$$

$$\bar{\mathcal{F}}_{\dot{\alpha}} = -\frac{1}{4}DD\bar{D}_{\dot{\alpha}}V(x, \theta, \bar{\theta}). \quad (1.4.38)$$

Note that $\mathcal{F}_a$ left chiral superfield and $\bar{\mathcal{F}}_{\dot{a}}$ is antichiral superfield because of the fact that $\bar{D}_{\dot{\alpha}}\bar{D}\bar{D}\mathcal{F}_a = 0$. Thus $\mathcal{F}_a$ satisfies $\bar{D}_{\dot{\alpha}}\mathcal{F}_a = 0$, similar $D_\alpha\mathcal{F}_a = 0$. Also they are invariant under super gauge transformation given by

$$V \rightarrow V - i(\Lambda - \Lambda^\dagger), \quad (1.4.39)$$

where Λ and $\Lambda^\dagger$ are chiral antichiral superfield. Under the above transformation, we have

$$\begin{aligned} \mathcal{F}_\alpha &\rightarrow \mathcal{F}_\alpha - \frac{i}{4}\bar{D}\bar{D}D_\alpha\Lambda + \frac{i}{4}\bar{D}\bar{D}D_\alpha\Lambda^\dagger = \mathcal{F}_\alpha - \frac{i}{4}\bar{D}\bar{D}D_\alpha\Lambda \\ &= \mathcal{F}_\alpha - \frac{i}{4}\bar{D}_{\dot{\alpha}}\{\bar{D}^{\dot{\alpha}}, D_\alpha\}\Lambda + \frac{i}{4}\bar{D}_{\dot{\alpha}}D_\alpha\bar{D}^{\dot{\alpha}}\Lambda \\ &= \mathcal{F}_\alpha - \frac{i}{4}\bar{D}_{\dot{\alpha}}\{\bar{D}^{\dot{\alpha}}, D_\alpha\}\Lambda \\ &= \mathcal{F}_\alpha - \frac{i}{2}(\sigma^\mu)_{\alpha}^{\dot{\alpha}}\partial_\mu\bar{D}_{\dot{\alpha}}\Lambda \\ &= \mathcal{F}_\alpha, \end{aligned} \quad (1.4.40)$$

where $\bar{D}_{\dot{\alpha}}\Lambda = 0$ because Λ is left-chiral.

We will now calculate $\mathcal{F}_\alpha$ in terms of component fields, where we use the vector superfield V in the Wess-Zumino gauge. To simplify the calculation we trade the coordinates $(x, \theta, \bar{\theta})$ for the coordinates (y, θ) , where $y^\mu = x^\mu + i\theta\sigma^\mu\bar{\theta}$. In this case V is given by

$$V_{WZ}(y) = \theta\sigma^\mu\bar{\theta}v_\mu(y) + i\theta\theta\bar{\theta}\bar{\lambda}(y) - \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}D(y) - i\bar{\theta}\bar{\theta}\theta\lambda(y). \quad (1.4.41)$$

By applying to $V_{WZ}(y)$ the SUSY derivatives

$$D_\alpha = \partial_\alpha + 2i\sigma^\mu_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_{\bar{\theta}}, \quad \bar{D}_{\dot{\alpha}} = \partial_{\dot{\alpha}}, \quad (1.4.42)$$

we find that

$$\begin{aligned} \mathcal{F}_\alpha(y, \theta) &= -\frac{1}{4}\bar{D}\bar{D}D_\alpha V_{WZ}(y) = -i\lambda_\alpha(y) + \theta_\alpha D(y) \\ &\quad + \frac{i}{2}(\theta\sigma^\mu\bar{\sigma}^\nu)_\alpha F_{\mu\nu}(y) - \theta\theta\sigma^\mu_{\alpha\dot{\alpha}}\partial_\mu\bar{\lambda}^{\dot{\alpha}}(y), \end{aligned} \quad (1.4.43)$$

where $F_{\mu\nu}(y) = \partial_\mu v_\nu - \partial_\nu v_\mu$. The SUSY transformations can be obtained from $(-i\xi^\alpha Q_\alpha - \bar{\xi}_{\dot{\alpha}}\bar{Q}^{\dot{\alpha}})\mathcal{F}$ as follows

$$\delta_\xi F_{\mu\nu} = (\xi\sigma_\nu\partial_\mu\bar{\lambda} - \bar{\xi}\bar{\sigma}_\nu\partial_\mu\lambda) - (\mu \leftrightarrow \nu), \quad (1.4.44)$$

$$\delta_\xi \lambda_\alpha = -\xi_\alpha D - \frac{i}{2}(\sigma^\mu\bar{\sigma}^\nu)_\alpha^\beta \xi_\beta F_{\mu\nu}, \quad (1.4.45)$$

$$\delta_\xi D = i\xi\sigma^\mu\partial_\mu\bar{\lambda} + i\bar{\xi}\bar{\sigma}^\mu\partial_\mu\lambda. \quad (1.4.46)$$

Since the Lagrangian must be Lorentz scalar, so we should consider $\mathcal{F}^\alpha\mathcal{F}_\alpha$. Thus to obtain a supersymmetric Lagrangian, we will need to extract its F term of $\mathcal{F}^\alpha\mathcal{F}_\alpha$ which contains the abelian field strength associated with v_μ . These are

$$\begin{aligned} \mathcal{F}^\alpha\mathcal{F}_\alpha &= i2\bar{\lambda}\sigma^\mu\partial_\mu\lambda\theta\theta + D^2\theta\theta - \frac{1}{2}(\sigma^{\mu\nu})^{\alpha\beta}(\sigma^{\rho\sigma})_{\alpha\beta}F_{\mu\nu}F_{\rho\sigma}\theta\theta \\ &= i2\bar{\lambda}\sigma^\mu\partial_\mu\lambda\theta\theta + D^2\theta\theta - \frac{1}{2}\text{Tr}(\sigma^{\mu\nu}\sigma^{\rho\sigma})F_{\mu\nu}F_{\rho\sigma}\theta\theta. \end{aligned} \quad (1.4.47)$$

Using the definition

$$\sigma^{\mu\nu} = \frac{i}{4}(\sigma^\mu \bar{\sigma}^\nu - \sigma^\nu \bar{\sigma}^\mu), \quad (1.4.48)$$

one can find

$$\begin{aligned} Tr(\sigma^{\mu\nu} \sigma^{\rho\sigma}) &= -\frac{1}{16}(\sigma^\mu \bar{\sigma}^\nu \sigma^\rho \bar{\sigma}^\sigma - \sigma^\mu \bar{\sigma}^\nu \sigma^\sigma \bar{\sigma}^\rho - \sigma^\nu \bar{\sigma}^\mu \sigma^\rho \bar{\sigma}^\sigma \\ &\quad + \sigma^\nu \bar{\sigma}^\mu \sigma^\sigma \bar{\sigma}^\rho) \\ &= -\frac{1}{2}(g^{\nu\rho} g^{\mu\sigma} - g^{\mu\rho} g^{\nu\sigma} - i\varepsilon^{\mu\nu\rho\sigma}). \end{aligned} \quad (1.4.49)$$

Substituting this into Eq. (1.4.47), we get

$$\begin{aligned} -\frac{1}{2}Tr(\sigma^{\mu\nu} \sigma^{\rho\sigma}) F_{\mu\nu} F_{\rho\sigma} &= \frac{1}{4}(F_{\mu\nu} F^{\nu\mu} - F_{\mu\nu} F^{\mu\nu} - i\varepsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}) \\ &= -\frac{1}{2}(F_{\mu\nu} F^{\nu\mu} + \frac{i}{2}\varepsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}), \end{aligned} \quad (1.4.50)$$

and thus

$$\mathcal{F}^\alpha \mathcal{F}_\alpha = i2\bar{\lambda}\sigma^\mu \partial_\mu \lambda \theta\theta + D^2 \theta\theta - \frac{1}{2}(F_{\mu\nu} F^{\nu\mu} \theta\theta + \frac{i}{2}\varepsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} \theta\theta). \quad (1.4.51)$$

The last term is a total derivative, so it will not be considered. Therefore, the expression of supersymmetric kinematic term of a $U(1)$ gauge boson v_μ and its fermionic partner λ_α can be written as follows

$$\frac{1}{4}\mathcal{F}^\alpha \mathcal{F}_\alpha|_F = -\frac{1}{4}F_{\mu\nu} F^{\nu\mu} + i\bar{\lambda}\bar{\sigma}^\mu \partial_\mu \lambda + \frac{1}{2}D^2. \quad (1.4.52)$$

In this case we have four bosonic degrees of freedom off-shell (3 from v_μ and one real D) and four fermionic λ_α , which reduce to two degrees of freedom each when equations of motion are applied. Thus, one finds that the tree level effective potential V is given by

$$V = \frac{1}{2}D^2. \quad (1.4.53)$$

This potential is also a positive definite with absolute minima at $D = 0$.

Finally, the supersymmetric Lagrangian density of a free abelian gauge supermultiplet is therefore

1.4.3.2 Supersymmetric Gauge Interactions

Here, we will generalize the results gotten in the abelian case to non-abelian gauge theory where the gauge invariant interactions of chiral and vector superfield will take place. We seen how superfield may be used to obtain supersymmetric Lagrangian for free abelian gauge field theory.

The gauge superfield V is now given by

$$V \equiv V_a T^a, \quad (1.4.54)$$

where the sum is over the number of generators of the gauge group, $n^2 - 1$, of the $SU(n)$. The generators are normalized as

$$Tr(T^a T^b) = \frac{1}{2} \delta^{ab}, \quad (1.4.55)$$

they satisfy

$$[T^a, T^b] = i f^{abc} T^c, \quad (1.4.56)$$

where f^{abc} are the antisymmetric constants of the gauge group.

Consider a chiral superfield transformation as

$$\Phi \rightarrow \Phi' = e^{i2g\Lambda} \Phi, \quad (1.4.57)$$

where $\Lambda = \Lambda_a T^a$ are constant chiral superfields. We take the transformation in Eq. (A.4.17) to generalize the gauge transformation of vector superfield as

$$e^{2gV} \rightarrow e^{2gV'} = e^{i2g\Lambda^\dagger} e^{2gV} e^{-i2g\Lambda}. \quad (1.4.58)$$

The new quantity $\Phi^\dagger e^{2gV} \Phi$ is invariant under local gauge transformation. Whereas, the field strength superfield $\mathcal{F}_\alpha$ given in Eq. (1.4.37) is now generalized as

$$\mathcal{F}_\alpha = -\frac{1}{4} \bar{D} \bar{D} (e^{-2gV} D_\alpha e^{2gV}), \quad (1.4.59)$$

where its transformation is

$$\mathcal{F}_\alpha \rightarrow \mathcal{F}'_\alpha = e^{2ig\Lambda} \mathcal{F}_\alpha e^{-2ig\Lambda}. \quad (1.4.60)$$

Therefore, $Tr(\mathcal{F}^\alpha \mathcal{F}_\alpha)$ is gauge invariant.

The Lagrangian for a supersymmetric non-abelian gauge theory containing a gauge superfield coupled to chiral superfield can be first written as

$$\mathcal{L} = \Phi_i (e^{2gV})_{ij} \Phi_j |_D + (\mathcal{W}(\Phi_i) + \frac{1}{16g^2} \mathcal{W}^{a\alpha} \mathcal{W}_\alpha^a + h.c.) |_F, \quad (1.4.61)$$

the kinetic term like the form

$$\begin{aligned} \Phi e^{2gV} \Phi |_D &= \partial_\mu \phi_i^\dagger \partial^\mu \phi_i + i \bar{\psi}_i \bar{\sigma}^\mu \partial_\mu \psi_i + F^\dagger F + g^2 \phi^\dagger \phi v^\mu v_\mu \\ &\quad - q \phi_i^\dagger \phi_i D - (\sqrt{2} g \psi_i \lambda \phi_i^\dagger + i g v^\mu \phi_i^\dagger \partial_\mu \phi_i + h.c) \\ &\quad + g \psi_i \bar{\sigma}^\mu \bar{\psi}_i v_\mu. \end{aligned} \quad (1.4.62)$$

To simplify this equation, consider the following terms

$$\begin{aligned} &\partial_\mu \phi_i^\dagger \partial^\mu \phi_i + g^2 \phi^\dagger \phi v^\mu v_\mu \\ &\quad - (\sqrt{2} g \psi_i \lambda \phi_i^\dagger + i g v^\mu \phi_i^\dagger \partial_\mu \phi_i + h.c) \\ &= \partial_\mu \phi_i^\dagger \partial^\mu \phi_i + g^2 \phi^\dagger \phi v^\mu v_\mu - i g v^\mu \phi_i^\dagger \partial_\mu \phi_i + i g v^\mu \phi_i \partial_\mu \phi_i^\dagger. \end{aligned} \quad (1.4.63)$$

These terms can be combined into $(D_\mu \phi_i)^\dagger (D^\mu \phi_i)$, where D_μ is the covariant derivative

$$D_\mu = \partial_\mu + i g v_\mu. \quad (1.4.64)$$

Also

$$\begin{aligned} i \bar{\psi}_i \bar{\sigma}^\mu \partial_\mu \psi_i + g \psi_i \bar{\sigma}^\mu \bar{\psi}_i v_\mu &= i \bar{\psi}_i \bar{\sigma}^\mu \partial_\mu \psi_i - g \bar{\psi}_i \bar{\sigma}^\mu \psi_i v_\mu \\ &= i \bar{\psi}_i \bar{\sigma}^\mu D_\mu \psi_i. \end{aligned} \quad (1.4.65)$$

Therefore, the kinematic term can be simplified to

$$\Phi e^{2gV} \Phi |_D = (D_\mu \phi_i)^\dagger (D^\mu \phi_i) + i \bar{\psi}_i \bar{\sigma}^\mu D_\mu \psi_i + F^\dagger F$$

$$-g\phi_i^\dagger\phi_i D - (\sqrt{2}g\psi_i\lambda\phi_i^\dagger + h.c). \quad (1.4.66)$$

Similarly, in terms of the component fields, we can write the most general renormalizable non-abelian gauge invariant Lagrangian as follows:

$$\begin{aligned} \mathcal{L} = & (D_\mu\phi_i)^\dagger(D^\mu\phi_i) + i\bar{\psi}_i\bar{\sigma}^\mu D_\mu\psi_i + |F|^2 - \frac{1}{4}F_{\mu\nu}^a F^{a\nu\mu} \\ & + i\bar{\lambda}^a\bar{\sigma}^\mu D_\mu\lambda^a + \frac{1}{2}D^a D^a - \left(\frac{\partial\mathcal{W}}{\partial\phi_i}F_i + \frac{1}{2}\frac{\partial^2\mathcal{W}}{\partial\phi_i\partial\phi_j}\psi_i\psi_j + h.c\right) \\ & + g_a D^a\phi_i^\dagger(T^a)_{ij}\phi_j + i\sqrt{2}\phi_i^\dagger\lambda^a(T^a)_{ij}\psi_j - ig_a\sqrt{2}\bar{\psi}_i\bar{\lambda}^a(T^a)_{ij}\phi_j, \end{aligned} \quad (1.4.67)$$

with

$$D_\mu\phi_i = \partial_\mu\phi_i + igv_\mu^a(T^a)_{ij}\phi_j, \quad (1.4.68)$$

$$D_\mu\psi_i = \partial_\mu\psi_i + igv_\mu^a(T^a)_{ij}\psi_j, \quad (1.4.69)$$

$$D_\mu\lambda^a = \partial_\mu\lambda^a - gf_{abc}v_\mu^b\lambda^c, \quad (1.4.70)$$

$$F_{\mu\nu}^a = \partial_\mu v_\nu^a - \partial_\nu v_\mu^a - gf_{abc}v_\mu^b v_\nu^c. \quad (1.4.71)$$

Eliminating the auxiliary fields by its equations of motion,

$$F_i^* = \frac{\partial\mathcal{W}}{\partial\phi_i}, \quad D^a = -g\phi_i^*(T^a)_{ij}\phi_j, \quad (1.4.72)$$

leads to the SUSY scalar potential of the form

$$V_{SUSY} = V_F + V_D, \quad (1.4.73)$$

where

$$V_D = \frac{1}{2}\sum_i |D^a|^2 = \frac{g}{2}(\phi_i^*(T^a)_{ij}\phi_j)^2, \quad (1.4.74)$$

$$V_F = \sum_i |F_i|^2 = \sum_i \left| \frac{\partial\mathcal{W}}{\partial\phi_i} \right|^2. \quad (1.4.75)$$

Finally, the SUSY Lagrangian in Eq. (1.4.2) then becomes

$$\mathcal{L}_{SUSY} = (D_\mu\phi_i)^\dagger(D^\mu\phi_i) + i\bar{\psi}_i\bar{\sigma}^\mu D_\mu\psi_i - \frac{1}{4}F_{\mu\nu}^a F^{a\nu\mu}$$

$$\begin{aligned}
 & + i\bar{\lambda}^a \bar{\sigma}^\mu D_\mu \lambda^a - \left(\frac{1}{2} \frac{\partial^2 \mathcal{W}}{\partial \phi_i \partial \phi_j} \psi_i \psi_j \right. \\
 & \left. - ig_a \sqrt{2} \phi^* \lambda^a (T^a)_{ij} \psi_j + h.c \right) - V(\phi_i, \phi_i^*).
 \end{aligned} \tag{1.4.76}$$

1.4.4 Soft Supersymmetry Breaking Lagrangian

The aspect of the theories we have seen, in the last subsection, so far is the basic techniques in the construction of a supersymmetric Lagrangian. Since all particles of a supersymmetric multiplet have the same mass due to the fact that the super charge commutes with the momentum operator (applying Q on a state does not change the eigenvalue of $P^\mu P_\mu = m^2$), supersymmetry can not be an exact symmetry of nature from the point of view of phenomenology because this degeneracy is not observed in nature. This implies *e.g.* there is no selectron of the same mass with electron and squarks with the same mass of quarks. Thus, supersymmetry must be broken to play a role in particle physics, one can break SUSY spontaneously (SSB). In this case, the Lagrangian preserves the symmetry but the vacuum state (ground state) does not, which implies that the charge generating the symmetry does not annihilate the vacuum *i.e.* $Q|0\rangle \neq 0$. Thus we get the spectrum for vacuum expectation value (VEV) of the Hamiltonian:

$$E_{\text{vev}} = \langle 0 | H | 0 \rangle > 0. \tag{1.4.77}$$

Furthermore, the only fields that may have a nonzero VEV without breaking Lorentz invariance are scalar fields, so the scalar potential given in Eq. (1.4.73) reduces the condition in Eq. (1.4.77) to

$$\langle 0 | V(\phi, \phi^\dagger) | 0 \rangle > 0. \tag{1.4.78}$$

Therefore, the auxiliary fields have non-vanishing VEV simultaneously, $\langle 0 | F_i | 0 \rangle \neq 0$ or $\langle 0 | D | 0 \rangle \neq 0$. We distinguish two ways to achieve this:

- The auxiliary fields F_i have non-zero VEV, $\langle 0 | F_i | 0 \rangle \neq 0$, irrespective of the value of $\langle 0 | D | 0 \rangle$. This is known as F-type SUSY breaking. In case of the superpotential

constructs from three chiral supermultiplets, this called O’Raifeartaigh model [46].

- The auxiliary fields D have a non-zero VEV. This case can be achieved through the Fayet-Iliopoulos mechanism [47], where at least one $U(1)$ gauge field must be introduced. This scenarios is referred to as D-type SUSY breaking.

These two mechanisms of SSB do not lead to phenomenological viable model and this is the MSSM. However, SUSY can be broken explicitly by additional terms in the Lagrangian that do not introduce quadratic divergences, these terms are called soft SUSY breaking terms.

In order of an effective theory of broken SUSY with only soft breaking terms. Girardello and Grisaru found [48]:

$$\begin{aligned}\mathcal{L}_{\text{soft}} = & \frac{1}{2}M_\lambda\lambda^a\lambda^a + \frac{1}{6}a^{ijk}\phi_i\phi_j\phi_k + \frac{1}{2}b^{ij}\phi_i\phi_j + h.c. \\ & - (m^2)_j^i\phi^{j\dagger}\phi_i.\end{aligned}\tag{1.4.79}$$

- gauginos mass M_λ .
- scalars mass m_{ij}^2 .
- bilinear couplings b_{ij} .
- trilinear scalar couplings a_{ijk} .

1.4.5 The Complete Lagrangian

Finally, we can present the full effective Lagrangian of a supersymmetric theory which can be grouped into two categories: supersymmetric Lagrangian in Eq. (1.4.76) and soft SUSY breaking Lagrangian in Eq. (1.4.79) resulting in

$$\begin{aligned}\mathcal{L} = & \mathcal{L}_{\text{SUSY}} + \mathcal{L}_{\text{soft}} \\ = & (D_\mu\phi_i)^\dagger(D^\mu\phi_i) + i\bar{\psi}_i\bar{\sigma}^\mu D_\mu\psi_i - \frac{1}{4}F_{\mu\nu}^a F^{a\mu\nu} \\ & + i\bar{\lambda}^a\bar{\sigma}^\mu D_\mu\lambda^a - \left(\frac{1}{2}\frac{\partial^2\mathcal{W}}{\partial\phi_i\partial\phi_j}\psi_i\psi_j\right. \\ & \left.- ig_a\sqrt{2}\phi^*\lambda^a(T^a)_{ij}\psi_j + h.c\right) - V(\phi_i, \phi_i^*)\end{aligned}\tag{1.4.80}$$

$$\begin{aligned}
 & + \frac{1}{2} M_\lambda \lambda^a \lambda^a + \frac{1}{6} a^{ijk} \phi_i \phi_j \phi_k + \frac{1}{2} b^{ij} \phi_i \phi_j + h.c. \\
 & - (m^2)_j^i \phi^{j\dagger} \phi_i
 \end{aligned} \tag{1.4.81}$$

1.5 Minimal Supersymmetric Standard Model

The minimal supersymmetric standard model is the minimal supersymmetric extension of the SM, which is based on the gauge symmetry group $SU(3) \otimes SU(2) \otimes U(1)$. The particles of the MSSM are the one from the SM, where each known particle has a superpartner. The superpartners of the SM fermions are the spin zero sfermion (squarks and slepton). The superpartner of the gauge bosons are the spin half gauginos. On the other hand the Higgs sector of the MSSM differs from that of the SM, *i.e.* the SM Higgs sector contains only a single doublet and the MSSM consists of two Higgs doublets H_u and H_d (the spin $\frac{1}{2}$ higgsinos) which couple at tree level to up and down type fermions separately.

The MSSM particle content, particles and superpartners which are placed in the chiral and gauge supermultiplets, is listed in the Table.1.1. The MSSM Lagrangian can be written as

$$\mathcal{L}_{MSSM} = \mathcal{L}_{kin} + \mathcal{L}_{int} + W + \mathcal{L}_{soft}. \tag{1.5.1}$$

All fields, gauge, fermion and scalar, involved have the canonical kinetic terms given by the Lagrangian

$$\begin{aligned}
 \mathcal{L}_{kin} = & (D_\mu \phi_i)^\dagger (D^\mu \phi_i) + i \bar{\psi}_i D_\mu \gamma^\mu \psi_i - \frac{1}{4} \sum_{a=1}^8 G_{\mu\nu}^a G^{a\mu\nu} \\
 & - \frac{1}{4} \sum_{a=1}^3 W_{\mu\nu}^a W^{a\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + i \bar{\lambda}^a \bar{\sigma}^\mu D_\mu \lambda^a,
 \end{aligned} \tag{1.5.2}$$

where $D_\mu = \partial_\mu + ig' Y B_\mu + ig \frac{\sigma^i}{2} W_\mu^i + ig_s \frac{\lambda^a}{2} G_\mu^a$. The fields ψ and λ have four and two components, respectively. Note that λ is the associated gaugino field.

1.5 Minimal Supersymmetric Standard Model

Bosons		Fermions		$SU_c(3)$	$SU_L(2)$	$U_Y(1)$
gluon	g^a	gluino	$\tilde{\lambda}^a$		(8,0,0)	
Weak	$(W^\pm, Z)$	wino, zino	$(\tilde{W}^\pm, \tilde{Z})$		(1,3,0)	
Hypercharge	B (γ)	bino	$\tilde{B}(\tilde{\gamma})$		(1,1,0)	
squarks	$\begin{cases} \tilde{Q}_i = (\tilde{u}, \tilde{d})_L \\ \tilde{U}_i = \tilde{u}_R \\ \tilde{D}_i = \tilde{d}_R \end{cases}$	quarks	$\begin{cases} Q_i = (u, d)_L \\ U_i = u_R^c \\ D_i = d_R^c \end{cases}$		(3, 2, 1/6)	
					(3*, 1, -2/3)	
					(3*, 1, 1/3)	
sleptons	$\begin{cases} \tilde{L}_i = (\tilde{\nu}_e, \tilde{e})_L \\ \tilde{E}_i = \tilde{e}_R \end{cases}$	leptons	$\begin{cases} L_i = (\nu_e, e)_L \\ E_i = e_R^c \end{cases}$		(1, 2, -1/2)	
					(1, 1, 1)	
Higgses	$\begin{cases} H_1 \\ H_2 \end{cases}$	higgsinos	$\begin{cases} \tilde{H}_1 \\ \tilde{H}_2 \end{cases}$		(1, 2, -1/2)	
					(1, 2, 1/2)	

Table.1.1. Particle Content of the MSSM.

The interaction terms among the fields are specified by SUSY and gauge invariance, they are

$$\mathcal{L}_{int} = (-ig_a \sqrt{2} \phi_i^* \lambda^a T^a \psi_i + h.c) - \frac{g_a}{2} (\phi_i^* T^a \phi)^2. \quad (1.5.3)$$

with T^a and g_a are the generators and coupling constants of the corresponding groups.

The soft terms which break SUSY explicitly, which are given in the soft Lagrangian in Eq.(1.4.79), for the MSSM can be written as follows:

$$\begin{aligned} \mathcal{L}_{soft} = & -\frac{1}{2} (M_1 \tilde{B} \tilde{B} + M_2 \tilde{W} \tilde{W} + M_3 \tilde{g} \tilde{g}) - m_{\tilde{q}_{ij}}^2 \tilde{q}_i^* \tilde{q}_j - m_{\tilde{u}_{ij}}^2 \tilde{u}_i^* \tilde{u}_j \\ & - m_{\tilde{d}_{ij}}^2 \tilde{d}_i^* \tilde{d}_j - m_{\tilde{l}_{ij}}^2 \tilde{l}_i^* \tilde{l}_j - m_{\tilde{e}_{ij}}^2 \tilde{e}_i^* \tilde{e}_j - m_{H_u}^2 |H_u|^2 - m_{H_d}^2 |H_d|^2 \\ & - [A_u^{ij} Y_u^{ij} \tilde{q}_i \tilde{u}_j H_u + A_d^{ij} Y_d^{ij} \tilde{q}_i \tilde{d}_j H_d + A_e^{ij} Y_e^{ij} \tilde{l}_i \tilde{e}_j H_d \\ & - B\mu H_u H_d + h.c], \end{aligned} \quad (1.5.4)$$

where the soft terms M_1 , M_2 and M_3 are the bino, wino and gluino mass terms, $m_{\tilde{q}}$, $m_{\tilde{u}}$, $m_{\tilde{d}}$, $m_{\tilde{l}}$ and $m_{\tilde{e}}$ are Hermitian 3×3 matrices (they can have complex entries) in flavour space so that the Lagrangian is real, while the trilinear couplings $A_f^A \equiv A_f Y_f$, with $f = u, d, e$, are complex 3×3 matrices with dimensions of mass. Also the Higgsino mass parameter is in general complex with a phase ϕ_μ . The soft SUSY terms induce about 100 free parameters which reduce the predictivity of the MSSM. At a specific high scale models, these parameters can be reduced. Thus they provide relations between these parameters. For instance, the constrained MSSM (mSUGRA) gives the constraints

$$\begin{aligned} m_{\tilde{q}_{ij}}^2 &= m_{\tilde{u}_{ij}}^2 = m_{\tilde{d}_{ij}}^2 = m_{\tilde{l}_{ij}}^2 = m_{\tilde{e}_{ij}}^2 = m_0^2 \delta_{ij}, \\ M_1 &= M_2 = M_3 = M_{1/2}, \\ A_u^{ij} &= A_d^{ij} = A_e^{ij} = A_0, \\ m_{H_u}^2 &= m_{H_d}^2 = m_0^2, \end{aligned} \tag{1.5.5}$$

where m_0 , $M_{1/2}$ and A_0 are the universal parameters at the grand unification theory (GUT) scale.

The superpotential for the MSSM contains the SM Yukawa interactions, which describes the interactions between Higgs bosons and matter superfields. It is given by

$$W_{MSSM} = Y_u^{ij} Q_{L_i} U_{L_j}^C H_u + Y_d^{ij} Q_{L_i} D_{L_j}^C H_d + Y_e^{ij} L_{L_i} E_{L_j}^C H_d + \mu H_d H_u, \tag{1.5.6}$$

where the indices i and j denote to the generations. The parameters Y_u^{ij} , Y_d^{ij} and Y_e^{ij} correspond to the Yukawa couplings present in the SM, which are non-diagonal 3×3 matrices in flavour space. The μ parameter has the dimension of mass, where the mu term provides the supersymmetric version to the masses of the Higgs bosons.

Also there are renormalizable and gauge invariant terms can be added in the superpotential. However each of these terms break baryon or lepton number, they are

given by

$$W' = \lambda_{ijk} L_i L_j E_k^C + \lambda'_{ijk} L_i Q_j D_k^C + \lambda''_{ijk} D_i^C D_j^C U_k^C + \mu' L_i H_u. \quad (1.5.7)$$

These terms are not present in the SM that automatically preserves baryon and lepton number. The above interactions would allow proton decay $p \rightarrow e^+ \pi^0$. In order to remove those interactions, a new symmetry should be imposed. The simplest one that works is R-parity defined as

$$R = (-1)^{3B+L+2S}, \quad (1.5.8)$$

where S is the spin, B and L are baryon and lepton number. It eliminates all interactions violating conservation of baryon and lepton numbers. On the other hand R-parity plays an important phenomenological role. One striking implication of invariance under R-parity is that there are no interactions coupling a single superpartner to two SM particles, and thus the lightest supersymmetric particle (LSP) is stable, *i.e.* it cannot decay to anything. This means that the universe may be filled with this particle, hence, it might be a possible candidate for dark matter. Also in the presence of R-parity the SUSY particles produced in pairs, decay to LSP plus SM particles (gives a signal of missing energy).

1.5.1 Electroweak Symmetry Breaking

Arguably the most important success of supersymmetry is that it can provide a natural mechanism for understanding Higgs physics and electroweak symmetry breaking.

So far we have seen the theory respects the gauge symmetry. Let us now discuss that the gauge symmetry of the MSSM can be broken to $SU(3) \otimes U(1)_{\text{em}}$, so that the gauge bosons W , Z and fermions get mass as they do in the SM, *i.e.* SUSY can provide a natural mechanism for understanding Higgs physics and electroweak symmetry breaking (EWSB) [49–53]. This means that we study the scalar potential

for the Higgs fields H_u and H_d , where the Higgs sector in the MSSM consists of two complex Higgs $SU(2)_L$ doublet $H_u = \begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix}$ and $H_d = \begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix}$ with $Y_{H_u} = -1$ and $Y_{H_d} = 1$. In the MSSM, the Higgs potential constructs from three sources [57, 58]:

$$V = V_D + V_F + V_{\text{soft}}, \quad (1.5.9)$$

where V_D is the D-term potential, V_F is the F-term potential and V_{soft} is the soft one, they are

$$V_D = \frac{g^2}{2} [4 |H_d^\dagger H_u|^2 - 2 |H_u|^2 |H_d|^2 + (|H_u|^2)^2 + (|H_d|^2)^2] + \frac{g'^2}{2} (|H_u|^2 - |H_d|^2)^2, \quad (1.5.10)$$

$$V_F = \mu^2 (|H_u|^2 + |H_d|^2), \quad (1.5.11)$$

$$V_{\text{soft}} = m_{H_u}^2 H_u^\dagger H_u + m_{H_d}^2 H_d^\dagger H_d + B\mu (H_u H_d + h.c.). \quad (1.5.12)$$

Therefore, the full scalar potential is

$$V(H_u, H_d) = (|\mu|^2 + m_{H_u}^2) (|H_u^+|^2 + |H_u^0|^2) + (|\mu|^2 + m_{H_d}^2) (|H_d^-|^2 + |H_d^0|^2) + [B\mu (H_u^\dagger H_d^- - H_u^0 H_d^0) + h.c.] + \frac{g^2 + g'^2}{8} [|H_u^+|^2 + |H_u^0|^2 - |H_d^-|^2 - |H_d^0|^2] \quad (1.5.13)$$

$$+ \frac{g^2}{2} |H_u^+ H_d^{0\dagger} + H_u^0 H_d^{-\dagger}|^2. \quad (1.5.14)$$

The neutral components of the two Higgs fields develop vacuum expectations values in terms of the $U(1)_{\text{em}}$ and CP preserving VEV's

$$\langle H_u^0 \rangle = \begin{pmatrix} 0 \\ v_u \end{pmatrix}, \quad \langle H_d^0 \rangle = \begin{pmatrix} v_d \\ 0 \end{pmatrix}, \quad (1.5.15)$$

the Higgs potential reduces to

$$V(v_u, v_d) = m_1^2 |v_u|^2 + m_2^2 |v_d|^2 - (m_3^2 v_u v_d + h.c.)$$

$$+ \frac{g^2 + g'^2}{8} (|v_u|^2 - |v_d|^2)^2. \quad (1.5.16)$$

where

$$m_1 = |\mu|^2 + m_{H_u}^2, \quad m_2 = |\mu|^2 + m_{H_d}^2, \quad m_3 = B\mu. \quad (1.5.17)$$

This potential must go to plus infinity. If we fix $v_u = v_d$, the last term in the Eq. (1.5.16) will vanish. In this case, the potential reduces to $V(v_u, v_d) = (m_1^2 + m_2^2 - 2m_3^2) |v_u|^2$. If we want this to have minimum (bounded from below), the overall coefficient must be positive. This implies that

$$m_1^2 + m_2^2 > 2m_3^2. \quad (1.5.18)$$

To find the minimum, we set the first derivatives of this potential to zero:

$$\frac{\partial V}{\partial v_u} = 2m_1^2 v_u - 2m_3^2 v_d + \frac{g^2 + g'^2}{2} v_u (v_u^2 - v_d^2) = 0, \quad (1.5.19)$$

$$\frac{\partial V}{\partial v_d} = 2m_2^2 v_d - 2m_3^2 v_u - \frac{g^2 + g'^2}{2} v_d (v_u^2 - v_d^2) = 0. \quad (1.5.20)$$

One possible solution of these equations is $v_u = v_d = 0$, *i.e.* no electroweak symmetry breaking. To ensure this does not occur, the origin must be a local maximum of the potential. The only way for the origin not be a minimum of the potential is to impose it to be a saddle point of the origin, this means that the determinant of the Higgs mass matrix of second derivative should be negative at the origin. Thus we write

$$\frac{\partial^2 V}{\partial v_u \partial v_d} = m_{H^0}^2 = \begin{pmatrix} m_1^2 & -m_3^2 \\ -m_3^2 & m_2^2 \end{pmatrix}, \quad (1.5.21)$$

therefore, $\det(m_{H^0}^2) < 0$ implies that

$$m_1^2 m_2^2 < m_3^4. \quad (1.5.22)$$

However, if $m_1^2 = m_2^2$, the condition in Eqs. (1.5.18) and (1.5.22) are not satisfied. Thus we must have non-vanishing soft SUSY breaking scalar masses m_{H_u} and m_{H_d}

$$m_1^2 \neq m_2^2 \Rightarrow m_{H_u}^2 \neq m_{H_d}^2. \quad (1.5.23)$$

Therefore, to break electroweak symmetry, we need also to break SUSY. This implies that there is a close connection between gauge symmetry breaking and SUSY breaking. This problem is solved at the GUT scale where $m_{H_u} = m_{H_d}$, but the running to lower energies makes that this degeneracy is lifted since H_u and H_d couple with different strength of fermions (top/bottom quarks and their SUSY partners).

1.5.2 Masses of the Gauge Bosons

The masses of the gauge bosons W and Z are generated after electroweak symmetry breaking, as in the SM, from the kinetic terms of H_u and H_d . The MSSM Lagrangian contains the following kinetic terms for the Higgs scalars:

$$\mathcal{L} = (D_\mu H_u)^\dagger (D^\mu H_u) + (D_\mu H_d)^\dagger (D^\mu H_d), \quad (1.5.24)$$

where

$$\begin{aligned} D_\mu &= \partial_\mu + ig \frac{\sigma^i}{2} W_\mu^i + i \frac{g'}{2} Y B_\mu \\ &= \partial_\mu + i \frac{g}{2} \begin{pmatrix} W_\mu^3 & W_\mu^1 - i W_\mu^2 \\ W_\mu^1 + i W_\mu^2 & -W_\mu^3 \end{pmatrix} + i \frac{g'}{2} B_\mu \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \end{aligned} \quad (1.5.25)$$

Recall that H_u and H_d at the minimum of the potential, they become

$$H_u^0 = \begin{pmatrix} 0 \\ v_u \end{pmatrix}, \quad H_d^0 = \begin{pmatrix} v_d \\ 0 \end{pmatrix}, \quad (1.5.26)$$

where the ratio of these VEV's is defined as

$$\tan \beta = \frac{v_u}{v_d} = \frac{v \sin \beta}{v \cos \beta}. \quad (1.5.27)$$

Using the fact that the hypercharge, respectively, of H_u and H_d are $-1, 1$, we get

$$D_\mu \begin{pmatrix} 0 \\ v_u \end{pmatrix} = i \frac{g}{2} v_u \begin{pmatrix} W_{1\mu} - i W_{2\mu} \\ -W_{3\mu} \end{pmatrix} - i \frac{g'}{2} v_u B_\mu \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad (1.5.28)$$

$$D_\mu \begin{pmatrix} v_d \\ 0 \end{pmatrix} = i \frac{g}{2} v_d \begin{pmatrix} W_{3\mu} \\ W_\mu^1 + i W_{2\mu} \end{pmatrix} + i \frac{g'}{2} v_d B_\mu \begin{pmatrix} 1 \\ 0 \end{pmatrix}. \quad (1.5.29)$$

Substituting these expressions in Eq. (1.5.24), we obtain

$$\begin{aligned} \mathcal{L} &= \frac{g^2}{4} (v_d^2 + v_u^2) (W_{1\mu} W_1^\mu + W_{2\mu} W_2^\mu + W_{3\mu} W_3^\mu) \\ &\quad + \frac{g'^2}{4} (v_d^2 + v_u^2) B_\mu B^\mu - \frac{g'g}{2} (v_d^2 + v_u^2) B_\mu W_{3\mu} \\ &= \frac{g^2(v_d^2 + v_u^2)}{4} (W_1^2 + W_2^2) + \frac{(v_d^2 + v_u^2)}{4} (g^2 W_3^2 - 2g'g B_\mu W_3^\mu + g'^2 B^2) \\ &= \frac{g^2(v_d^2 + v_u^2)}{4} (W_1^2 + W_2^2) + \frac{(v_d^2 + v_u^2)(g^2 + g'^2)}{4} \frac{(gW_3^\mu - g'B^\mu)^2}{g^2 + g'^2}, \end{aligned} \quad (1.5.30)$$

where

$$W_1^2 + W_2^2 = (W_{1\mu} + i W_{2\mu})(W_1^\mu - i W_2^\mu) \equiv W_\mu W^{\mu\dagger} \quad (1.5.31)$$

When we define

$$Z_\mu = \frac{gW_{3\mu} - g'B_\mu}{\sqrt{g^2 + g'^2}}, \quad W_\mu^+ = \frac{(W_1^\mu + i W_2^\mu)}{\sqrt{2}}. \quad (1.5.32)$$

Then the gauge bosons masses are defined as

$$M_Z^2 = \frac{(v_d^2 + v_u^2)(g^2 + g'^2)}{2}, \quad (1.5.33)$$

$$M_W^2 = \frac{g^2(v_d^2 + v_u^2)}{2}. \quad (1.5.34)$$

We see that these relations are exactly the same as in the standard model with v^2 replaced by $v^2 = \sqrt{v_d^2 + v_u^2} \simeq 174$.

The minimization conditions $\frac{\partial V}{\partial H_u^0} = \frac{\partial V}{\partial H_d^0} = 0$ lead to the following equations:

$$(|\mu|^2 + m_{H_u}^2) = B\mu \frac{v_d}{v_u} + \frac{1}{4} \frac{(g^2 + g'^2)(v_d^2 - v_u^2)}{v_u}, \quad (1.5.35)$$

$$(|\mu|^2 + m_{H_d}^2) = B\mu \frac{v_u}{v_d} - \frac{1}{4} \frac{(g^2 + g'^2)(v_d^2 - v_u^2)}{v_d}. \quad (1.5.36)$$

Using the Eq. (1.5.27), the two above equations become

$$(|\mu|^2 + m_{H_u}^2) = B\mu \cot \beta + \frac{1}{2} m_Z^2 \cos 2\beta, \quad (1.5.37)$$

$$(|\mu|^2 + m_{H_d}^2) = B\mu \tan \beta - \frac{1}{2}m_Z^2 \cos 2\beta. \quad (1.5.38)$$

Finally, one obtains two minimization conditions that can be written as following

$$|\mu|^2 = \frac{m_{H_d}^2 - m_{H_u}^2 \tan^2 \beta}{\tan^2 \beta - 1} - \frac{M_Z^2}{2}, \quad (1.5.39)$$

$$B\mu = \frac{(m_{H_d}^2 - m_{H_u}^2) \tan 2\beta + m_Z^2 \sin 2\beta}{2}. \quad (1.5.40)$$

These two equations are known as the electroweak symmetry breaking (EWSB) conditions. From these expressions we see that naturalness requires that both μ and B are electroweak scale parameters. However, the μ -term is a supersymmetric term in the Lagrangian and could take any value between EWSB and Planck scales. This leads to the so-called μ -problem. On the other hand, these relations are very important since the requirement of radiative symmetry breaking leads to additional constraints and lowers the number of free parameters. Thus, two parameters (usually $|\mu|$ and B) from the free parameters of the MSSM ($m_0, M_{1/2}, A_0, B, \mu$ and $\tan \beta$) can be fixed.

1.5.3 MSSM Mass Spectrum

1.5.3.1 Charginos

The mass spectrum for the charged gauginos is more complicated because once $SU(2)_L \times U(1)_Y$ is broken the charged winos $\tilde{W}^\pm$ and the charged higgsinos $\tilde{H}_u^\pm$, and $\tilde{H}_d^\pm$ mix amongst themselves to form two mass-eigenstates with charge ± 1 called charginos. The chargino mass matrix can be derived from:

(i) the supersymmetric contribution, which arise from the term $\mu \tilde{h}_u^+ \tilde{h}_d^-$ in the superpotential and leads to a fermion mass term

$$\mathcal{L} \supset -\frac{1}{2}\mu \bar{\psi}_{\tilde{h}_u^+} \psi_{\tilde{h}_d^-} - \frac{1}{2}\mu \bar{\psi}_{\tilde{h}_d^-} \psi_{\tilde{h}_u^+}. \quad (1.5.41)$$

On other hand, the coupling of charged gauginos-higgsino to matter fields arise from

$$\mathcal{L} \supset -g_a \sqrt{2} (\phi_i^\dagger T^a \lambda^a \psi_i) + h.c., \quad (1.5.42)$$

where ϕ_i and ψ_i are the two Higgs supermultiplets containing Higgs and Higgsino fields respectively. Then the Eq. (1.5.42) can be written as,

$$\begin{aligned}
 \mathcal{L} &\supset -\frac{\sqrt{2}}{2}g \begin{pmatrix} H_u^{+*} & H_u^{0*} \end{pmatrix} (\tau^1 \widetilde{W}^1 + \tau^2 \widetilde{W}^2) \begin{pmatrix} \widetilde{H}_u^+ \\ \widetilde{H}_u^0 \end{pmatrix} + h.c. \\
 &\quad -\frac{\sqrt{2}}{2}g \begin{pmatrix} H_d^{0*} & H_d^{-*} \end{pmatrix} (\tau^1 \widetilde{W}^1 + \tau^2 \widetilde{W}^2) \begin{pmatrix} \widetilde{H}_d^0 \\ \widetilde{H}_d^- \end{pmatrix} + h.c. \\
 &= -\frac{1}{2}\sqrt{2}g \begin{pmatrix} H_u^{+*} & H_u^{0*} \end{pmatrix} \begin{bmatrix} 0 & \widetilde{W}^1 - i\widetilde{W}^2 \\ \widetilde{W}^1 + i\widetilde{W}^2 & 0 \end{bmatrix} \begin{pmatrix} \widetilde{H}_u^+ \\ \widetilde{H}_u^0 \end{pmatrix} + h.c. \\
 &\quad -\frac{1}{2}\sqrt{2}g \begin{pmatrix} H_d^{0*} & H_d^{-*} \end{pmatrix} \begin{bmatrix} 0 & \widetilde{W}^1 - i\widetilde{W}^2 \\ \widetilde{W}^1 + i\widetilde{W}^2 & 0 \end{bmatrix} \begin{pmatrix} \widetilde{H}_d^0 \\ \widetilde{H}_d^- \end{pmatrix} + h.c. \quad (1.5.43)
 \end{aligned}$$

After electroweak symmetry breaking, the Higgs H_u^0 and H_d^0 has acquired a VEVs v_u and v_u respectively. Then the above equation yields,

$$\begin{aligned}
 \mathcal{L} &\supset -\frac{1}{2}gv_u \frac{(\widetilde{W}^1 + i\widetilde{W}^2)}{\sqrt{2}} \widetilde{H}_u^+ - gv_d \frac{(\widetilde{W}^1 - i\widetilde{W}^2)}{\sqrt{2}} \widetilde{H}_d^- + h.c. \\
 &= -\frac{1}{2}gv_u \widetilde{W}^- \widetilde{H}_u^+ - \frac{1}{2}gv_d \widetilde{W}^+ \widetilde{H}_d^- + h.c. \quad (1.5.44)
 \end{aligned}$$

Thus the chargino mass terms in the Lagrangian contribution from the SUSY are

$$\begin{aligned}
 \mathcal{L}_{mass}^{SUSY} &= -\frac{1}{2}\mu \widetilde{h}_u^+ \widetilde{h}_d^- - \frac{1}{2}gv_u \widetilde{W}^- \widetilde{H}_u^+ - \frac{1}{2}gv_d \widetilde{W}^+ \widetilde{H}_d^- + h.c. \\
 &= -\frac{1}{2} \begin{pmatrix} \widetilde{W}^- & \widetilde{H}_d^- \end{pmatrix} \begin{pmatrix} 0 & gv_u \\ gv_d & \mu \end{pmatrix} \begin{pmatrix} \widetilde{W}^+ \\ \widetilde{H}_u^+ \end{pmatrix} + h.c. \quad (1.5.45)
 \end{aligned}$$

We deduce that

$$M_{\widetilde{\chi}^\pm}^{SUSY} = \begin{pmatrix} 0 & gv_u \\ gv_d & \mu \end{pmatrix}. \quad (1.5.46)$$

(ii) The soft supersymmetry breaking contribution, which is only generates a gaugino mass

$$\mathcal{L}_{mass}^{soft} = -\frac{1}{2}M_2 \widetilde{W} \widetilde{W}$$

$$= -\frac{1}{2} \begin{pmatrix} \widetilde{W}^- & \widetilde{H}_d^- \end{pmatrix} \begin{pmatrix} M_2 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \widetilde{W}^+ \\ \widetilde{H}_u^+ \end{pmatrix}, \quad (1.5.47)$$

then

$$M_{\widetilde{\chi}^\pm}^{soft} = \begin{pmatrix} M_2 & 0 \\ 0 & 0 \end{pmatrix}. \quad (1.5.48)$$

Finally, the total chargino mass is,

$$\begin{aligned} M_{\widetilde{\chi}^\pm} &= M_{\widetilde{\chi}^\pm}^{soft} + M_{\widetilde{\chi}^\pm}^{SUSY} = \begin{pmatrix} M_2 & gv_u \\ gv_d & \mu \end{pmatrix} \\ &= \begin{pmatrix} M_2 & \sqrt{2}m_W \sin \beta \\ \sqrt{2}m_W \cos \beta & \mu \end{pmatrix}. \end{aligned} \quad (1.5.49)$$

Where $v_u = v \sin \beta$ and $v_d = v \cos \beta$.

The chargino mass matrix is non-symmetric, so it can be diagonalized by two unitary 2×2 matrices U and V ².

The mass eigenstates are related to the gauge eigenstates, such that

$$\begin{pmatrix} \widetilde{\chi}_1^+ \\ \widetilde{\chi}_2^+ \end{pmatrix} = V \begin{pmatrix} \widetilde{W}^+ \\ \widetilde{H}_u^+ \end{pmatrix}, \quad \begin{pmatrix} \widetilde{\chi}_1^- \\ \widetilde{\chi}_2^- \end{pmatrix} = U \begin{pmatrix} \widetilde{W}^- \\ \widetilde{H}_d^- \end{pmatrix}. \quad (1.5.50)$$

Then, the chargino diagonalized mass matrix is,

$$U^\dagger M_{\widetilde{\chi}^\pm} V = M_{\chi^\pm}^{diag}. \quad (1.5.51)$$

The explicit forms for the chargino mixing matrices are $U = O_-$ while $V = O_+$ if $\det \mathcal{M}_{\chi^\pm} \geq 0$, and $V = \sigma_3 O_+$ if $\det \mathcal{M}_{\chi^\pm} \leq 0$. $O_\pm$ is given by [54, 55]

$$O_\pm = \begin{pmatrix} \cos \phi_\pm & \sin \phi_\pm \\ -\sin \phi_\pm & \cos \phi_\pm \end{pmatrix}, \quad (1.5.52)$$

where σ_3 is the Pauli matrix and $O_\pm$ are rotation matrices with angles $\phi_\pm$ given by

$$\tan 2\phi_- = 2\sqrt{2}M_W \frac{-\mu \sin \beta + M_2 \cos \beta}{M_2^2 - \mu^2 - 2M_W^2 \cos 2\beta}, \quad (1.5.53)$$

²Note that some authors use Z_+ and Z_- instead of U and V . We see this in chapter 4.

$$\tan 2\phi_+ = 2\sqrt{2}M_W \frac{-\mu \cos \beta + M_2 \sin \beta}{M_2^2 - \mu^2 - 2M_W^2 \cos 2\beta}. \quad (1.5.54)$$

Thus, the mass matrix $\mathcal{M}_{\chi^\pm}$ has two eigenstates $\tilde{\chi}_1^\pm$ and $\tilde{\chi}_2^\pm$ (the charginos) with the following mass eigenvalues

$$\begin{aligned} M_{2,1}^2 &= 1/2(M_2^2 + \mu^2 + 2M_W^2 \\ &\pm \sqrt{(M_2^2 - \mu^2)^2 + 4M_W^4 \cos^2 2\beta + 4M_W^2(M_2^2 + \mu^2 - 2M_2\mu \sin 2\beta)}) \end{aligned} \quad (1.5.55)$$

The lightest chargino $\tilde{\chi}_1^\pm$ is often of order M_Z and has a ground choice of being the lightest charged SUSY particles. Today's negative SUSY searches provides the following lower mass limit on chargino mass through the search for the process $e^+e^- \rightarrow \tilde{\chi}^+\tilde{\chi}^-$:

$$M_{\tilde{\chi}^\pm} > 94 \text{ GeV}. \quad (1.5.56)$$

1.5.3.2 Neutralinos

The uncharged fermionic superpartners $\tilde{B}$ (bino) and $\tilde{W}^0$ (wino) and the neutral higgsinos $\tilde{H}_{1,2}^0$ (Higgsinos) mix after electroweak symmetry breaking to form four neutral mass-eigenstates called neutralinos. The relevant terms for the neutralino mass matrix are derived from the soft SUSY breaking terms, the SUSY gauge interaction terms and the Yukawa interaction terms, which can be deduced from the superpotential. In the $(-i\tilde{B}, -i\tilde{W}^0, \tilde{H}_1^0, \tilde{H}_2^0)$ basis, the Lagrangian contribution from the soft SUSY breaking neutral gaugino masses is,

$$\begin{aligned} \mathcal{L}_{mass}^{soft} &= -\frac{1}{2}M_1\tilde{B}\tilde{B} - \frac{1}{2}M_2\tilde{W}^0\tilde{W}^0 \\ &= -\frac{1}{2} \begin{pmatrix} \tilde{B}^* & \tilde{W}^{0*} & \tilde{H}_u^{0*} & \tilde{H}_d^{0*} \end{pmatrix} \begin{pmatrix} M_1 & 0 & 0 & 0 \\ 0 & M_2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \tilde{B} \\ \tilde{W}^0 \\ \tilde{H}_u^0 \\ \tilde{H}_d^0 \end{pmatrix} \end{aligned} \quad (1.5.57)$$

Then, the soft neutral gaugino mass matrix is

$$M_{\tilde{N}}^{\text{soft}} = \begin{pmatrix} M_1 & 0 & 0 & 0 \\ 0 & M_2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \quad (1.5.58)$$

Also, the supersymmetric neutral higgsino mass terms is

$$\mathcal{L} = \frac{1}{2} \mu \tilde{h}_u^0 \tilde{h}_d^0 \quad (1.5.59)$$

In addition to this, from Higgs-higgsino-gaugino couplings terms we derive neutral higgsino-gaugino mixing, such that

$$\begin{aligned} \mathcal{L} &\supset -\frac{\sqrt{2}}{2} \begin{pmatrix} H_u^{+*} & H_u^{0*} \end{pmatrix} (g' \tau^0 \tilde{B} + g \tau^3 \tilde{W}^0) \begin{pmatrix} \tilde{H}_u^+ \\ \tilde{H}_u^0 \end{pmatrix} + h.c. \\ &\quad -\frac{\sqrt{2}}{2} \begin{pmatrix} H_d^{0*} & H_d^{-*} \end{pmatrix} (g' \tau^0 \tilde{B} + g \tau^3 \tilde{W}^0) \begin{pmatrix} \tilde{H}_d^0 \\ \tilde{H}_d^- \end{pmatrix} + h.c. \\ &= -\frac{\sqrt{2}}{2} \begin{pmatrix} H_u^{+*} & H_u^{0*} \end{pmatrix} \begin{bmatrix} g' \tilde{B} + g \tilde{W}^0 & 0 \\ 0 & g' \tilde{B} - g \tilde{W}^0 \end{bmatrix} \begin{pmatrix} \tilde{H}_u^+ \\ \tilde{H}_u^0 \end{pmatrix} + h.c. \\ &\quad -\frac{\sqrt{2}}{2} \begin{pmatrix} H_d^{0*} & H_d^{-*} \end{pmatrix} \begin{bmatrix} -g' \tilde{B} + g \tilde{W}^0 & 0 \\ 0 & -g' \tilde{B} - g \tilde{W}^0 \end{bmatrix} \begin{pmatrix} \tilde{H}_d^0 \\ \tilde{H}_d^- \end{pmatrix} + h.c. \end{aligned} \quad (1.5.60)$$

When the Higgs scalars replaced by their VEVs, the above equation becomes as follows:

$$\begin{aligned} \mathcal{L} &\supset -\frac{\sqrt{2}}{2} v_u (g' \tilde{B} - g \tilde{W}^0) \tilde{H}_u^0 + \frac{\sqrt{2}}{2} v_d (-g' \tilde{B} + g \tilde{W}^0) \tilde{H}_d^0 + h.c. \\ &= -\frac{1}{2} m_Z s_W [-c_\beta (\tilde{B} \tilde{H}_d^0 + \tilde{H}_d^0 \tilde{B}) + s_\beta (\tilde{B} \tilde{H}_u^0 + \tilde{H}_u^0 \tilde{B})] \\ &\quad -\frac{1}{2} m_Z c_W [c_\beta (\tilde{W}^0 \tilde{H}_d^0 + \tilde{H}_d^0 \tilde{W}^0) - s_\beta (\tilde{W}^0 \tilde{H}_u^0 + \tilde{H}_u^0 \tilde{W}^0)] + h.c., \end{aligned} \quad (1.5.61)$$

with $\sin \theta_W = s_W = \frac{g'}{\sqrt{g'^2 + g^2}}$, $\cos \theta_W = c_W = \frac{g}{\sqrt{g'^2 + g^2}}$, $s_\beta = \sin \beta$ and $c_\beta = \cos \beta$.

The SUSY neutralino mass term reads,

$$\begin{aligned}
 \mathcal{L}_{mass}^{SUSY} &= -\frac{1}{2}(-\mu\tilde{h}_u^0\tilde{h}_d^0) - \frac{1}{2}m_{ZSW}[-c_\beta(\tilde{B}\tilde{H}_d^0 + \tilde{H}_d^0\tilde{B}) + s_\beta(\tilde{B}\tilde{H}_u^0 + \tilde{H}_u^0\tilde{B})] \\
 &\quad - \frac{1}{2}m_{ZCW}[c_\beta(\tilde{W}^0\tilde{H}_d^0 + \tilde{H}_d^0\tilde{W}^0) - s_\beta(\tilde{W}^0\tilde{H}_u^0 + \tilde{H}_u^0\tilde{W}^0)] + h.c. \\
 &= -\frac{1}{2} \begin{pmatrix} \tilde{B}^* & \tilde{W}^{0*} & \tilde{H}_d^{0*} & \tilde{H}_u^{0*} \end{pmatrix} M_N^{SUSY} \begin{pmatrix} \tilde{B} \\ \tilde{W}^0 \\ \tilde{H}_d^0 \\ \tilde{H}_u^0 \end{pmatrix} + h.c., \quad (1.5.62)
 \end{aligned}$$

with

$$M_N^{SUSY} = \begin{pmatrix} 0 & 0 & -m_{ZC_\beta S_W} & m_{ZS_\beta S_W} \\ 0 & 0 & m_{ZC_W S_\beta} & -m_{ZC_W C_\beta} \\ -m_{ZC_\beta S_W} & m_{ZC_W C_\beta} & 0 & -\mu \\ m_{ZS_\beta S_W} & -m_{ZC_W S_\beta} & -\mu & 0 \end{pmatrix}. \quad (1.5.63)$$

So, the total neutralino mass is,

$$M_N = M_N^{SUSY} + M_N^{soft} = \begin{pmatrix} M_1 & 0 & -m_{ZC_\beta S_W} & m_{ZS_\beta S_W} \\ 0 & M_2 & m_{ZC_W S_\beta} & -m_{ZC_W C_\beta} \\ -m_{ZC_\beta S_W} & m_{ZC_W C_\beta} & 0 & -\mu \\ m_{ZS_\beta S_W} & -m_{ZC_W S_\beta} & -\mu & 0 \end{pmatrix}, \quad (1.5.64)$$

where $s_\beta \equiv \sin \beta$, $c_\beta \equiv \cos \beta$, $c_W \equiv \cos \theta_W$, and $s_W \equiv \sin \theta_W$. As mentioned above, M_1 and M_2 are the $U(1)_Y$ and $SU(2)_L$ soft SUSY breaking gaugino masses, respectively. The Hermitian matrix M_N can be diagonalized by unitary transformation of the neutralino fields to obtain mass eigenstates, so that:

$$\tilde{N}_i = N_{ij}\psi_j^0, \quad (1.5.65)$$

with $\psi^0 = (\tilde{B}, \tilde{W}^0, \tilde{H}_d^0, \tilde{H}_u^0)$ is the gauge-eigenstate basis, so that

$$N^\dagger M_N N = M_N^{diag}. \quad (1.5.66)$$

In the limit of large $|\mu|$ values, the masses of the neutralino states simplify to [59]

$$\begin{aligned}
 m_{\chi_1^0} &\simeq M_1 - \frac{m_Z^2}{\mu^2}(M_1 + \mu \sin 2\beta)s_W^2, \\
 m_{\chi_2^0} &\simeq M_2 - \frac{m_Z^2}{\mu^2}(M_2 + \mu \sin 2\beta)c_W^2, \\
 m_{\chi_3^0} &\simeq |\mu| + \frac{1}{2}\frac{m_Z^2}{\mu^2}\epsilon_\mu(1 - \sin 2\beta)(\mu + M_2s_W^2 + M_1c_W^2), \\
 m_{\chi_4^0} &\simeq |\mu| + \frac{1}{2}\frac{m_Z^2}{\mu^2}\epsilon_\mu(1 + \sin 2\beta)(\mu - M_2s_W^2 - M_1c_W^2).
 \end{aligned} \tag{1.5.67}$$

In the limit $|\mu| \rightarrow \infty$, $m_{\chi_1^0}$ corresponds to a pure bino with mass $m_{\chi_1^0} \simeq M_1$, $m_{\chi_2^0}$ corresponds to pure wino with mass $m_{\chi_2^0} \simeq M_2$ while $m_{\chi_3^0}$ and $m_{\chi_4^0}$ are pure higgsinos with masses $m_{\chi_3^0} \simeq m_{\chi_4^0} \simeq |\mu|$.

The matrix elements of the diagonalizing matrix, N_{ij} ³ with $i, j = 1, 2, 3, 4$, are given by [56]

$$N_{i1} = \left(1 + \left(\frac{N_{i2}}{N_{i1}}\right)^2 + \left(\frac{N_{i3}}{N_{i1}}\right)^2 + \left(\frac{N_{i4}}{N_{i1}}\right)^2\right)^{\frac{1}{2}}, \tag{1.5.68}$$

$$\frac{N_{i2}}{N_{i1}} = -\frac{1}{\tan \theta_W} \frac{M_1 - \epsilon_i m_{\chi_i^0}}{M_2 - \epsilon_i m_{\chi_i^0}}, \tag{1.5.69}$$

$$\frac{N_{i3}}{N_{i1}} = \frac{\mu(M_1 - \epsilon_i m_{\chi_i^0})(M_2 - \epsilon_i m_{\chi_i^0}) - m_Z^2 \sin \beta \cos \beta [(M_1 - M_2)c_W^2 + M_2 - \epsilon_i m_{\chi_i^0}]}{m_Z(M_2 - \epsilon_i m_{\chi_i^0})s_W(\mu \cos \beta + \epsilon_i m_{\chi_i^0} \sin \beta)}, \tag{1.5.70}$$

$$\frac{N_{i4}}{N_{i1}} = \frac{-\epsilon_i m_{\chi_i^0}(M_1 - \epsilon_i m_{\chi_i^0})(M_2 - \epsilon_i m_{\chi_i^0}) - m_Z^2 \cos^2 \beta [(M_1 - M_2)c_W^2 + M_2 - \epsilon_i m_{\chi_i^0}]}{m_Z(M_2 - \epsilon_i m_{\chi_i^0})s_W(\mu \cos \beta + \epsilon_i m_{\chi_i^0} \sin \beta)}, \tag{1.5.71}$$

where ϵ_i is the sign of the i th eigenvalue of the neutralino mass matrix, which in the large $|\mu|$ limit are: $\epsilon_1 = \epsilon_2 = 1$ and $\epsilon_4 = -\epsilon_3 = \epsilon_\mu$. The lightest eigenvalue of this matrix and the corresponding eigenstates say $\tilde{\chi}$ has good chance of being the lightest supersymmetric particle (LSP).

³Note that some authors Z_N rather than N .

1.5.3.3 Sfermions

The scalar partners of quarks get their masses from the following contributions:

1. The soft SUSY breaking scalar mass terms m_{ij}^2 , which are the most important contribution. Note that these terms are in general non-universal, even if they are assumed to be universal at GUT scale, due to their different evolution down to low energy.
2. A contribution comes from the gauge piece of the scalar potential once the doublets $H_{u,d}$ get VEVs.
3. The supersymmetric contribution, which is equal to the mass squared of the respective fermionic partners. This contribution is essentially negligible except for the quarks of the third generation.
4. A contribution due to the soft breaking trilinear terms, after the Higgs doublets acquire VEVs. In most of SUSY breaking scenarios, this term is proportional to the product of the trilinear coupling A and the mass of the fermionic partner. Therefore, it is negligible for the first two generations. This mass contribution mixes left-handed with right-handed scalars.

We will show in detail the origin of each element of squarks-mass matrix. The complete scalar potential is given by

$$V = V_F + V_D + V_{\text{soft}}, \quad (1.5.72)$$

where

$$V_F = |Y_u \tilde{u}^c \tilde{u} + \mu H_d|^2 + |Y_u \tilde{q} H_u|^2 + \dots \quad (1.5.73)$$

$$V_D = \frac{1}{2} g^2 \left(\frac{1}{2} \tilde{u}_L^* \tilde{u}_L + \frac{1}{2} H_u^{0*} H_u^0 - \frac{1}{2} H_d^{0*} H_d^0 + \dots \right)^2$$

$$+\frac{1}{2}g'^2\left(\frac{1}{6}\tilde{u}_L^*\tilde{u}_L-\frac{2}{3}\tilde{u}_R^*\tilde{u}_R-\frac{1}{2}H_u^{0*}H_u^0+\frac{1}{2}H_d^{0*}H_d^0+\dots\right)^2. \quad (1.5.74)$$

So we can distinguish two cases, the mass terms arise from:

(i) The supersymmetric mass terms arising from V_F and V_D terms. For up-type squarks and after gauge symmetry breaking we get the follows

$$\begin{aligned} V_F &= -v_d(\mu^*Y_u\tilde{u}_R^*\tilde{u}_L+\mu Y_u^*\tilde{u}_L^*\tilde{u}_R)+v_u^2Y_u^2\tilde{u}_L^*\tilde{u}_L+v_u^2Y_u^2\tilde{u}_R^*\tilde{u}_R+\dots \\ &= -m_u\cot\beta(\mu^*\tilde{u}_R^*\tilde{u}_L+\mu\tilde{u}_L^*\tilde{u}_R)+m_u^2\tilde{u}_L^*\tilde{u}_L+m_u^2\tilde{u}_R^*\tilde{u}_R+\dots \end{aligned} \quad (1.5.75)$$

$$\begin{aligned} V_D &= -\frac{1}{2}g'^2\left(\frac{1}{6}(v_u^2-v_d^2)\tilde{u}_L\tilde{u}_L^*-\frac{2}{3}(v_u^2-v_d^2)\tilde{u}_R^*\tilde{u}_R+c.c.\dots\right) \\ &\quad +\frac{1}{2}g^2\left(\frac{1}{2}(v_u^2-v_d^2)\tilde{u}_L\tilde{u}_L^*+c.c+\dots\right) \\ &= \frac{1}{2}(v_u^2-v_d^2)\left(\frac{1}{2}g^2-\frac{1}{6}g'^2\right)\tilde{u}_L^*\tilde{u}_L+\frac{1}{3}g'^2(v_u^2-v_d^2)\tilde{u}_R^*\tilde{u}_R+c.c+\dots \\ &= -\frac{1}{6}\cos 2\beta(m_Z^2-4m_W^2)\tilde{u}_L^*\tilde{u}_L+\frac{2}{3}m_Z^2\cos 2\beta\sin\theta_W\tilde{u}_R^*\tilde{u}_R+c.c+\dots \end{aligned} \quad (1.5.76)$$

thus, we can write

$$\mathcal{L}_{mass}^{D-F} = -\begin{pmatrix} \tilde{u}_L^* & \tilde{u}_R^* \end{pmatrix} M_{D-F}^2 \begin{pmatrix} \tilde{u}_L \\ \tilde{u}_R \end{pmatrix}, \quad (1.5.77)$$

where

$$M_{D-F}^2 = \begin{pmatrix} v_u^2Y_u^2-\frac{1}{6}\cos 2\beta(m_Z^2-4m_W^2) & -v_d\mu Y_u^* \\ -v_d\mu^*Y_u & v_u^2Y_u^2+\frac{2}{3}m_Z^2\cos 2\beta\sin\theta_W \end{pmatrix} \quad (1.5.78)$$

(ii) The soft supersymmetry-breaking scalar mass terms is

$$\begin{aligned} \mathcal{L}_{mass}^{soft} &= -m_{\tilde{q}}^2\tilde{u}_L^*\tilde{u}_L-m_{\tilde{u}_R}^2\tilde{u}_R^*\tilde{u}_R-v_u(Y_u^*A_u^*\tilde{u}_L^*\tilde{u}_R+Y_uA_u\tilde{u}_R\tilde{u}_L^*) \\ &= -\begin{pmatrix} \tilde{u}_L^* & \tilde{u}_R^* \end{pmatrix} M_{soft}^2 \begin{pmatrix} \tilde{u}_L \\ \tilde{u}_R \end{pmatrix}, \end{aligned} \quad (1.5.79)$$

with

$$M_{soft}^2 = \begin{pmatrix} m_{\tilde{q}}^2 & v_uY_uA_u^* \\ v_uY_uA_u & m_{\tilde{u}_R}^2 \end{pmatrix}, \quad (1.5.80)$$

therefore, the total mass matrix of up-type squarks is

$$\begin{aligned} M_{\tilde{u}}^2 &= M_{soft}^2 + M_{D-F}^2 \\ &= \begin{pmatrix} m_{\tilde{q}}^2 + v_u^2 Y_u^2 - \frac{1}{6} \cos 2\beta (m_Z^2 - 4m_W^2) & Y_u^* (v_u A_u^* - v_d \mu) \\ Y_u (v_u A_u - v_d \mu^*) & m_{\tilde{u}_R}^2 + v_u^2 Y_u^2 + \frac{2}{3} m_Z^2 \cos 2\beta \sin \theta_W \end{pmatrix}, \end{aligned} \quad (1.5.81)$$

by a similar way, we can write the mass matrix of down-type squarks

$$M_{\tilde{d}}^2 = \begin{pmatrix} m_{\tilde{q}}^2 + v_d^2 Y_d^2 - \frac{1}{6} \cos 2\beta (2m_W^2 + m_Z^2) & Y_d^* (v_d A_d^* - v_u \mu) \\ Y_d (v_d A_d - v_u \mu^*) & m_{\tilde{u}_R}^2 + v_d^2 Y_d^2 + \frac{2}{3} m_Z^2 \cos 2\beta \sin \theta_W \end{pmatrix}. \quad (1.5.82)$$

It is necessary to work in the mass eigenstate bases of quarks and squarks. For the quarks, mass eigenstates are related to the gauge eigenstates through 3×3 unitary matrices $V_{L,R}^{u,d}$ as follows:

$$\begin{aligned} u' &= V_L^u u + V_R^u C \bar{u}^c{}^T, \\ d' &= V_L^d d + V_R^d C \bar{d}^c{}^T, \end{aligned} \quad (1.5.83)$$

and thus, their diagonal 3×3 mass matrices are related to the Yukawa couplings via

$$m_u = v \sin \beta V_R^u Y_u^T V_L^{u\dagger}, \quad m_d = v \cos \beta V_R^d Y_d^T V_L^{d\dagger}. \quad (1.5.84)$$

The Kobayashi-Maskawa matrix (V_{CKM}) is defined as in the SM and so we have $V_{CKM} = V_L^u V_L^{d\dagger}$. In the same way, the squarks are rotated parallel to their corresponding quarks forming the so-called “super-CKM” basis

$$\tilde{U} = \begin{pmatrix} V_L^u \tilde{u} \\ V_R^u \tilde{u}^c{}^* \end{pmatrix}, \quad \tilde{D} = \begin{pmatrix} V_L^d \tilde{d} \\ V_R^d \tilde{d}^c{}^* \end{pmatrix}. \quad (1.5.85)$$

Hence, the mass matrix given in Eqs (1.5.3.3) and (1.5.82) take the following form:

$$\mathcal{M}_{\tilde{U}}^2 = \begin{pmatrix} (M_{\tilde{u}}^2)_{LL} + m_u^2 - \frac{\cos 2\beta}{6} (M_Z^2 - 4M_W^2) & (M_{\tilde{u}}^2)_{LR} - \cot \beta \mu m_u \\ (M_{\tilde{u}}^2)_{LR}^\dagger - \cot \beta \mu^* m_u & (M_{\tilde{u}}^2)_{RR} + m_u^2 + \frac{2 \cos 2\beta}{3} M_Z^2 \sin^2 \theta_W \end{pmatrix} \quad (1.5.86)$$

$$\mathcal{M}_{\tilde{d}}^2 = \begin{pmatrix} (M_{\tilde{d}}^2)_{LL} + m_d^2 - \frac{\cos 2\beta}{6} (M_Z^2 + 2M_W^2) & (M_{\tilde{d}}^2)_{LR} - \tan \beta \mu m_d \\ (M_{\tilde{d}}^2)_{LR}^\dagger - \tan \beta \mu^* m_d & (M_{\tilde{d}}^2)_{RR} + m_d^2 - \frac{\cos 2\beta}{3} M_Z^2 \sin^2 \theta_W \end{pmatrix} \quad (1.5.87)$$

where

$$\begin{aligned} (M_{\tilde{u}}^2)_{LL} &= V_L^u M_{\tilde{q}}^2 V_L^{u\dagger} & (M_{\tilde{u}}^2)_{RR} &= V_R^u M_{\tilde{u}^c}^{2T} V_R^{u\dagger} & (M_{\tilde{u}}^2)_{LR} &= v \sin \beta V_L^u A_u^* V_R^{u\dagger} \\ (M_{\tilde{d}}^2)_{LL} &= V_L^d M_{\tilde{q}}^2 V_L^{d\dagger} & (M_{\tilde{d}}^2)_{RR} &= V_R^d M_{\tilde{d}^c}^{2T} V_R^{d\dagger} & (M_{\tilde{d}}^2)_{LR} &= v \cos \beta V_L^d A_d^* V_R^{d\dagger}. \end{aligned} \quad (1.5.88)$$

The mass matrices $M_{\tilde{u}}^2$ and $M_{\tilde{d}}^2$ can be diagonalized to real matrices by two unitary matrices Γ^U and Γ^D ⁴ respectively

$$M_{\tilde{u}}^2 = \Gamma^U M_{\tilde{U}}^2 \Gamma^{U\dagger}, \quad (1.5.89)$$

$$M_{\tilde{d}}^2 = \Gamma^D M_{\tilde{D}}^2 \Gamma^{D\dagger}, \quad (1.5.90)$$

where

$$M_{\tilde{u}}^2 = \text{diag}(m_{\tilde{u}_1}^2, m_{\tilde{u}_2}^2, m_{\tilde{u}_3}^2, m_{\tilde{u}_4}^2, m_{\tilde{u}_5}^2, m_{\tilde{u}_6}^2), \quad (1.5.91)$$

$$M_{\tilde{d}}^2 = \text{diag}(m_{\tilde{d}_1}^2, m_{\tilde{d}_2}^2, m_{\tilde{d}_3}^2, m_{\tilde{d}_4}^2, m_{\tilde{d}_5}^2, m_{\tilde{d}_6}^2). \quad (1.5.92)$$

The squark mass eigenstates $\tilde{u}$ and $\tilde{d}$ are related to $\tilde{U}$ and $\tilde{D}$ respectively via

$$\tilde{u} = \Gamma^U \tilde{U}, \quad \tilde{d} = \Gamma^D \tilde{D}. \quad (1.5.93)$$

In mSUGRA, where the soft SUSY breaking are universal, the mass matrix $M_{\tilde{f}}^2$ is given by [54]

$$\mathcal{M}_{\tilde{f}}^2 = \begin{pmatrix} m_{\tilde{f}}^2 + m_{LL}^2 & m_f X_f \\ m_f X_f & m_{\tilde{f}}^2 + m_{RR}^2 \end{pmatrix}, \quad (1.5.94)$$

with

$$\begin{aligned} m_{LL}^2 &= m_{\tilde{f}_L}^2 + (I_f^{3L} - Q_f s_W^2) M_Z^2 c_{2\beta} \\ m_{RR}^2 &= m_{\tilde{f}_R}^2 + Q_f s_W^2 M_Z^2 c_{2\beta} \\ X_f &= A_f - \mu (\tan \beta)^{-2I_f^{3L}} \end{aligned} \quad (1.5.95)$$

They are diagonalized by 2×2 rotation matrices, which turn the gauge eigenstates $\tilde{f}_L$ and $\tilde{f}_R$ into the mass eigenstates $\tilde{f}_1$ and $\tilde{f}_2$

$$R^{\tilde{f}} = \begin{pmatrix} \cos \theta_f & \sin \theta_f \\ -\sin \theta_f & \cos \theta_f \end{pmatrix}, \quad \sin 2\theta_f = \frac{2m_f X_f}{m_{\tilde{f}_1}^2 - m_{\tilde{f}_2}^2}, \quad \cos 2\theta_f = \frac{m_{LL}^2 - m_{RR}^2}{m_{\tilde{f}_1}^2 - m_{\tilde{f}_2}^2}. \quad (1.5.96)$$

⁴ Also some authors use Z^U and Z^D instead of Γ^U and Γ^D

Therefor the sfermion masses are then given by

$$m_{\tilde{f}_{1,2}}^2 = m_f^2 + \frac{1}{2} \left[m_{LL}^2 + m_{RR}^2 \mp \sqrt{(m_{LL}^2 - m_{RR}^2)^2 + 4m_f^2 X_f^2} \right]. \quad (1.5.97)$$

For $\tilde{t}$ and $\tilde{b}$ the mass matrices are given by

$$M_{\tilde{t}}^2 = \begin{pmatrix} m_{\tilde{q}}^2 + m_t^2 - \frac{\cos 2\beta}{6}(M_Z^2 - 4M_W^2) & m_t(A_t - \mu \cot \beta) \\ m_t(A_t - \mu \cot \beta) & m_{\tilde{t}_R}^2 + m_t^2 + \frac{2}{3}M_Z^2 \sin^2 \theta_W \cos 2\beta \end{pmatrix}. \quad (1.5.98)$$

$$M_{\tilde{b}}^2 = \begin{pmatrix} m_{\tilde{q}}^2 + m_b^2 - \frac{\cos 2\beta}{6}(M_Z^2 + 2M_W^2) & m_b(A_b - \mu \tan \beta) \\ m_b(A_b - \mu \tan \beta) & m_{\tilde{b}_R}^2 + m_b^2 - \frac{1}{3}m_Z^2 \sin^2 \theta_W \cos 2\beta \end{pmatrix}, \quad (1.5.99)$$

where eigenvalues are denoted by $m_{\tilde{t}_1(\tilde{b}_1)}^2$ and $m_{\tilde{t}_2(\tilde{b}_2)}^2$, with $m_{\tilde{t}_1(\tilde{b}_1)}^2 < m_{\tilde{t}_2(\tilde{b}_2)}^2$.

The experimental limits on squark masses are rather involved and depends on the effects of cascade decays. Assume GUT relations between gaugino masses, the gauge coupling, $A_0 = 0$, $\tan \beta = 3$, the following limits are reported for $M_{\tilde{q}}$, the degenerate mass of all squarks except the stop and sbottom:

$$M_{\tilde{q}} > 379 \text{ GeV}. \quad (1.5.100)$$

The limits on the stop and sbottom mass are given by

$$M_{\tilde{t}} > 95.7 \text{ GeV}, \quad M_{\tilde{b}} > 89 \text{ GeV}. \quad (1.5.101)$$

As in the squarks case, the slepton and sneutrino mass matrices take the form

$$\begin{aligned} \mathcal{M}_{\tilde{L}}^2 &= \begin{pmatrix} (M_L^2)_{LL} + m_l^2 + \frac{\cos 2\beta}{2}(M_Z^2 - 2M_W^2) & (M_L^2)_{LR} - \tan \beta \mu m_l \\ (M_L^2)_{LR}^\dagger - \tan \beta \mu^* m_l & (M_L^2)_{RR} + m_l^2 - \cos 2\beta M_Z^2 \sin^2 \theta_W \end{pmatrix} \\ \mathcal{M}_{\tilde{N}}^2 &= V_L^e M_{\tilde{\ell}}^2 V_L^{e\dagger} + \frac{\cos 2\beta}{2} M_Z^2, \end{aligned} \quad (1.5.102)$$

where

$$(M_L^2)_{LL} = V_L^e M_{\tilde{\ell}}^2 V_L^{e\dagger} \quad (M_L^2)_{RR} = V_R^e M_{\tilde{e}}^2 V_R^{e\dagger} \quad (M_L^2)_{LR} = v \cos \beta V_L^e A_e^* V_R^{e\dagger}. \quad (1.5.103)$$

The two matrices $\mathcal{M}_L^2$ and $\mathcal{M}_N^2$ can be diagonalized by two unitary matrices $\Gamma^{N,E}$

$$\begin{aligned} M_\nu^2 &= \Gamma^N M_N^2 \Gamma^{N\dagger}, \\ M_e^2 &= \Gamma^E M_L^2 \Gamma^{E\dagger}. \end{aligned} \quad (1.5.104)$$

Also in mSUGRA, the stau mass matrix is given by

$$\mathcal{M}_\tau^2 = \begin{pmatrix} M_\ell^2 + m_\tau^2 + \frac{\cos 2\beta}{2}(M_Z^2 - 2M_W^2) & m_\tau(A_\tau - \mu \tan \beta) \\ m_\tau(A_\tau - \mu \tan \beta) & M_{e^c}^2 + m_\tau^2 - \cos 2\beta M_Z^2 \sin^2 \theta_W \end{pmatrix}. \quad (1.5.105)$$

The current experimental constraints on the slepton masses are given by

$$M_{\tilde{e}} > 107 \text{ GeV}, \quad M_{\tilde{\mu}} > 94 \text{ GeV}, \quad (1.5.106)$$

$$M_{\tilde{\tau}} > 81.9 \text{ GeV}, \quad M_{\tilde{\nu}} > 45 \text{ GeV}. \quad (1.5.107)$$

1.5.3.4 Higgs Sector

The Higgs sector of the MSSM contains two Higgs doublets. There are 2 CP-even Higgs bosons, 1 CP-odd and 2 charged Higgs particles. The mass matrix for the CP-even Higgs particles is given by

$$\mathcal{M}_R^2 = \begin{pmatrix} -m_3^2 \tan \beta + M_Z^2 \cos^2 \beta & m_3^2 - M_Z^2 \sin \beta \cos \beta \\ m_3^2 - M_Z^2 \sin \beta \cos \beta & -m_3^2 \cot \beta + M_Z^2 \sin^2 \beta \end{pmatrix}. \quad (1.5.108)$$

After diagonalizing the $\mathcal{M}_R^2$ by the rotation matrix

$$\begin{pmatrix} H \\ h \end{pmatrix} = \begin{pmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} H_1^0 \\ H_2^0 \end{pmatrix}, \quad (1.5.109)$$

we get the light and heavy neutral Higgs boson masses

$$M_{h,H}^2 = \frac{1}{2} \left(M_A^2 + M_Z^2 \mp \sqrt{(M_A^2 + M_Z^2)^2 - 4M_A^2 M_Z^2 \cos^2 2\beta} \right). \quad (1.5.110)$$

While the CP-odd Higgs mass squared is given by

$$M_A^2 = -m_3^2 (\tan \beta + \cot \beta) = -\frac{2m_3^2}{\sin 2\beta}. \quad (1.5.111)$$

In the case of the charged Higgs boson, the mass matrix can be written as

$$M_{H^\pm} = \begin{pmatrix} m_1^2 + \frac{1}{4}g_1^2(v_1^2 - v_2^2) + \frac{1}{4}g_2^2(v_1^2 + v_2^2) & -m_{12}^2 - \frac{1}{2}g_2^2 v_1 v_2 \\ -m_{12}^2 - \frac{1}{2}g_2^2 v_1 v_2 & m_2^2 + \frac{1}{4}g_1^2(v_2^2 - v_1^2) + \frac{1}{4}g_2^2(v_1^2 + v_2^2) \end{pmatrix}, \quad (1.5.112)$$

where m_1, m_2, m_{12} denote the soft SUSY-breaking parameters in the Higgs sector, and g_1, g_2 are the $U(1)$ and $SU(2)$ gauge couplings, respectively. The mass eigenstates $H^\pm$ and $G^\pm$ are related to the initial Higgs fields by the rotation matrix Z_H

$$\begin{pmatrix} H^\pm \\ G^\pm \end{pmatrix} = Z^H \begin{pmatrix} H_1^\pm \\ H_2^\pm \end{pmatrix}, \quad (1.5.113)$$

$$Z_H = \begin{pmatrix} -\sin \beta & \cos \beta \\ \cos \beta & \sin \beta \end{pmatrix}. \quad (1.5.114)$$

These charged scalars mass matrix has two eigenvalues 0 and $m_W^2 + m_A^2$. The massless state corresponds to the superposition

$$G^\pm = \cos \beta H_1^\pm + \sin \beta H_2^\pm, \quad (1.5.115)$$

which are eaten by $W^\pm$ bosons and disappear from the Lagrangian. The massive state is

$$H^\pm = \cos \beta \phi_2^\pm - \sin \beta \phi_1^\pm, \quad (1.5.116)$$

where

$$m_{H^\pm}^2 = m_W^2 + m_A^2. \quad (1.5.117)$$

1.6 SUSY searches at the LHC

SUSY has been used extensively in many modern developments in mathematical and theoretical physics. It is important to understand that, for now, SUSY is a purely mathematical concept. We do not know if it has a role to play in the description of nature, if it exists in our observed universe at TeV scale, then SUSY particles

should be discovered at the Large Hadron Collider (LHC) at the Centre Européen de Recherche Nucleaire (CERN), otherwise it will be just a mathematical theory without relation to the reality. The summary of the search results of LHC Run1 and Run2 at $\sqrt{s} = 7, 8, 13$ TeV, collected by the ATLAS experiment in 2017, is given in Fig.1.1. The results of CMS at $\sqrt{s} = 7, 8$ are given in Fig.1.2. The second Run of the LHC at 13 TeV, shows the exclusion limits of SUSY masses at 95% CL ,*e.g.* the light chargino and neutralino ~ 1 TeV while the masses of the gluinos have a lower bound 2 TeV. In the near future LHC offers the exciting possibility of finally testing whether SUSY plays a role in the fundamental laws of physics.

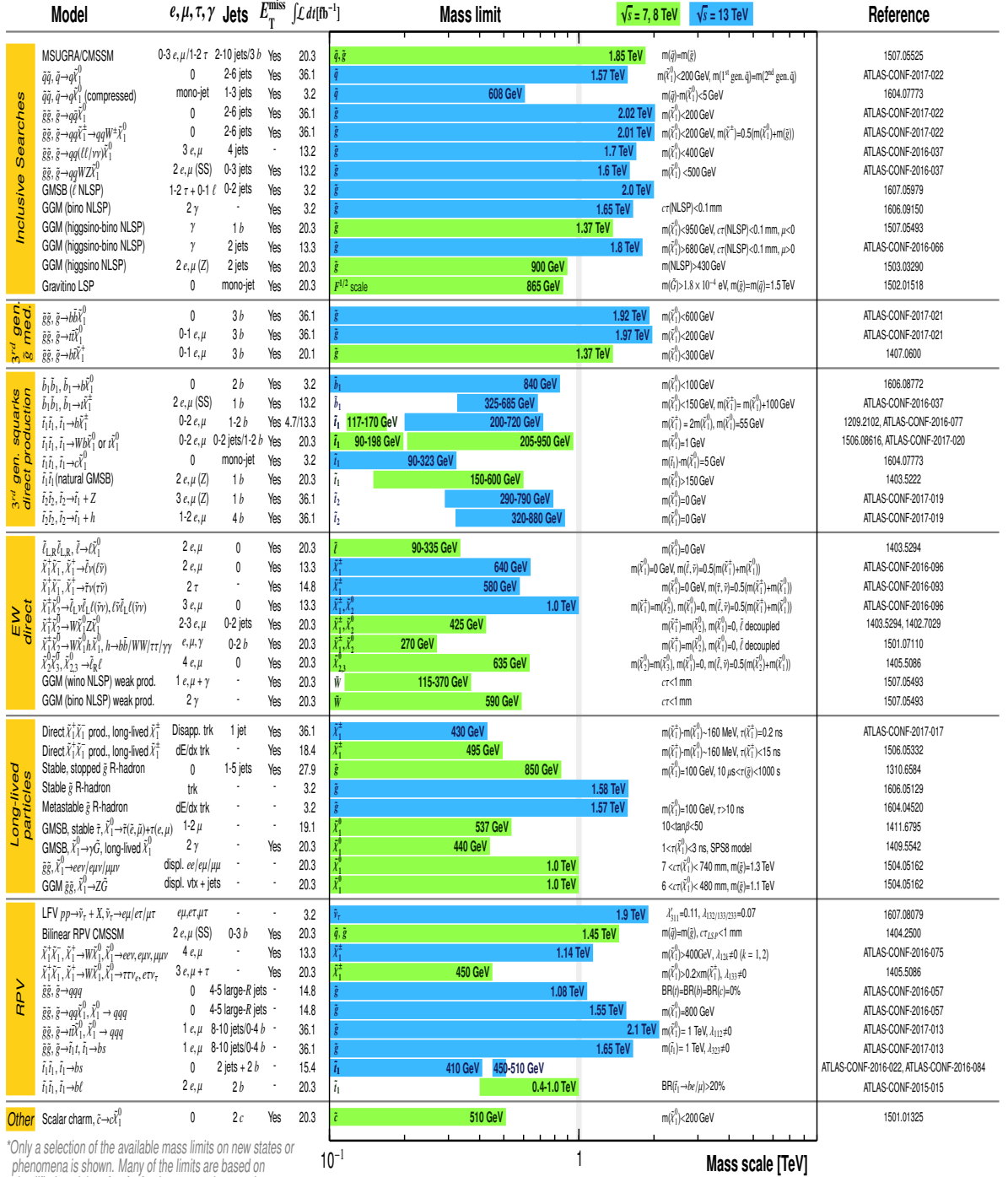
1.6 SUSY searches at the LHC

ATLAS SUSY Searches* - 95% CL Lower Limits

Status: March 2017

ATLAS Preliminary

$\sqrt{s} = 7, 8, 13 \text{ TeV}$



*Only a selection of the available mass limits on new states or phenomena is shown. Many of the limits are based on simplified models, c.f. refs. for the assumptions made.

Figure 1.1: ATLAS summary of SUSY particles search and limits [60]

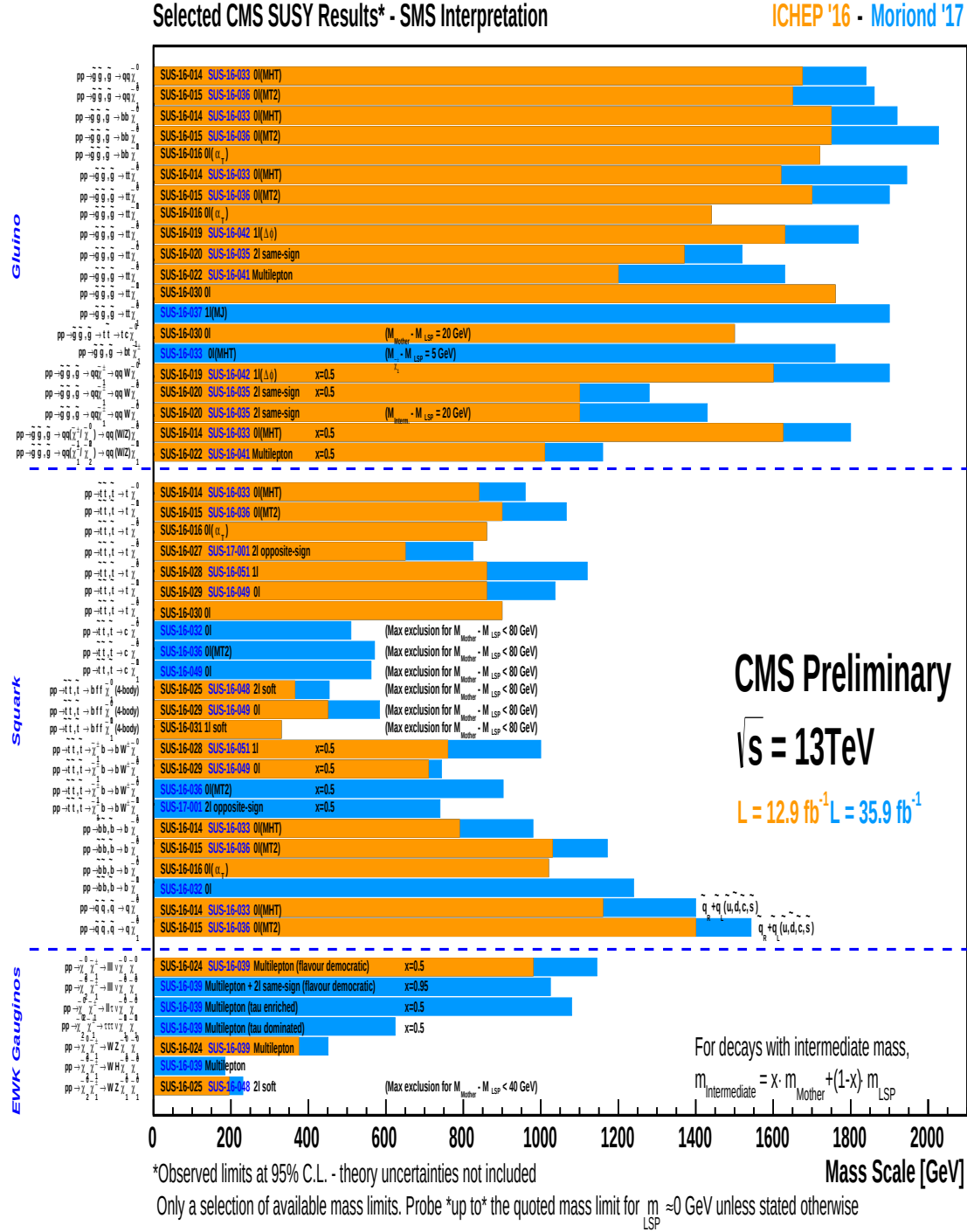


Figure 1.2: CMS summary of SUSY particles search and limits 2017 [61]

2

CP Violation in Supersymmetric Models

All measurements of CP violation to date are consistent with the predictions of SM. In fact, it is now established that the Kobayashi-Maskawa (KM) mechanism plays a major role in the CP violation measured in the quark sector. However, the amount of CP violation present in the SM is not sufficient to explain the matter-antimatter asymmetry observed in our Universe. Therefore, the universe requires additional sources of CP violation, and such sources are generated by extensions to the SM. SUSY predicts new sources of CP violation. Therefore, probing any deviation from the SM expectations for CP violating processes at LHCb would be a clear hint for SUSY.

2.1 CP Transformation

The C transformation (charge transformation) changes particles to its anti-particles and vice versa: *e.g.* $C|p\rangle = |\bar{p}\rangle$, $C|e^-\rangle = |e^+\rangle$, $C|\pi^-\rangle = |\pi^+\rangle$. It inverts all discrete additive quantum numbers of particles: Baryon number, Lepton number, Isospin,

Strangeness, Charm, etc., as well as Charge, but leaves particles spins ($\vec{s}$) and momenta ($\vec{p}$) unchanged, that is

$$C|N, \vec{p}, \vec{s}\rangle = \eta_C|-N, \vec{p}, \vec{s}\rangle, \quad (2.1.1)$$

where η_C refers to arbitrary phase. If the interaction Hamiltonian conserves the charge transformation, the operator C must satisfy $[C, \mathcal{H}] = 0$. But we know that C is not a good symmetry of Nature. Therefore, there is no operator C which represents correctly the C transformation.

The P transformation (parity transformation) is equivalent to a spatial point reflection. It changes the sign of the coordinate system and thus a right-handed coordinate system into a left-handed one or vice versa, a mirror reflection. We have

$$P|N, \vec{p}, \vec{s}\rangle = \eta_P|N, -\vec{p}, -\vec{s}\rangle, \quad (2.1.2)$$

where η_P is a phase factor which is identified with the intrinsic parity. In quantum theory, $[P, \mathcal{H}] = 0$ means that parity is a good symmetry of Nature. This contradicts with what we will see later, that parity is not a symmetry of the weak interactions.

The CP transformation combine charge conjugate C with parity P . Namely, the conceptual problem of distinguishing matter from antimatter can only acquire a solution if we are able to eliminate the convention of what is left and what is right from the particle physics world. If CP were an exact symmetry, the laws of Nature would be the same for matter and for antimatter. In fact, the most phenomena, the gravitational, electromagnetic and strong interactions, are symmetric under C and P transformations, and therefore CP-symmetric. While the weak interactions violate C and P .

Let us give an example of this phenomena. The charged pion π^+ decays predominantly to $\mu^+\nu_\mu$. The muon neutrino from the decay is left-handed (Fig 2.1.a). If we take the C -conjugate process, the decay is $\pi^- \rightarrow \mu^-\bar{\nu}_\mu$, and we get a left-handed

antineutrino $\bar{\nu}_{\mu L}$ (Fig 2.1.c). The problem here is that left-handed antineutrinos have never been observed, and thus C-symmetry is violated. If we flip, i.e. mirror, the process, we get a right-handed neutrino $\nu_{\mu L}$, which never occurs (no right-handed neutrinos have been observed). This violates P-symmetry (Fig 2.1.b). This leads

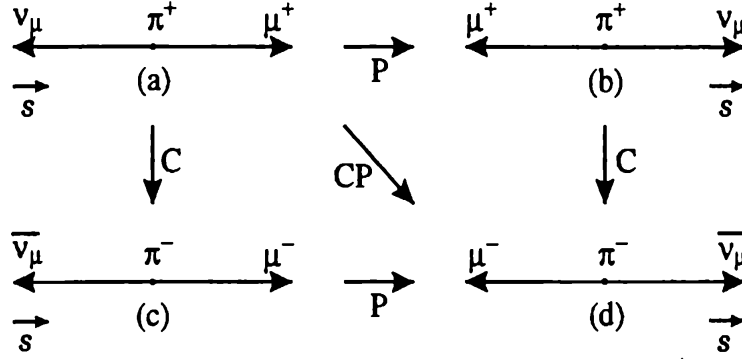


Figure 2.1: C, P, and CP transformation of the decay $\pi^+ \rightarrow \mu^+ \nu_\mu$.

to the fact that both of C and of P are violated¹. P is violated because the neutrinos have a definite handedness. If P were conserved, the neutrinos should be either left-handed or right-handed, both in $\pi^+ \rightarrow \mu^+ \nu_\mu$ and in $\pi^- \rightarrow \mu^- \bar{\nu}_\mu$. C is violated because the neutrinos in the two observed decays have opposite handedness, and therefore the decays are not C-conjugates of each other.

On the other hand, if we take both the P and C conjugation of the process, we get a right-handed antineutrino (Fig 2.1.d). So, when we simultaneously interchange $\pi^+ \leftrightarrow \pi^-$, $\mu^+ \leftrightarrow \mu^-$ and $\nu_\mu \leftrightarrow \bar{\nu}_\mu$ (C transformation), and also interchange left and right-handedness of the neutrinos and of the muons (P transformation), the decay rates are equal. This means that C and P are violated, but the combined symmetry CP is not.

¹For a general and theoretical treatment, see [62]

2.2 CP Violation in Meson Decay

Before discussing the theoretical formalism and experimental measurement, we first introduce some notations and formalism. We consider the general case of $M^0 - \bar{M}^0$ mixing, where M^0 is a meson, with CP transformation defined by

$$\text{CP}|M^0\rangle = e^{i\xi_M}|\bar{M}^0\rangle, \quad (2.2.1)$$

$$\text{CP}|\bar{M}^0\rangle = e^{-i\xi_M}|M^0\rangle, \quad (2.2.2)$$

where ξ_M is the CP transformation phase. This corresponds to a quark basis

$$|M^0\rangle \equiv |q_L, \bar{q}_H\rangle \quad \text{and} \quad |\bar{M}^0\rangle \equiv |q_H, \bar{q}_L\rangle, \quad (2.2.3)$$

where q_L and q_H refer to light and heavy quarks respectively, it has the advantage that all phases appearing in the matrix elements arise from KM phase.

The time evolution of a general combination of the two states

$$|\psi(t)\rangle = a(t)|M^0\rangle + b(t)|\bar{M}^0\rangle, \quad (2.2.4)$$

is described by Schrödinger equation

$$i\frac{\partial}{\partial t}|\psi(t)\rangle = H_{\text{eff}}|\psi(t)\rangle, \quad (2.2.5)$$

where H_{eff} is the effective Hamiltonian for the process. From Eqs. (2.2.4) and (2.2.5), one write the Schrödinger equation in the form

$$i\frac{\partial}{\partial t} \begin{pmatrix} a(t) \\ b(t) \end{pmatrix} = H_{\text{eff}} \begin{pmatrix} a(t) \\ b(t) \end{pmatrix} = \left(M - \frac{i}{2}\Gamma \right) \begin{pmatrix} a(t) \\ b(t) \end{pmatrix}, \quad (2.2.6)$$

where

$$H_{\text{eff}} = \begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix} = \begin{pmatrix} M_{11} - \frac{i}{2}\Gamma_{11} & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{21} - \frac{i}{2}\Gamma_{21} & M_{22} - \frac{i}{2}\Gamma_{22} \end{pmatrix}, \quad (2.2.7)$$

where M and Γ are hermitian matrices, such that, M is the mass matrix and Γ is the decay matrix. They are associated with $(M^0, \bar{M}^0) \leftrightarrow (M^0, \bar{M}^0)$.

2.2 CP Violation in Meson Decay

Diagonal elements of M and Γ are associated with flavor-conserving transition $M^0 \rightarrow M^0$ and $\bar{M}^0 \rightarrow \bar{M}^0$, while off-diagonal elements are associated with flavor-changing transitions $M^0 \leftrightarrow \bar{M}^0$. Furthermore CPT invariance implies that

$$M_{11} = M_{22} \quad \text{and} \quad \Gamma_{11} = \Gamma_{22}, \quad (2.2.8)$$

where M_{11} and Γ_{11} are real. If CP is conserved, then the off-diagonal elements would be equal

$$M_{12} = M_{21}^* = M_{12}^* \quad \text{and} \quad \Gamma_{12} = \Gamma_{21} = \Gamma_{12}^*. \quad (2.2.9)$$

Diagonalising the matrix H gives the eigenvalues

$$\lambda_{1,2} = H_{11} \pm \sqrt{H_{12}H_{21}}, \quad (2.2.10)$$

where the mass and decay rates of the two eigenstates are given by

$$m_{1,2} = m \pm \frac{1}{2}\Delta m = \Re\{H_{11} \pm \sqrt{H_{12}H_{21}}\}, \quad (2.2.11)$$

$$\Gamma_{1,2} = \Gamma \pm \frac{1}{2}\Delta\Gamma = -\Im\{H_{11} \pm \sqrt{H_{12}H_{21}}\}. \quad (2.2.12)$$

To specify the eigenstates, M^0 and $\bar{M}^0$, in the light M_1^0 and heavy M_2^0 mass eigenstates, we introduce two complex parameters, p , q , we write

$$|M_{1,2}^0\rangle = p|M^0\rangle \pm q|\bar{M}^0\rangle, \quad (2.2.13)$$

where the complex coefficients, p and q , obey the normalization condition $|p|^2 + |q|^2 = 1$, so the eigenstates of the effective Hamiltonian can be written as

$$|M_{1,2}^0\rangle = \frac{1}{\sqrt{|p|^2 + |q|^2}} (p|M^0\rangle \pm q|\bar{M}^0\rangle). \quad (2.2.14)$$

It is convenient to write the effective Hamiltonian eigenstates, M_L^0 and M_H^0 , in the CP eigenstates, M_+^0 and M_-^0 , as follows

$$|M_{1,2}^0\rangle = \frac{1}{\sqrt{2(1 + |\eta|^2)}} ((1 + \eta)|M^0\rangle \pm (1 - \eta)|\bar{M}^0\rangle) \quad (2.2.15)$$

$$= \frac{1}{\sqrt{(1+|\eta|^2)}} (|M_{\mp}^0\rangle + \eta|\bar{M}_{\pm}^0\rangle), \quad (2.2.16)$$

where

$$|M_{\pm}^0\rangle = \frac{1}{\sqrt{2}} (|M^0\rangle \mp |\bar{M}^0\rangle), \quad (2.2.17)$$

are the CP eigenstates

$$CP|M_{\pm}^0\rangle = \pm|M_{\pm}^0\rangle, \quad (2.2.18)$$

and

$$\begin{aligned} \eta &= \frac{\sqrt{H_{12}} - \sqrt{H_{21}}}{\sqrt{H_{12}} + \sqrt{H_{21}}} = \frac{1 - \sqrt{H_{21}/H_{12}}}{1 + \sqrt{H_{21}/H_{12}}} \\ &= \frac{1 - q/p}{1 + q/p} = \frac{p - q}{p + q}, \end{aligned} \quad (2.2.19)$$

where

$$\frac{q}{p} = \sqrt{\frac{H_{21}}{H_{12}}} = \sqrt{\frac{H_{12}^*}{H_{12}}} = \sqrt{\frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}}. \quad (2.2.20)$$

On the other hand, the mass difference and the width difference between the physical states are given by

$$\begin{aligned} \Delta m &= m_1 - m_2 = 2\Re \left[\sqrt{H_{12}H_{12}^*} \right] \\ &= 2\Re \left[\sqrt{(M_{12} - \frac{i}{2}\Gamma_{12})(M_{12}^* - \frac{i}{2}\Gamma_{12}^*)} \right], \end{aligned} \quad (2.2.21)$$

$$\begin{aligned} \Delta\Gamma &= \Gamma_1 - \Gamma_2 = -4\Im \{ \sqrt{H_{12}H_{12}^*} \} \\ &= -4\Im \left[\sqrt{(M_{12} - \frac{i}{2}\Gamma_{12})(M_{12}^* - \frac{i}{2}\Gamma_{12}^*)} \right]. \end{aligned} \quad (2.2.22)$$

In these terms, it is convenient to rewrite the Eq (2.2.20) as [63, 64]

$$\frac{q}{p} = \frac{2M_{12}^* - i\Gamma_{12}^*}{\Delta m - \frac{i}{2}\Delta\Gamma_{12}}. \quad (2.2.23)$$

Also, the Eqs. (2.2.11) and (2.2.12) can be written as

$$m_{1,2} = m \pm \Re \left[\sqrt{(M_{12} - \frac{i}{2}\Gamma_{12})(M_{12}^* - \frac{i}{2}\Gamma_{12}^*)} \right], \quad (2.2.24)$$

$$\Gamma_{1,2} = \Gamma \mp 2\Im \left[\sqrt{(M_{12} - \frac{i}{2}\Gamma_{12})(M_{12}^* - \frac{i}{2}\Gamma_{12}^*)} \right]. \quad (2.2.25)$$

To discuss CP violation in mixing, it is useful to write Eq. (2.2.23) to first order in $|\Gamma_{12}/M_{12}|$ [63]:

$$\frac{q}{p} = \frac{M_{12}^*}{|M_{12}|} \left[1 - \frac{1}{2}\Im \left(\frac{\Gamma_{12}}{M_{12}} \right) \right]. \quad (2.2.26)$$

If CP were conserved, then $p = q$ giving $\eta = 0$ from Eq (2.2.19) which corresponds to $\Im \left[\frac{\Gamma_{12}}{M_{12}} \right] = 0$. In this case, the mass eigenstates are orthogonal

$$\langle M_1^0 | M_2^0 \rangle = |p|^2 - |q|^2 = 0. \quad (2.2.27)$$

and so $|M_1^0\rangle$ would be CP odd and $|M_2^0\rangle$ would be CP even.

2.3 Classification of CP violation

We define decay amplitudes of the meson M and its CP conjugate $\bar{M}$ to final state f (multi-particle final state) and to the CP-conjugate decay channel $\bar{f}$ as

$$A_f = \langle f | \mathcal{H} | M \rangle, \quad \bar{A}_f = \langle f | \mathcal{H} | \bar{M} \rangle, \quad (2.3.1)$$

$$A_{\bar{f}} = \langle \bar{f} | \mathcal{H} | M \rangle, \quad \bar{A}_{\bar{f}} = \langle \bar{f} | \mathcal{H} | \bar{M} \rangle, \quad (2.3.2)$$

where $\mathcal{H}$ is the weak interaction Hamiltonian. As we saw in the previous section, the action of CP violation on these states introduces phases ξ_M and ξ_f that relate the decay amplitudes A_f and $\bar{A}_f$ according to

$$\text{CP} |M\rangle = e^{i\xi_M} |\bar{M}\rangle, \quad \text{CP} |f\rangle = e^{i\xi_f} |\bar{f}\rangle, \quad (2.3.3)$$

$$\text{CP} |\bar{M}\rangle = e^{-i\xi_M} |M\rangle, \quad \text{CP} |\bar{f}\rangle = e^{-i\xi_f} |f\rangle, \quad (2.3.4)$$

which leads to the CP-invariance conditions for the decay amplitudes

$$\bar{A}_{\bar{f}} = e^{\xi_f - \xi_M} A_f, \quad A_{\bar{f}} = e^{\xi_f + \xi_M} \bar{A}_f. \quad (2.3.5)$$

This implies that A_f and $\bar{A}_{\bar{f}}$ have the same magnitude and arbitrary relative phase, where the arbitrary phases should be eliminated. Thus from Eq (2.3.5), the condition for CP conservation in the decay amplitudes is

$$|A_f| = |\bar{A}_{\bar{f}}|, \quad |A_{\bar{f}}| = |\bar{A}_f|. \quad (2.3.6)$$

This means that the probabilities of the decays of M^0 to f and $\bar{M}^0$ to $\bar{f}$ must be equal. In this regards, we distinguish three types of CP violation in the quark sector:

I. CP violation in decay (direct CP violation): This type of CP violation occurs when the decay rate of a M to a final state f differs from the decay rate of an $\bar{M}$ to the CP-conjugated final state $\bar{f}$:

$$\Gamma(M \rightarrow f) \neq \Gamma(\bar{M} \rightarrow \bar{f}), \quad (2.3.7)$$

this is satisfied when

$$|\bar{A}_{\bar{f}}/A_f| \neq 1. \quad (2.3.8)$$

This shows up from a clash between the phases of two interfering decay amplitudes in the total decay amplitude, and requires at least two terms have different weak phases and different strong phases in addition to the spurious CP transformation in Eq (2.3.5). Those spurious phases are due to arbitrary choice of phase convention, and do not originate from any dynamics or induce any CP violation, they can be set to zero for simplicity [62, 65].

Weak phases change sign under CP, *i.e.* there is CP-odd phase in an amplitude. Consider, for instance, the transitions of i and $\bar{i}$ to f and $\bar{f}$, then the transition amplitudes must be the sum of two or more interfering amplitudes with different weak phases and different strong phases, we have

$$\langle f | CP | i \rangle = A e^{i(\delta + \phi)}, \quad \langle \bar{f} | CP | \bar{i} \rangle = A e^{i(\delta - \phi)}, \quad (2.3.9)$$

2.3 Classification of CP violation

where A is a real positive number, ϕ is a weak (CP-odd) phase, δ is a strong (CP-even) phase. As CP invariance holds when the amplitudes are as in Eq (2.3.9), let us suppose that there are two contribution, $A = A_1 + A_2$, we write

$$A_f = \langle f|CP|i \rangle = |A_1|e^{i(\delta_1+\phi_1)} + |A_2|e^{i(\delta_2+\phi_2)}, \quad (2.3.10)$$

$$\bar{A}_{\bar{f}} = \langle \bar{f}|CP|\bar{i} \rangle = |A_1|e^{i(\delta_1-\phi_1)} + |A_2|e^{i(\delta_2-\phi_2)}. \quad (2.3.11)$$

There are now two interfering amplitudes; they contain CP-even phases δ_1 and δ_2 , and CP-odd phases ϕ_1 and ϕ_2 associated with moduli $|A_1|$ and $|A_2|$, respectively. These two interfering amplitudes may arise from two different Feynman diagrams for the processes.

In order to obtain CP violation, the interfering amplitudes must have different strong phases ($\delta_1 \neq \delta_2$) and different weak phases ($\phi_1 \neq \phi_2$). Thus, from Eqs (2.3.10) and (2.3.11), one defines a more relevant quantity, the CP asymmetry, as

$$\frac{|A_f|^2 - |\bar{A}_{\bar{f}}|^2}{|A_f|^2 + |\bar{A}_{\bar{f}}|^2} = -\frac{2|A_1||A_2|\sin(\delta_2 - \delta_1)\sin(\phi_2 - \phi_1)}{|A_1|^2 + |A_2|^2 + 2|A_1||A_2|\cos(\delta_2 - \delta_1)\cos(\phi_2 - \phi_1)}. \quad (2.3.12)$$

It is now evident that in order for the asymmetry to be large, the difference between the weak phases ($\phi_2 - \phi_1$) and strong phases ($\delta_2 - \delta_1$) should be close to $\pi/2$ in addition to small difference between the moduli of the two interfering amplitudes, i.e., $A_1 \approx A_2$. In the other hand, the weak phase difference ($\phi_2 - \phi_1$) is the most interest quantity in theory. It can be extracted from the asymmetry requires, however, that the amplitude ratio $|A_1/A_2|$ and the strong phase difference ($\phi_2 - \phi_1$) should be known. It is difficult to calculate these quantities since they depend on non-perturbative hadron parameters, but in some cases can be obtained from experiment.

In charged Boson decays [65],

$$A_f^\pm \equiv \frac{\Gamma(M^- \rightarrow f^-) - \Gamma(M^+ \rightarrow f^+)}{\Gamma(M^- \rightarrow f^-) + \Gamma(M^+ \rightarrow f^+)}, \quad (2.3.13)$$

are purely an effect of CP violation in decay. In terms of the decay amplitudes,

$$A_{f^\pm} \equiv \frac{|\bar{A}_{f^-}/A_{f^+}|^2 - 1}{|\bar{A}_{f^-}/A_{f^+}|^2 + 1}. \quad (2.3.14)$$

Note that CP violation in decay has been observed in the neutral K system ($\Re \varepsilon'_K \neq 0$).

II. CP violation in mixing (indirect CP violaion): In this case the oscillation from meson to anti-meson ($M^0 \rightarrow \bar{M}^0$) is different from the oscillation from anti-meson to meson ($\bar{M}^0 \rightarrow M^0$), i.e., this implies

$$|q/p| \neq 1. \quad (2.3.15)$$

This occurs when the mass eigenstates are not CP eigentates, and requires a relative phase between M_{12} and Γ_{12} , *i.e.* $\Im(M_{12}^* \Gamma_{12}) \neq 0$ [64, 66, 67]. This type of CP violation can be isolated in semi-leptonic meson decays $M, \bar{M} \rightarrow l^\pm X$ (experimentally this is searched for in the semi-leptonic decay of both the B and $\bar{B}^0$, where the $\bar{b}$ -quark inside the B^0 meson decays weakly to a positively charged lepton, and vice versa), this effect could be observed through the asymmetrie of decays induced by mixing [65]:

$$A_{SL}(t) \equiv \frac{d\Gamma/dt[\bar{M}^0(t) \rightarrow l^+ X] - d\Gamma/dt[M^0(t) \rightarrow l^- X]}{d\Gamma/dt[\bar{M}^0(t) \rightarrow l^+ X] + d\Gamma/dt[M^0(t) \rightarrow l^- X]}. \quad (2.3.16)$$

Also, in terms of q and p , we have

$$A_{SL} = \frac{1 - |q/p|^4}{1 + |q/p|^4}. \quad (2.3.17)$$

Using Eq. (2.2.26) and (2.2.23) to calculate the CP asymmetry in semileptonic neutral-meson decays, in Eq. (2.3.16), one can find

$$A_{SL} = -\Im \frac{\Gamma_{12}}{M_{12}} = -\left| \frac{\Gamma_{12}}{M_{12}} \right| \sin(\phi_M - \phi_\Gamma), \quad (2.3.18)$$

where the extraction of the weak phase $\phi_M - \phi_\Gamma$ is much difficult since $\left| \frac{\Gamma_{12}}{M_{12}} \right|$ depends on long distance physics. This involves large hadronic uncertainties, in particular in the hadronization models for Γ_{12} .

Note that CP violation in mixing has been observed in the neutral K system ($\Re \varepsilon_K \neq 0$).

III. CP violation in the interference between decays with and without mixing: For decays of neutral mesons into a CP eigenstate a third type of CP violation is possible in interference between decays with and without mixing, respectively, $M^0 \rightarrow f$ and $M^0 \rightarrow \bar{M}^0 \rightarrow f$, which is defined by [62, 65]

$$\arg(\lambda_f) + \arg(\lambda_{\bar{f}}) \neq 0, \quad (2.3.19)$$

with

$$\lambda_f = \frac{q}{p} \frac{\bar{A}_f}{A_f}. \quad (2.3.20)$$

In this case CP violation arises from a clash between the phase of q/p , Eq. (2.3.15) and the phases of the decay amplitudes, Eq. (2.3.7), lead to $|\lambda_f| \neq 1$,

We now consider the case of decays to a CP eigenstate, *i.e.*, the case $f = \bar{f}$. Then, we only have one direct CP invariance condition:

$$|A_f| = |\bar{A}_f|, \quad (2.3.21)$$

then the condition in Eq. (2.3.19) becomes

$$\arg(\lambda_{f_{CP}}) \neq 0. \quad (2.3.22)$$

Thus, the condition for interference CP violation is

$$\Im \lambda_{f_{CP}} \neq 0. \quad (2.3.23)$$

This effect of CP violation can be observed in neutral meson decays into CP eigenstates through the asymmetry,

$$A_{f_{CP}}(t) \equiv \frac{d\Gamma/dt[\bar{M}_{phys}^0(t) \rightarrow f_{CP}] - d\Gamma/dt[M_{phys}^0(t) \rightarrow f_{CP}]}{d\Gamma/dt[\bar{M}_{phys}^0(t) \rightarrow f_{CP}] + d\Gamma/dt[M_{phys}^0(t) \rightarrow f_{CP}]}. \quad (2.3.24)$$

If CP violation effect in mixing is negligible, $|q/p| = 1$. If, in addition, the decay amplitudes achieve $|\bar{A}_{f_{CP}}| = |A_{f_{CP}}|$, so that the contribution from CP violation in decay is also negligible. Therefore, the interference between decays with and without

mixing is the only source of asymmetry. For decays that are dominated by a single weak phase, $A_f = |A_f|e^{i\phi_f}$, and $q/p = |q/p|e^{i\phi_M}$, we obtain for a final CP eigenstate,

$$\lambda_{CP} = \eta_{f_{CP}} e^{i(\phi_M + 2\phi_f)}, \quad (2.3.25)$$

where $\eta_{f_{CP}} = \pm 1$ is the CP eigenstate. In this regards, the CP asymmetries in decays to a final CP eigenvalue f are given by

$$A_{f_{CP}}(t) = \text{Im}(\lambda_{f_{CP}}) \sin(\Delta m_M t), \quad (2.3.26)$$

$$\text{Im}(\lambda_{f_{CP}}) = \eta_{f_{CP}} \sin(\phi_M + 2\phi_f). \quad (2.3.27)$$

Note that CP violation in the interference of decays with and without mixing has been observed for the neutral K system ($\text{Im}(\varepsilon_K) \neq 0$).

2.4 Recent Measurements of CP Violating Observables

In this section we list the recent measured observables where CP violation has been observed at a level above 5σ [65, 68]:

- Indirect CP violation in $K \rightarrow \pi\pi$ and $K \rightarrow \pi l \nu$ decays, and in $K_L \rightarrow \pi^+ \pi^- e^+ e^-$ decay, is given by

$$|\varepsilon| = (2.228 \pm 0.011) \times 10^{-3}. \quad (2.4.1)$$

- Direct CP violation in $K \rightarrow \pi\pi$ decays is given by

$$\text{Re}(\varepsilon'/\varepsilon) = (1.65 \pm 0.26) \times 10^{-3}. \quad (2.4.2)$$

- CP violation in the interference of mixing and decay in the tree-dominated $b \rightarrow c\bar{c}s$ transitions, such as $B^0 \rightarrow \psi K^0$, is given by

$$S_{\psi K^0} = +0.691 \pm 0.017. \quad (2.4.3)$$

- CP violation in the interference of mixing and decay in modes governed by tree-dominated $b \rightarrow c\bar{u}d$ transitions is given by

$$S_{D_{cp}^{(*)}h^0} = +0.63 \pm 0.11. \quad (2.4.4)$$

- CP violation in the interference of mixing and decay in various related to $b \rightarrow c\bar{c}d$ transitions is given by

$$S_{\psi\pi^0} = -0.93 \pm 0.15, \quad (2.4.5)$$

$$S_{D^+D^-} = -0.98 \pm 0.17, \quad (2.4.6)$$

$$S_{D^{*+}D^{*-}} = -0.71 \pm 0.09. \quad (2.4.7)$$

- CP violation in the interference of mixing and decay in various related to $b \rightarrow q\bar{q}s$ transitions is given by

$$S_{\phi K^0} = +0.94_{-0.13}^{+0.11}, \quad (2.4.8)$$

$$S_{\eta' K^0} = +0.63 \pm 0.06, \quad (2.4.9)$$

$$S_{f_0 K^0} = +0.69_{-0.12}^{+0.10}, \quad (2.4.10)$$

$$S_{K^+K^-K^0} = +0.68_{-0.10}^{+0.09}. \quad (2.4.11)$$

- CP violation in the interference of mixing and decay in $B^0 \rightarrow \phi^+\phi^-$ mode is given by

$$S_{\pi^+\pi^-} = -0.66 \pm 0.06. \quad (2.4.12)$$

- Direct CP violation in the $B^0 \rightarrow \phi^+\phi^-$ mode is given by

$$C_{\pi^+\pi^-} = -0.31 \pm 0.05. \quad (2.4.13)$$

- Direct CP violation in the $\bar{B}^0 \rightarrow K^-\pi^+$ mode is given by

$$A_{\bar{B}^0 \rightarrow K^-\pi^+} = -0.82 \pm 0.006. \quad (2.4.14)$$

- Direct CP violation in $B^+ \rightarrow D_+ K^+$ decays (D_+ is the CP-even neutral D state) is given by

$$A_{B^+ \rightarrow D_+ K^+} = +0.195 \pm 0.027. \quad (2.4.15)$$

- Direct CP violation in the $\bar{B}_s^0 \rightarrow K^+ \pi^-$ mode is given by

$$A_{\bar{B}_s^0 \rightarrow K^+ \pi^-} = +0.26 \pm 0.04. \quad (2.4.16)$$

- Direct CP violation in $B^+ \rightarrow K^+ K^- \pi^+$ decays is given by

$$A_{B^+ \rightarrow K^+ K^- \pi^+} = -0.118 \pm 0.022. \quad (2.4.17)$$

2.5 CP Violation in the Standard Model

The SM Lagrangian consists of three parts. The first one, which is known as the gauge sector, is the part of the Lagrangian that contains the interactions of fermions with the gauge bosons and also the interactions of the gauge bosons among themselves. The second part is the Yukawa sector, which contains the interactions of fermions with the scalar Higgs doublet. The third part is the Higgs sector, which contains the scalar potential of the Higgs field. It is remarkable that the gauge sector is CP conserving and hence there is no complex phase in this sector. Also the Higgs sector of one Higgs doublet models can be made real by field redefinitions. Therefore, there is no CP violating phase in this sector either. Finally, the Yukawa couplings are generally complex and they are indeed the source of CP violation in the SM. The Yukawa interactions are given by

$$-\mathcal{L}_{Yukawa}^{quarks} = Y_{ij}^u \bar{Q}_{Li} \Phi u'_{Rj} + Y_{ij}^d \bar{Q}_{Li} \tilde{\Phi} d'_{Rj} + h.c. \quad (2.5.1)$$

So, $\mathcal{L}_{Yukawa}$ remains unchanged under the CP operation if $Y_{ij} = Y_{ij}^*$.

2.5 CP Violation in the Standard Model

As is known, the fermion masses are induced by the Yukawa interactions after EWSB. For the up and down types of quarks, one finds

$$\mathcal{L}_{mass} = \bar{u}_L^i M_u^{ij} u_R'^j + \bar{d}_L^i M_d^{ij} d_R'^j + h.c., \quad (2.5.2)$$

where the mass matrices $M_{u,d}$ are defined in terms of the Yukawa matrices as $M_{u,d} = \frac{v}{\sqrt{2}} Y_{u,d}$. These mass matrices are generic 3×3 matrices. The physical (or mass) eigenstates, where the mass matrices are diagonal, are obtained by the following bi-unitary rotations:

$$u_{Li}' \rightarrow V_{ij}^u u_{Lj}, \quad u_{Ri}' \rightarrow U_{ij}^u u_{Rj}, \quad (2.5.3)$$

$$d_{Li}' \rightarrow V_{ij}^d d_{Lj}, \quad d_{Ri}' \rightarrow U_{ij}^d d_{Rj}, \quad (2.5.4)$$

where $V^{u,d}$, $U^{u,d}$ satisfy the following diagonalization conditions:

$$M_u^{\text{diag}} \rightarrow V_u^* M_u U_u^\dagger, \quad M_d^{\text{diag}} \rightarrow V_d^* M_d U_d^\dagger. \quad (2.5.5)$$

When introducing these transformations in the interaction between fermions and charged gauge boson, the electroweak charged current can be written as

$$\begin{pmatrix} \bar{u}' & \bar{c}' & \bar{t}' \end{pmatrix} \gamma^\mu (1 - \gamma^5) \begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} \bar{u} & \bar{c} & \bar{t} \end{pmatrix} \gamma^\mu (1 - \gamma^5) V_u^\dagger V_d \begin{pmatrix} d \\ s \\ b \end{pmatrix}, \quad (2.5.6)$$

then the charged current Lagrangian in the basis of quark mass eigenstates is

$$\mathcal{L}_W = \frac{g}{\sqrt{2}} \bar{u}_L^i \gamma^\mu V_{\text{CKM}}^{ij} d_L^j W^{+\mu} + \frac{g}{\sqrt{2}} \bar{d}_L^i \gamma^\mu V_{\text{CKM}}^{*ij} u_L^j W^{-\mu}, \quad (2.5.7)$$

where the V_{CKM} is the Cabibbo Kobayashi Maskawa (CKM) mixing matrix, defined as

$$V_{\text{CKM}} = V_L^{u\dagger} V_L^d = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}. \quad (2.5.8)$$

The CP-transformed expression,

$$\mathcal{L}_W^{CP} = \frac{g}{\sqrt{2}} \bar{d}_L^j \gamma^\mu V_{CKM}^{ij} u_L^i W^{-\mu} + \frac{g}{\sqrt{2}} \bar{u}_L^j \gamma^\mu V_{CKM}^{*ij} u_L^i W^{+\mu}. \quad (2.5.9)$$

Then we can conclude that the Lagrangian is unchanged if $V_{CKM}^{ij} = V_{CKM}^{*ij}$. Therefore, the complex nature of the CKM matrix is the origin of CP violation in the SM.

Let us now discuss the properties of the unitary CKM matrix V_{CKM} . We start by counting the number of free parameters for the CKM-matrix. Consider the Standard Model with N flavours. Then the CKM matrix has N^2 complex elements, and thus $2N^2$ real parameters. These can be expressed as a set of mixing angles and complex phases. Unitarity ($V^\dagger V = 1$) implies N^2 constraints: (i) N unitary conditions (unity of the diagonal elements) (ii) $N^2 - N$ orthogonality relations (vanishing off-diagonal elements). The overall phase of all quarks can be rotated freely, we may eliminate $2N - 1$ of these complex phases, one finds that the CKM matrix has

$$2N^2 - N^2 - (2N - 1) = (N - 1)^2 \quad (2.5.10)$$

complex phases. The mixing angles are the parameters one would find if you restricted V_{CKM} to be an orthogonal matrix. A general $N \times N$ orthogonal matrix can be constructed from $\frac{1}{2}N(N - 1)$ angles describing the rotations among the N dimensions. Thus, The remaining physical complex phases in the N -flavour CKM matrix is

$$(N - 1)^2 - \frac{1}{2}N(N - 1) = \frac{1}{2}(N - 2)(N - 1). \quad (2.5.11)$$

Note that the mismatch between the rotations of the up and down types of quark fields is the source of the mixing matrix V_{CKM} , which is generally non-diagonal. Therefore, flavour changes in the charged current are naturally implemented in the SM.

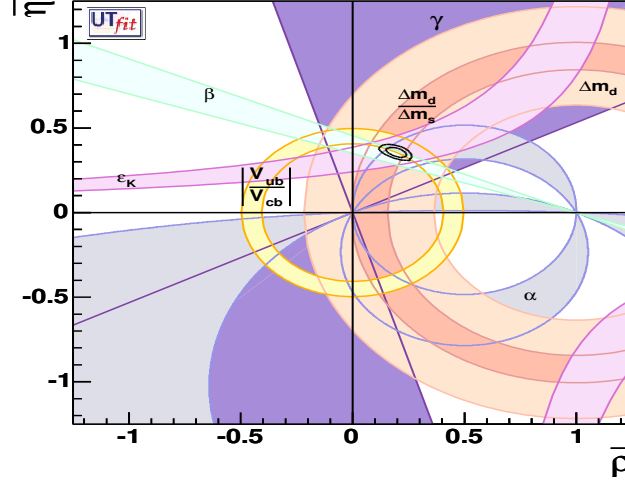


Figure 2.2: Experimental constraints on the $\rho - \eta$ plane as obtained by the UTfit collaboration [69].

Similarly, it is straightforward to show that there is no such mismatch in the neutral current ($\bar{u}uZ$ and $\bar{d}dZ$), hence it remains flavour diagonal.

The CKM matrix is a unitary matrix, since $V_{\text{CKM}}V_{\text{CKM}}^\dagger = (V_u^\dagger V_d)(V_d^\dagger V_u) = 1$. Thus, for three generations of quarks ($N = 3$), the CKM matrix generally depends on three real angles and six phases. However, five of these phases can be absorbed into the redefinition of quark phases, leaving a single physical phase. Therefore, in the SM with three families, CP violation is accommodated by a single complex phase (δ_{KM}) in the CKM mixing matrix. Also, one can conclude that, with two quark generations, no CP violation can be generated. Indeed, a 2×2 unitary matrix ($N = 2$) is a real matrix, where all phases are rotated away by some field redefinitions.

The standard parameterization of the the CKM matrix is given by [70]

$$V_{\text{CKM}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{12}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, \quad (2.5.12)$$

where $c_{ij} = \cos \theta_{ij}$, $s_{ij} = \sin \theta_{ij}$ and δ is the CP violating phase. From the fact that one can parametrize V_{CKM} by three real and only one physical phase, another useful parameterization of the CKM matrix was introduced by Wolfenstein [71, 72],

$$V_{\text{CKM}} = \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda + \frac{1}{2}A^2\lambda^5[1 - 2(\rho + i\eta)] & 1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4(1 + 4A^2) & A\lambda^2 \\ A\lambda^3[1 - (1 - \frac{1}{2}\lambda^2)(\rho + i\eta)] & -A\lambda^2 + \frac{1}{2}A\lambda^4[1 - 2(\rho + i\eta)] & 1 - \frac{1}{2}A^2\lambda^4 \end{pmatrix}, \quad (2.5.13)$$

where (λ, A, ρ, η) are the four mixing parameters with $\lambda = |V_{us}| \approx 0.23$ playing the role of an expansion parameter and η representing the CP violating phase.

The parameter A is determined by measuring V_{cb} from the inclusive leptonic b decays and the exclusive decay $B^0 \rightarrow D^{*+}l^-\bar{\nu}_l$. The (ρ, η) plane can be constrained from the combination of many experimental observables as in Fig. 2.2. Note that $\bar{\rho} = \rho(1 - \lambda^2/2)$ and $\bar{\eta} = \eta(1 - \lambda^2/2)$ are usually used instead of ρ and η to keep unitarity exact. The best determination of the Wolfenstein parameters is [69]

$$\rho = 0.135_{-0.016}^{+0.031}, \quad \eta = 0.349_{-0.017}^{+0.015}, \quad A = 0.814_{-0.022}^{+0.021}, \quad \lambda = 0.2257_{-0.0010}^{+0.0009}. \quad (2.5.14)$$

The uncertainty is still not negligible; in particular, the theoretical results suffer from significant uncertainties due to the hadronic matrix elements involved in the semileptonic decays, as we will show in the next chapter.

The unitarity of the CKM matrix leads to the following relations:

$$V_{ud}V_{us}^* + V_{cd}V_{cs}^* + V_{td}V_{ts}^* = 0, \quad (2.5.15)$$

$$V_{us}V_{ub}^* + V_{cs}V_{cb}^* + V_{ts}V_{tb}^* = 0, \quad (2.5.16)$$

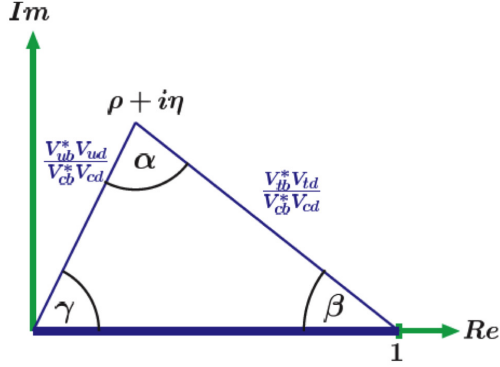


Figure 2.3: The unitarity triangle of the CKM matrix.

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0. \quad (2.5.17)$$

Each of these relations can be represented graphically by a triangle. The last relation is commonly used to define what is called a 'unitarity triangle' (UT). One can display this triangle in terms of the following three angles:

$$\alpha = \arg \left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*} \right) \simeq \arg \left(-\frac{1 - \rho - i\eta}{\rho + i\eta} \right), \quad (2.5.18)$$

$$\beta = \arg \left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*} \right) \simeq \arg \left(\frac{1}{1 - \rho - i\eta} \right), \quad (2.5.19)$$

$$\gamma = \arg \left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} \right) \simeq \arg(\rho + i\eta), \quad (2.5.20)$$

which satisfy the constraint $\alpha + \beta + \gamma = \pi$, as shown in Fig. 2.3.

Multiplying Eq. (2.5.17) by $V_{us}^*V_{cs}$ and taking the imaginary part, one finds that

$$\text{Im}[V_{us}^*V_{cs}V_{ud}V_{cd}^*] = -\text{Im}[V_{us}^*V_{cs}V_{ub}V_{cb}^*]. \quad (2.5.21)$$

Also, from other orthogonal relations, one can show that the quartets $V_{ij}V_{kl}V_{il}^*V_{kj}^*$ have equal imaginary parts up to a sign. Therefore, one defines the Jarlskog invariant [73, 74]:

$$J = \text{Im}[V_{us}V_{cb}V_{ub}^*V_{cs}^*], \quad (2.5.22)$$

as an invariant measure for the amount of CP violation in the SM. Using the Wolfenstein parameterizations for V_{CKM} , one can show that $J \simeq A^2 \lambda^6 \eta \lesssim 10^{-4}$. It is remarkable that the smallness of J is due to the smallness of the off-diagonal elements of V_{CKM} , in particular $|V_{ub}|$, and not because of a small value of the CP violating phase. In addition, the area of the UT is given by $|J|/2$. Thus, the area of this triangle measures the amount of CP violation.

Finally, one should also recall (as previously intimated) that the SM has another source of CP violation. This arises from the term $\frac{\theta}{16\pi^2} F_{\mu\nu} \tilde{F}^{\mu\nu}$ in the QCD Lagrangian. For non-zero θ , a large contribution to the Electric Dipole Moment (EDM) of the neutron is obtained. The current experimental limits of the EDMs imply that $\theta < 10^{-10}$. Such a small value induces a fine-tuning problem, known as the strong CP problem. The most elegant solution to overcome the strong CP problem is the PQ mechanism [75], where the smallness of the θ parameter is explained dynamically through a spontaneous breaking of a $U(1)$ symmetry, called $U(1)_{\text{PQ}}$.

2.6 CP Violation in $K^0 - \bar{K}^0$ Mixing

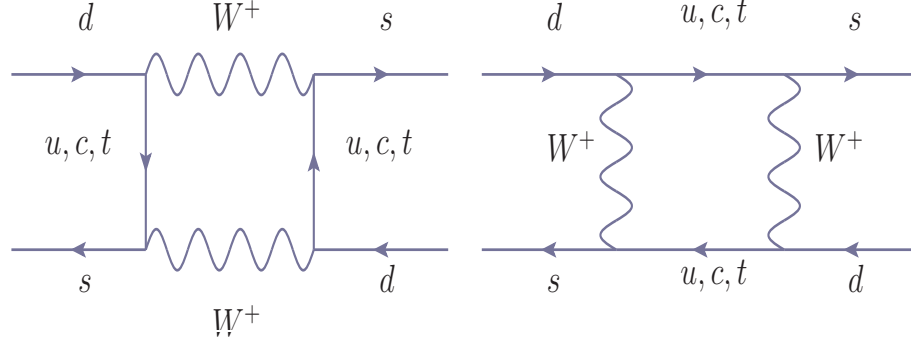
CP violation was discovered in $K \rightarrow \pi\pi$ in 1964 [76] demonstrated that CP symmetry is violated in weak interactions. In this section we discuss the $K^0 - \bar{K}^0$ mixing in the SM.

In the K^0 and $\bar{K}^0$ system, the flavour eigenstates are $K^0 = (\bar{s}d)$ and $\bar{K}^0 = (s\bar{d})$ which in the SM may mix via weak interactions through the box diagrams in Fig. 2.4. These states are not definite CP eigenstates but for a chosen phase, we have

$$CP|K^0\rangle = -|\bar{K}^0\rangle, \quad CP|\bar{K}^0\rangle = -|K^0\rangle. \quad (2.6.1)$$

The CP even eigenstate K_1 and the CP odd eigenstate K_2 can be written as linear combinations of K^0 and $\bar{K}^0$ as follows

$$K_{1,2} = \frac{1}{\sqrt{2}} (|K^0\rangle \mp |\bar{K}^0\rangle), \quad (2.6.2)$$


 Figure 2.4: Box diagrams contributing to $K^0 - \bar{K}^0$ mixing in the SM.

with $CP|K_{1,2}\rangle = \pm|K_{1,2}\rangle$. The mass eigenstates K_S and K_L are related to the CP eigenstates by diagonalizing the mixing matrix

$$K_S = pK^0 + q\bar{K}^0, \quad (2.6.3)$$

$$K_L = pK^0 - q\bar{K}^0, \quad (2.6.4)$$

where L and S refer to heavy and light mass eigenstates (long and short mass eigenstates), respectively, and

$$p = \frac{1 + \varepsilon}{\sqrt{2(1 + |\varepsilon|^2)}}, \quad q = \frac{1 - \varepsilon}{\sqrt{2(1 + |\varepsilon|^2)}}, \quad \frac{q}{p} = \frac{1 - \varepsilon}{1 + \varepsilon}, \quad (2.6.5)$$

where ε is the CP violating parameter in the $K^0 - \bar{K}^0$ system, the mass eigenstates can now be written as

$$K_S = \frac{K_1 + \varepsilon K_2}{\sqrt{1 + |\varepsilon|^2}}, \quad K_L = \frac{K_2 + \varepsilon K_1}{\sqrt{1 + |\varepsilon|^2}}. \quad (2.6.6)$$

From Eq. (2.2.23), one finds

$$\frac{1 - \varepsilon}{1 + \varepsilon} = \frac{2M_{12}^* - i\Gamma_{12}^*}{\Delta m - \frac{i}{2}\Delta\Gamma_{12}}. \quad (2.6.7)$$

For $\Gamma_{12} \ll \Delta m$, then ε can be approximated to a useful expression for theoretical evaluation:

$$\varepsilon = \frac{\Im M_{12}}{\sqrt{2}\Delta m_K}. \quad (2.6.8)$$

Thus, if M_{12} and Γ_{12} are real, then ε will vanish and $K_{L,S}$ correspond to $K_{1,2}$. The $K_L - K_S$ mass difference is experimentally measured to be [70, 77]

$$\Delta m_K = m_{K_L} - m_{K_S} = (3.490 \pm 0.006) \times 10^{-15} \text{ GeV}, \quad (2.6.9)$$

while the experimental value of ε (the measure for indirect CP violation) is give by [70, 77]

$$\varepsilon \equiv \frac{A(K_L \rightarrow (\pi\pi)_{I=0})}{A(K_S \rightarrow (\pi\pi)_{I=0})}, \quad (2.6.10)$$

$$\varepsilon = (2.28 \pm 0.013) \times 10^{-3}, \quad (2.6.11)$$

where I refers to isospin. In addition, the direct CP violation is characterized by a complex parameter ε' defined as

$$\varepsilon' \equiv \frac{1}{\sqrt{2}} \left(\frac{A(K_L \rightarrow (\pi\pi)_{I=2})}{A(K_S \rightarrow (\pi\pi)_{I=0})} - \frac{A(K_S \rightarrow (\pi\pi)_{I=2})}{A(K_S \rightarrow (\pi\pi)_{I=0})} \frac{A(K_L \rightarrow (\pi\pi)_{I=0})}{A(K_S \rightarrow (\pi\pi)_{I=0})} \right), \quad (2.6.12)$$

where the isospin amplitudes A_I in $K \rightarrow \pi\pi$ decays are introduced through [64]

$$A(K^+ \rightarrow \pi^+ \pi^0) = \sqrt{\frac{3}{2}} A_2 e^{i\delta_2}, \quad (2.6.13)$$

$$A(K^0 \rightarrow \pi^+ \pi^-) = \sqrt{\frac{2}{3}} A_0 e^{i\delta_0} + \sqrt{\frac{1}{3}} A_2 e^{i\delta_2}, \quad (2.6.14)$$

$$A(K^0 \rightarrow \pi^0 \pi^0) = \sqrt{\frac{2}{3}} A_0 e^{i\delta_0} - 2\sqrt{\frac{1}{3}} A_2 e^{i\delta_2}. \quad (2.6.15)$$

This direct CP violation can be measured via the ratio ε'/ε which is given by

$$\Re(\varepsilon'/\varepsilon) = \frac{1}{6(1 + \omega/\sqrt{2})} \left(1 - \left| \frac{\eta_{00}}{\eta_{+-}} \right|^2 \right), \quad (2.6.16)$$

where

$$\eta_{00} = \frac{A(K_L \rightarrow \pi^0 \pi^0)}{A(K_S \rightarrow \pi^0 \pi^0)}, \quad \eta_{+-} = \frac{A(K_L \rightarrow \pi^+ \pi^-)}{A(K_S \rightarrow \pi^+ \pi^-)}, \quad \omega = \frac{\Re A_2}{\Re A_0}. \quad (2.6.17)$$

Theoretically, ε can be calculated from Eq. (2.6.8),

$$\varepsilon = \frac{\Im M_{12}}{\sqrt{2}\Delta m_K}, \quad (2.6.18)$$

where

$$M_{12} = \langle K^0 | \mathcal{H}_{\text{eff}}^{\Delta S=2} | \bar{K}^0 \rangle. \quad (2.6.19)$$

The SM contribution to $\mathcal{H}_{\text{eff}}^{\Delta S=2}$ is obtained from box diagrams in Fig. 2.4. It can be expressed via operator product expansion (OPE) as

$$\mathcal{H}_{\text{eff}}^{\Delta S=2} = \sum_i C_i(\mu) Q_i, \quad (2.6.20)$$

where $C_i(\mu)$ is the Wilson coefficients and Q_i are the relevant local operators.

Let us now calculate The amplitude of the $K^0 - \bar{K}^0$ transition which is generated through the box diagrams (Fig.2.4). The amplitude in the SM is

$$\begin{aligned} \mathcal{M} &= \int \frac{d^d k}{(2\pi)^d} \left(\bar{s} \frac{ig}{2\sqrt{2}} \gamma^\mu (1 - \gamma_5) V_{\alpha s}^* \frac{i(\not{k} + m_i)}{k^2 - m_i^2} \frac{ig}{2\sqrt{2}} V_{\alpha d} \gamma^\nu (1 - \gamma_5) d \right) \\ &\quad \times \left(\bar{s} \frac{ig}{2\sqrt{2}} \gamma^\rho (1 - \gamma_5) V_{\beta s}^* \frac{i(\not{k} + m_\beta)}{k^2 - m_\beta^2} \frac{ig}{2\sqrt{2}} V_{\beta d} \gamma^\sigma (1 - \gamma_5) d \right) \\ &\quad \times \left(\frac{-i}{k^2 - M_W^2} (g_{\nu\rho} - \frac{k_\nu k_\rho}{M_W^2}) \frac{-i}{k^2 - M_W^2} (g_{\mu\sigma} - \frac{k_\mu k_\sigma}{M_W^2}) \right) \\ &= \frac{g^4}{64} (V_{\alpha s}^* V_{\alpha d}) (V_{\beta s}^* V_{\beta d}) \int \frac{d^d k}{(2\pi)^d} \left(\bar{s} \gamma^\mu (1 - \gamma_5) (\not{k} + m_i) \gamma^\nu (1 - \gamma_5) d \right) \\ &\quad \times \left(\bar{s} \gamma^\rho (1 - \gamma_5) (\not{k} + m_j) \gamma^\sigma (1 - \gamma_5) d \right) \left(g_{\nu\rho} - \frac{k_\nu k_\rho}{M_W^2} \right) \left(g_{\mu\sigma} - \frac{k_\mu k_\sigma}{M_W^2} \right) \\ &\quad \times \frac{1}{(k^2 - m_i^2)(k^2 - m_j^2)(k^2 - M_W^2)}. \end{aligned} \quad (2.6.21)$$

To simplify the amplitude formula, we use the following relations:

$$(V_{is}^* V_{id})(V_{js}^* V_{jd}) = \lambda_i \lambda_j, \quad \text{with } i, j = u, c, t, \quad (2.6.22)$$

$$(1 - \gamma_5)(\not{k} + m_i) \gamma^\nu (1 - \gamma_5) = 2\not{k} \gamma^\nu (1 - \gamma_5) = 2\not{k} \gamma_L^\nu, \quad (2.6.23)$$

$$(1 - \gamma_5)(\not{k} + m_j)\gamma^\sigma(1 - \gamma_5) = 2\not{k}\gamma_L^\sigma, \quad (2.6.24)$$

$$\left(g_{\nu\rho} - \frac{k_\nu k_\rho}{M_W^2}\right) \left(g_{\mu\sigma} - \frac{k_\mu k_\sigma}{M_W^2}\right) = g_{\nu\rho}g_{\mu\sigma} - g_{\mu\sigma} \frac{k_\nu k_\rho}{M_W^2} - g_{\nu\rho} \frac{k_\mu k_\sigma}{M_W^2} + \frac{k_\nu k_\rho k_\mu k_\sigma}{M_W^4}, \quad (2.6.25)$$

$$\frac{g^4}{16} = 2G_F^2 M_W^4. \quad (2.6.26)$$

Therefore, the amplitude now can be written as

$$\begin{aligned} \mathcal{M} &= 2G_F^2 M_W^4 \lambda_i \lambda_j \int \frac{d^d k}{(2\pi)^d} \frac{1}{(k^2 - m_i^2)(k^2 - m_j^2)(k^2 - M_W^2)^2} \\ &\quad \times \left((\bar{s}\gamma^\mu \not{k}\gamma_L^\nu d)(\bar{s}\gamma^\rho \not{k}\gamma_L^\sigma d) \left(g_{\nu\rho}g_{\mu\sigma} - g_{\mu\sigma} \frac{k_\nu k_\rho}{M_W^2} - g_{\nu\rho} \frac{k_\mu k_\sigma}{M_W^2} + \frac{k_\nu k_\rho k_\mu k_\sigma}{M_W^4} \right) \right) \\ &= 2G_F^2 M_W^4 \lambda_i \lambda_j \int \frac{d^d k}{(2\pi)^d} \frac{1}{(k^2 - m_i^2)(k^2 - m_j^2)(k^2 - M_W^2)^2} \\ &\quad \times \left((\bar{s}\gamma^\mu \not{k}\gamma_L^\nu d)(\bar{s}\gamma_{\nu L} \not{k}\gamma_\mu d) - \frac{1}{M_W^2} (\bar{s}\gamma_L^\mu \not{k}\not{k}d)(\bar{s}\not{k}\not{k}\gamma_{\mu L} d) - \frac{1}{M_W^2} (\bar{s}\not{k}\not{k}\gamma_L^\nu d)(\bar{s}\gamma_{\nu L} \not{k}\not{k}d) \right. \\ &\quad \left. + \frac{1}{M_W^4} (\bar{s}\not{k}\not{k}\not{k}_L d)(\bar{s}\not{k}\not{k}\not{k}_L d) \right) \end{aligned} \quad (2.6.27)$$

$$\begin{aligned} &= 2G_F^2 M_W^4 \lambda_i \lambda_j \int \frac{d^d k}{(2\pi)^d} \frac{1}{(k^2 - m_i^2)(k^2 - m_j^2)(k^2 - M_W^2)} \\ &\quad \times \left(k^\alpha k^\beta (\bar{s}\gamma^\mu \gamma_\alpha \gamma_L^\nu d)(\bar{s}\gamma_{\nu L} \gamma_\beta \gamma_\mu d) - \frac{2k^4}{M_W^2} (\bar{s}\gamma_L^\nu d)(\bar{s}\gamma_{\nu L} d) \right. \\ &\quad \left. + \frac{k^4}{M_W^4} k^\alpha k^\beta (\bar{s}\gamma_{\alpha L} d)(\bar{s}\gamma_{\beta L} d) \right). \end{aligned} \quad (2.6.28)$$

Using the following useful formulae,

$$\int \frac{d^4 k}{(2\pi)^4} \frac{k^\alpha k^\beta}{f(k^2)} = \frac{g^{\alpha\beta}}{4} \int \frac{d^4 k}{(2\pi)^4} \frac{k^2}{f(k^2)}, \quad (2.6.29)$$

$$(\gamma^\mu \gamma^\beta \gamma_L^\nu) \otimes (\gamma_{\nu L} \gamma_\beta \gamma_\mu) = 4\gamma_L^\nu \otimes \gamma_{\nu L}, \quad (2.6.30)$$

$$\gamma^\mu \gamma^\beta \gamma^\nu = g^{\alpha\beta} \gamma^\nu + g^{\beta\nu} \gamma^\nu \gamma^\mu - g^{\mu\nu} \gamma^\beta + i\varepsilon^{\mu\beta\nu\sigma} \gamma_\sigma, \quad (2.6.31)$$

one finds

$$\mathcal{M} = 2G_F^2 M_W^4 \lambda_i \lambda_j (\bar{s}\gamma_L^\nu d)(\bar{s}\gamma_{\nu L} d) \int \frac{d^d k}{(2\pi)^d} \frac{k^2 - \frac{2k^4}{M_W^2} + \frac{k^6}{4M_W^4}}{(k^2 - m_i^2)(k^2 - m_j^2)(k^2 - M_W^2)^2}. \quad (2.6.32)$$

After evaluating the loop integral over the momentum k for both box diagrams, we obtain

$$\mathcal{M} = -\frac{iG_F^2 M_W^2}{4\pi^2} \lambda_i \lambda_j (\bar{s} \gamma_L^\nu d) (\bar{s} \gamma_{\nu L} d) \left(\frac{x_i^2 (4 - 8x_i + x_i^2) \ln x_i}{4(x_i - x_j)(1 - x_i)^2} - \frac{x_j^2 (4 - 8x_i + x_j^2) \ln x_j}{4(x_i - x_j)(1 - x_j)^2} - \frac{3}{4(1 - x_i)(1 - x_j)} \right), \quad (2.6.33)$$

where $x_i = m_i^2/M_W^2$. If we consider the sum runs over the quarks c and t , we get

$$\mathcal{M} = -\frac{iG_F^2 M_W^2}{4\pi^2} (\bar{s} \gamma_L^\nu d) (\bar{s} \gamma_{\nu L} d) [\lambda_c^2 S(x_c) + \lambda_t^2 S(x_t) + 2\lambda_c \lambda_t S(x_c, x_t)] \quad (2.6.34)$$

where the loop functions are given by

$$S(x_i) = \frac{4x_i - 11x_i^2 + x_i^3}{4(1 - x_i)^2} - \frac{3x_i^2 \ln x_i}{2(1 - x_i)^2}, \quad (2.6.35)$$

$$S(x_c, x_t) = x_c \left(\ln \frac{x_t}{x_c} - \frac{3x_t}{4(1 - x_t)} - \frac{3x_t^2 \ln x_t}{4(1 - x_t)^2} \right). \quad (2.6.36)$$

Then, the effective Hamiltonian is given in term of the operator, $O_1 = (\bar{s} \gamma_L^\nu d) (\bar{s} \gamma_{\nu L} d)$, by

$$\mathcal{H}_{eff} = \frac{G_F^2 M_W^2}{4\pi^2} (\bar{s} \gamma_L^\nu d) (\bar{s} \gamma_{\nu L} d) [\lambda_c^2 S(x_c) + \lambda_t^2 S(x_t) + 2\lambda_c \lambda_t S(x_c, x_t)], \quad (2.6.37)$$

where $\mathcal{M} = -i\mathcal{H}_{eff}$. Therefore, the corresponding Wilson coefficient is given by

$$C_1 = \frac{G_F^2 M_W^2}{4\pi^2} [\lambda_c^2 S(x_c) + \lambda_t^2 S(x_t) + 2\lambda_c \lambda_t S(x_c, x_t)]. \quad (2.6.38)$$

In the other hand, including leading and next-to-leading (NLO) QCD corrections lead [64, 78, 79]

$$C_1 = \frac{G_F^2 M_W^2}{4\pi^2} [\eta_1 \lambda_c^2 S(x_c) + \eta_2 \lambda_t^2 S(x_t) + \eta_3 2\lambda_c \lambda_t S(x_c, x_t)], \quad (2.6.39)$$

where η_i are the QCD correction factors whose NLO values [79, 80] $\eta_1 = 1.38 \pm 0.20$, $\eta_2 = 0.57 \pm 0.04$ and $\eta_3 = 0.47 \pm 0.04$.

So, M_{12} is given by

$$M_{12} = \frac{G_F^2 M_W^2}{4\pi^2} \langle K^0 | (\bar{s} \gamma_L^\nu d) (\bar{s} \gamma_{\nu L} d) | \bar{K}^0 \rangle \mathcal{F}^*, \quad (2.6.40)$$

where

$$\mathcal{F}^* = \eta_1 \lambda_c^2 S(x_c) + \eta_2 \lambda_t^2 S(x_t) + \eta_3 2\lambda_c \lambda_t S(x_c, x_t). \quad (2.6.41)$$

The matrix element of the operator Q_1 is parameterized by the renormalization group invariant parameter B_K as²

$$\langle K^0 | (\bar{s} \gamma_L^\nu d) (\bar{s} \gamma_{\nu L} d) | \bar{K}^0 \rangle = \frac{1}{3} f_K^2 m_K B_K, \quad (2.6.42)$$

where $F_K = 160$ MeV is the K -meson decay constant. Therefore, we find that M_{12}^{SM} is given by

$$M_{12}^{SM} = \frac{G_F^2 M_W^2}{12\pi^2} f_K^2 m_K B_K \mathcal{F}^*. \quad (2.6.43)$$

Finally, one finds the SM prediction for ε is given by

$$\varepsilon \simeq 1.7 \times 10^{-3}. \quad (2.6.44)$$

This prediction lies in the ballpark of the measured values. However, a precise prediction cannot be made due to the strong interaction and the corresponding hadronization, which are involved in the hadronic matrix element, and CKM uncertainties.

2.7 CP Violation in the MSSM

In SUSY theories there are several other possible sources for CP violation arising from the complexity of the soft SUSY breaking terms, namely the phases associated with the gaugino masses, the Hermitian squark and slepton mass matrices, the trilinear couplings and the phase associated with bilinear coupling. In addition to all this, there is another CP phase of SUSY origin preserving the μ parameter. In general, these phases can be classified into the following two categories [81]:

²The matrix element can be calculated by using the vacuum insertion approximation (VIA), which reduces the problem of calculating the matrix element of a quartic operator into the problem of calculating the matrix elements of two bilinear operators [62].

- (i) Flavour-independent phases: such as the phases of the μ -parameter, B -parameter, gaugino masses M_a , and the phases of diagonal elements of the trilinear couplings.
- (ii) Flavour-dependent phases: such as the phases of the off-diagonal elements of A_{ij}^f , $i \neq j$ and phases in the Hermitian squark and slepton mass matrices $(m_{ij}^f)^2$.

In general, these phases are not all independent and one can eliminate some of them by field redefinitions. For instance, in the case of a cMSSM (with universal soft SUSY breaking terms), the SUSY phases arise from the parameters $M_{1/2}$, A , and B in addition to μ . So let us assume that

$$M_{1/2} = |M_{1/2}|e^{i\alpha_M}, \quad A = |A|e^{i\alpha_A}, \quad B = |B|e^{i\alpha_B}, \quad \mu = |\mu|e^{i\alpha_\mu}. \quad (2.7.1)$$

However, one can make $M_{1/2}$ real by using an R -transformation with R charge $Q_R = 1$ for lepton and quark superfields and $Q_R = 0$ for vector and Higgs superfields. Thus, under such an R transformation, the vector superfield transforms as

$$V(x, \theta, \bar{\theta}) \rightarrow V(x, e^{i\alpha_M/2}\theta, e^{-i\alpha_M/2}\bar{\theta}). \quad (2.7.2)$$

Under this transformation the gaugino $\lambda(x)$ transforms as $\lambda(x)e^{-i\alpha_M/2}$, which makes $M_{1/2}$ real. Note that in the case of non-universal gaugino masses with three different complex phases, one phase only can be rotated by the R transformation and two phases remain physical instead.

Also, under this R transformation, the Higgs and quark superfields transform as

$$H_i(x, \theta) \rightarrow H_i(x, e^{i\alpha_M/2}\theta), \quad (2.7.3)$$

$$Q(x, \theta) \rightarrow e^{i\alpha_M/2}Q(x, e^{i\alpha_M/2}\theta). \quad (2.7.4)$$

Since the Higgs is a chiral superfield, it has the form

$$H_i(x, \theta) = H_i(x) + \sqrt{2}\theta\tilde{H}_i + \theta^2 F_i(x), \quad (2.7.5)$$

where $\tilde{H}_i(x)$ are the Higgsino fields, which transform under R as $\tilde{H}_i(x)e^{i\alpha_M/2}$. Therefore, the μ term in the superpotential leads to

$$\mu H_u H_d|_{\theta\theta} = |\mu| e^{i\alpha_\mu} \tilde{H}_u \tilde{H}_d \rightarrow |\mu| e^{i(\alpha_\mu + \alpha_M)} \tilde{H}_u \tilde{H}_d. \quad (2.7.6)$$

Also, the bilinear coupling transforms as

$$B\mu H_u H_d \rightarrow |B\mu| e^{i(\alpha_B + \alpha_\mu)} H_u H_d. \quad (2.7.7)$$

Thus, by re-defining the superfields H_u and H_d such that $H_i \rightarrow e^{-i(\alpha_B + \alpha_\mu)/2} H_i$, one finds that $B\mu H_u H_d$ is real, which ensures that the Higgs VEVs are real. Moreover, due to this re-definition, the phase of the μ term is changed to $-\phi_B = \alpha_M - \alpha_B$. Similarly, one can show that, under R transformation and superfield re-definitions, the phase of the trilinear coupling is changed to $\phi_A = \alpha_A - \alpha_M$. Therefore, only two phases are physical.

It is clear that in the case of non-universal soft-SUSY breaking terms, the number of CP violating phases increases. These new phases have significant implications and can modify the SM predictions in CP violating phenomena. In particular, they would give large contributions to the EDMs of the electron, neutron, and mercury atom.

2.8 Flavour Changing Neutral Currents

Flavour changing neutral currents (FCNC's) provide an important test for any new physics beyond SM. In the SM, there are no tree-level FCNCs. If we look at the terms in the Lagrangian corresponding to the Z -boson current, we have terms of the form $Z^\mu \bar{u}_L \gamma_\mu u_L$. Since the Dirac matrix does not affect the flavour structure, so when one applies the unitary transformations between two left-handed up (down) quarks will cancel one another identically. Hence, there are no transitions between quarks of different flavours. That is certainly true at tree level since there are no explicit terms in the Lagrangian. This implies that the baryon (B) and lepton (L) numbers

are automatically conserved, however, the problem of tree level B and L violation in supersymmetric theories is solved by introducing an additional discrete symmetry, the R-parity. In addition, low energy supersymmetry predicts new particles carrying flavour numbers with mass of order TeV that may lead to potentially large FCNC rates. FCNCs, however, do occur in the SM via loop-mediated processes, although suppressed by what is called the Glashow- Iliopoulos-Maiani (GIM) mechanism [82].

As an illustrative example, Let us do a quick qualitative analysis for $K^0 - \bar{K}^0$ mixing in the SM. The diagrams contributing to this process are shown in Fig. 2.4. This loop diagram generates a four-point fermion term in the effective lagrangian, $s\bar{d}\bar{s}d$. Let us estimate the coefficient of this operator and hence the amplitude for this transition: (i) The process contains a momentum integral whose mass dimension is -2. The characteristic mass scale comes from the loop momenta scale which is given by the mass of the W boson, and hence we have a factor of $1/M_W^2$. (ii) Each vertex gives a factor of the SU(2) coupling g and the appropriate factor of the CKM matrix, which one can see amounts to a factor $g^4 \left(\sum_i V_{id}^\dagger V_{is} \right)^2$. (iii) The loop integral gives us the factor of $1/(4\pi)^2$.

We know that the up-type quark propagators, in the loop, contain factors of quark masses m_i^2 in their denominators. The quark masses are small compared to the energy scales of the system, $m_i \ll M_W$, so let us Taylor expand those terms with respect to m_i^2 to insert the appropriate factors into amplitude. This argument breaks down for the top quark since its mass is larger than M_W . However, the smallness of off-diagonal element of the CKM matrix, V_{td} , makes the contribution of the top quark negligible. We can estimate the amplitude to be (for more detail, see [83]):

$$\mathcal{M} \approx \frac{2g^4}{(4\pi)^2 M_W^2} \left(\sum_i V_{id}^\dagger V_{is} m_i \right)^2. \quad (2.8.1)$$

This leads us to say that the amplitude is suppressed by squared factor of the smallness of off-diagonal elements of the CKM matrix and the smallness of the quark masses

relative to the W boson mass. This is the GIM mechanism.

2.9 Electric Dipole Moments and SUSY CP Problem

The most stringent constraints on models of new physics with additional sources of CP violation come from continued efforts to measure the EDMs of the neutron, electron, and mercury atom. The current experimental limits on EDMs are given by [84–86]

$$d_n < 2.9 \times 10^{-26} \text{ e cm}, \quad (2.9.1)$$

$$d_e < 8.7 \times 10^{-29} \text{ e cm}, \quad (2.9.2)$$

$$d_{Hg} < 3.1 \times 10^{-29} \text{ e cm}. \quad (2.9.3)$$

With the expected improvements in experimental precision, the EDM is likely to be one of the most important tests for physics beyond the SM for some time to come, and EDMs will remain a difficult hurdle for CP violating SUSY theories.

The EDM of a pair of opposite charges of magnitude q is defined as

$$\mathbf{D} = q \mathbf{d}, \quad (2.9.4)$$

where $\mathbf{d}$ is the displacement vector pointing from the negative charge to the positive charge. For a classical charge distribution, the EDM is given by

$$\mathbf{D} = \int \mathbf{r} \rho(\mathbf{r}) d^3r. \quad (2.9.5)$$

In the case of elementary particles, the EDM must be proportional to the spin $\mathbf{S}$, since elementary particles have no intrinsic vectorial degree of freedom except their spin. Thus, one can write $\mathbf{D} = d \mathbf{S}$, where the proportional constant d is an intrinsic property of the particle that represents the size of the EDM. Since $\mathbf{D}$ and $\mathbf{S}$ have the same transformation under discrete symmetries, under parity, P , transformation

one has $\mathbf{D} \rightarrow \mathbf{D}$ while the electric field $\mathbf{E}$ transforms as $\mathbf{E} \rightarrow -\mathbf{E}$. Also, under time reversal, T , transformation one has $\mathbf{D} \rightarrow -\mathbf{D}$ and $\mathbf{E} \rightarrow \mathbf{E}$. Therefore, the interaction $\mathbf{D} \cdot \mathbf{E}$ violates both P and T . In this case, if d is different from zero, then both P and T are violated, and hence, according to the CPT theorem, the CP symmetry is also violated.

In a quantum field theory, the P and T violating effective interaction between a spin-1/2 particle and the EM field is given by ³.

$$\mathcal{L}_{\text{eff}} = iF_3(q^2)\bar{\psi}(p_2)\sigma_{\mu\nu}\gamma_5\psi(p_1)F^{\mu\nu}, \quad (2.9.6)$$

where $q^2 = (p_2 - p_1)^2$. The magnitude of the EDM of ψ , d_ψ , corresponds to the on shell limit of the form factor F_3 , *i.e.*, $d_\psi = F_3(0)$. The renormalizability condition rules out the possibility of having this type of interaction at tree level. However, it can be generated at loop level.

The electron EDM is labelled d_e . It can be measured experimentally from the EDM of the thallium atom. The electron EDM calculation involves little uncertainty, therefore it allows us to extract reliable bounds on the CP-violating SUSY phases.

The neutron EDM has contributions from a number of CP-violating operators involving quarks, gluons, and photons. The most important ones include the electric and chromoelectric dipole operators and the Weinberg three-gluon operator:

$$\mathcal{L} = -\frac{i}{2}d_q^E\bar{q}\sigma_{\mu\nu}\gamma_5qF^{\mu\nu} - \frac{i}{2}d_q^C\bar{q}\sigma_{\mu\nu}\gamma_5T^aqG^{a\mu\nu} - \frac{1}{6}d^Gf_{abc}G_{a\mu\rho}G_{b\nu}^\rho G_{c\lambda\sigma}\epsilon^{\mu\nu\lambda\sigma}, \quad (2.9.7)$$

where $G_{\mu\nu}^a$ is the gluon field strength and T^a and f_{abc} are the $SU(3)$ generators and group structure coefficients, respectively. Given these operators, it is, however, a non-trivial task to evaluate the neutron EDM, since assumptions about the neutron's internal structure are necessary. Usually, two models, namely the quark chiral model and the quark parton model, are considered to relate the quark EDMs to the neutron

³It is worth noting that I have some contributions in the Ref [87]. For this reason some sections in this chapter are similar.

EDM. Neither of these models is sufficiently reliable by itself; however, the power of the combined analysis should provide an insight into implications of the bound on the neutron EDM. It is important to note that, in any case, the neutron EDM calculations involve uncertain hadronic parameters such as the quark masses, and thus these calculations have a status of estimates at best. However, the major conclusions of this section are independent of the specifics of the neutron model.

The chiral quark model is a non-relativistic model that relates the neutron EDM to the EDMs of valence quarks with the help of the $SU(6)$ coefficients:

$$d_n = \frac{4}{3}d_d - \frac{1}{3}d_u . \quad (2.9.8)$$

The quark EDMs can be *estimated*, via Naive Dimensional Analysis (NDA), as [88]

$$d_q = \eta^E d_q^E + \eta^C \frac{e}{4\pi} d_q^C + \eta^G \frac{e\Lambda}{4\pi} d_q^G , \quad (2.9.9)$$

where the QCD correction factors are given by $\eta^E = 1.53$, $\eta^C \simeq \eta^G \simeq 3.4$, while $\Lambda \simeq 1.19$ GeV is the chiral symmetry breaking scale. The parameters $\eta^{C,G}$ involve considerable uncertainties stemming from the fact that the strong coupling constant at low energies is unknown. Another weak side of the model is that it neglects the sea quark contributions, which play an important role in the nucleon spin structure.

The parton quark model is based on the isospin symmetry and known contributions of different quarks to the spin of the proton. The quantities Δ_q , defined as $\langle n | \frac{1}{2} \bar{q} \gamma_\mu \gamma_5 q | n \rangle = \Delta_q S_\mu$, where S_μ is the neutron spin, are related by the isospin symmetry to the quantities $(\Delta_q)_p$, which are measured in deep inelastic scattering (and other) experiments, *i.e.*, $\Delta_u = (\Delta_d)_p$, $\Delta_d = (\Delta_u)_p$, and $\Delta_s = (\Delta_s)_p$. The main assumption of the model is that the quark contributions to the neutron EDM are weighted by the same factors Δ_i , *i.e.*,

$$d_n = \eta^E (\Delta_d d_d^E + \Delta_u d_u^E + \Delta_s d_s^E) . \quad (2.9.10)$$

In our numerical analysis we use the standard values for these quantities: $\Delta_d = 0.746$, $\Delta_u = -0.508$, and $\Delta_s = -0.226$.

The major difference from the chiral quark model is a large strange quark contribution. In particular, due to the large strange and charm quark masses, the strange quark contribution dominates in most regions of the parameter space. This leads to considerable numerical differences between the predictions of the two models. It has been shown that the mercury EDM is primarily sensitive to the chromoelectric dipole moments of the quarks [89]:

$$d_{Hg} = -e (d_d^C - d_u^C - 0.012d_s^C) \times 3.2 \times 10^{-2} . \quad (2.9.11)$$

Although the coefficient for the d_s^C is suppressed with respect to those for the chromoelectric EDM of down and up quarks, this contribution is still important since, d_s^C is enhanced by a heavier quark mass and large mixing in the second generation.

2.9.1 SM contributions to EDMs

As mentioned above, in the SM, CP violation comes from the CKM mixing matrix. Since $W^\pm$'s couple only to left-handed quarks and leptons, the one-loop contributions to d_e or d_q must have an external mass insertion. However, the CKM phase at the two $W^\pm$ vertices is cancelled. Hence, within the SM, a CP violating EDM cannot be generated at the one-loop level. At the two-loop level, it is possible to generate a non-self conjugate diagram, depicted in Fig. 2.5, that may contribute to the EDM of the electron and neutron. However, in 1978 it was shown that the sum of all two-loop contributions vanishes identically [90].

This cancellation was first shown by Donoghue for the electron and independently by Shabalin for quarks. However, there is no symmetry to explain this cancellation. In fact, the vanishing of the EDM in the SM at one and two loops is only due to the unitarity of the CKM matrix and to the purely left-handed chirality of the charged

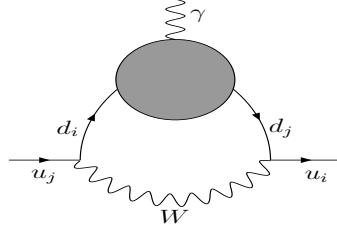


Figure 2.5: Possible two-loop SM contributions to the EDMs.

current. In this respect, one expects a non-vanishing three-loop contribution. Donoghue estimated this contribution to the electron EDM [91] and found that

$$d_e^{\text{SM}} < 10^{-50} \text{ e cm.} \quad (2.9.12)$$

Also, Shabalin showed that the corresponding neutron EDM is given by [90].

$$d_n^{\text{SM}} \leq 10^{-34} \text{ e cm.} \quad (2.9.13)$$

2.9.2 SUSY contribution to EDMs

In SUSY models, the new CP violating phases may generate EDMs for quarks and electrons at the one-loop level. The electron EDM arises due to CP-violating one-loop diagrams involving chargino and neutralino exchange, as shown in Fig. 2.6. Therefore, the SUSY electron EDM is given by

$$d_e = d_e^{x^+} + d_e^{x^0} \quad (2.9.14)$$

while the SUSY contributions to the EDMs of the individual quarks result from the one-loop gluino, chargino, and/or neutralino exchange diagrams, shown in Fig. 2.6. Thus,

$$d_q^{E,C} = d_q^{\tilde{g} (E,C)} + d_q^{x^+ (E,C)} + d_q^{x^0 (E,C)}. \quad (2.9.15)$$

In addition, one finds contributions from the two-loop gluino-quark-squark diagrams, shown in Fig. 2.7, which generate d^G .

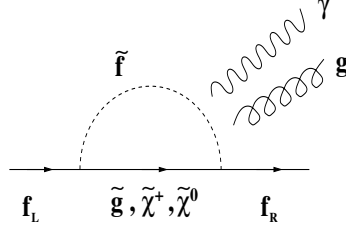


Figure 2.6: Leading SUSY contributions to the EDMs. The photon and gluon lines are to be attached to the loop in all possible ways.

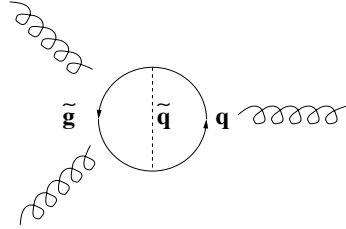


Figure 2.7: Two-loop of gluino-quark-squark contribution to the EDMs.

The explicit computation of the SUSY contributions to the EDMs of electron and up and down quarks, using the full one-loop results, is straightforward. Here, we provide detailed calculations for the electron EDM, and results for quarks can be derived similarly. As mentioned, the effective Lagrangian that defines the EDM of an electron is given by

$$\mathcal{L} = -\frac{i}{2} d_e \bar{\psi} \sigma_{\mu\nu} \gamma_5 \psi F^{\mu\nu}, \quad (2.9.16)$$

which can be written for $\psi(x) = u(p)e^{-ip \cdot x}$ and $A_\mu(x) = \varepsilon_\mu(q)e^{-iq \cdot x}$ as

$$\mathcal{L} = -\frac{i}{2} d_e \bar{u}(p+q) \sigma_{\mu\nu} \varepsilon^\mu(q) q^\nu \gamma_5 u(p). \quad (2.9.17)$$

Using Gordon decomposition, one can write this Lagrangian as

$$\mathcal{L} = -2im_e d_e \bar{u}(p+q) (\varepsilon \cdot \gamma) \gamma_5 u(p) + 2id_e \bar{u}(p+q) (\varepsilon \cdot p) \gamma_5 u(p). \quad (2.9.18)$$

As usual, the term proportional to the $\varepsilon.\gamma$ structure contributes to the wave function renormalization. Thus, in our calculation of an invariant amplitude, we need only to concentrate on the $\varepsilon.p$ term:

$$\mathcal{L} = 2id_e \bar{u}(p+q)(\varepsilon.p)\gamma_5 u(p). \quad (2.9.19)$$

The Feynman diagram of chargino contribution to the electron EDM is shown in Fig. 2.6. From the electron-chargino-sneutrino interaction, one finds that the $e_L - e_R$ transition amplitude is given by

$$\begin{aligned} \mathcal{M} = & \sum_{i=1}^2 \int \frac{d^4 k}{(2\pi)^4} \bar{u}(p+q) \frac{e}{\sin \theta_W} \left[(V^{i1})^* P_R + \frac{m_e}{\sqrt{2}M_W \cos \beta} U^{i2} P_L \right] \\ & (\Gamma_\nu^{1I}) \frac{1}{(\not{p} - \not{k} + \not{q}) - m_{\tilde{\chi}_i^+}} (e\varepsilon\gamma) \frac{1}{(\not{p} - \not{k}) - m_{\tilde{\chi}_i^+}} \left(\frac{e}{\sin \theta_W} \right) \\ & \left[\left(V^{i1} P_L + \frac{m_e}{\sqrt{2}M_W \cos \beta} (U^{2i})^* P_R \right) (\Gamma_\nu^{1I})^* \right] \frac{1}{k^2 - m_{\tilde{\nu}^2}} u(p), \end{aligned} \quad (2.9.20)$$

which leads to

$$\begin{aligned} \mathcal{M} = & \frac{1}{8\pi^2} \frac{e^3}{\sin^2 \theta_W} \frac{m_e}{\sqrt{2}M_W \cos \beta} (\varepsilon.p) \bar{u}(p+q) \gamma_5 u(p) \\ & \sum_{i=1}^2 \frac{m_{\tilde{\chi}_i^+}}{m_{\tilde{\nu}}^2} \text{Im} \left[U^{21*} V^{i1*} \right] A \left(\frac{m_{\tilde{\chi}_i^+}^2}{m_{\tilde{\nu}}^2} \right), \end{aligned} \quad (2.9.21)$$

where the loop function $A(x)$ is given by

$$A(r) = \frac{1}{2(1-r)^2} \left[3 - r + \frac{2 \ln r}{1-r} \right]. \quad (2.9.22)$$

Thus one finds that the electron EDM is given by

$$d_e^{\tilde{\chi}^+}/e = \frac{\alpha_{em}}{4\pi \sin^2 \theta_W} \frac{m_e}{\sqrt{2}M_W \cos \beta} \sum_{i=1}^2 \frac{m_{\tilde{\chi}_i^+}}{m_{\tilde{\nu}}^2} \text{Im} \left[U^{2i*} V^{i1*} \right] A \left(\frac{m_{\tilde{\chi}_i^+}^2}{m_{\tilde{\nu}}^2} \right). \quad (2.9.23)$$

Similarly, one can show that the neutralino contribution to the electron (and also to the up and down quark) EDM is given by [92]

$$d_f^{\tilde{\chi}^0}/e = \frac{\alpha_{em}}{4\pi \sin^2 \theta_W} \sum_{k=1}^2 \sum_{i=1}^4 \text{Im}(\eta_{fik}) \frac{m_{\tilde{\chi}_i^0}}{M_{\tilde{f}_k}^2} Q_{\tilde{f}} B \left(\frac{m_{\tilde{\chi}_i^0}^2}{M_{\tilde{f}_k}^2} \right), \quad (2.9.24)$$

where η_{fik} is the neutralino vertex, which is given by

$$\begin{aligned} \eta_{fik} &= \left[-\sqrt{2}\{\tan\theta_W(Q_f - I_{3_f})N_{1i} + I_{3_f}N_{2i}\}D_{f1k}^* - \kappa_f N_{bi}D_{f2k}^* \right] \\ &\times \left[\sqrt{2}\tan\theta_W Q_f N_{1i}D_{f2k} - \kappa_f N_{bi}D_{f1k} \right], \end{aligned} \quad (2.9.25)$$

where I_3 is the third component of the isospin, $b = 3$ (4) for $I_3 = -1/2$ ($1/2$), and N is the unitary matrix diagonalising the neutralino mass matrix: $N^T M_{\chi^0} N = \text{diag}(m_{\chi_1^0}, m_{\chi_2^0}, m_{\chi_3^0}, m_{\chi_4^0})$. Here, we use our convention from Chapter 1, where the mass matrix eigenvalues are positive and ordered as $m_{\chi_1^0} > m_{\chi_2^0} > \dots$. The loop function $B(r)$ is defined by

$$B(r) = \frac{1}{2(r-1)^2} \left(1 + r + \frac{2r \ln r}{1-r} \right). \quad (2.9.26)$$

In a similar way, one can derive the following expression for the one-loop gluino contributions to the the EDM of the up and down quarks [92]:

$$d_{\tilde{q}}^{\tilde{g}(E)}/e = \frac{-2\alpha_s}{3\pi} \sum_{k=1}^2 \text{Im} \left[R_{2k}^{\tilde{q}} R_{1k}^{\tilde{q}*} \right] \frac{M_{\tilde{g}}}{M_{\tilde{q}_k}^2} Q_{\tilde{q}} B \left(\frac{M_{\tilde{g}}^2}{M_{\tilde{q}_k}^2} \right). \quad (2.9.27)$$

Here, $R^{\tilde{q}}$ is defined by $R^{\tilde{q}} M_{\tilde{q}}^2 R^{\tilde{q}\dagger} = \text{diag}(M_{\tilde{q}_1}^2, M_{\tilde{q}_2}^2)$. Finally, the chargino contributions to the EDM of the up and down quarks are given by

$$d_u^{\tilde{\chi}^+(E)}/e = \frac{-\alpha_{em}}{4\pi \sin^2 \theta_W} \sum_{i,k=1}^2 \text{Im}(\Gamma_{uik}) \frac{m_{\tilde{\chi}_i^+}}{M_{\tilde{d}_k}^2} \left[Q_{\tilde{d}} B \left(\frac{m_{\tilde{\chi}_i^+}^2}{M_{\tilde{d}_k}^2} \right) + (Q_u - Q_{\tilde{d}}) A \left(\frac{m_{\tilde{\chi}_i^+}^2}{M_{\tilde{d}_k}^2} \right) \right], \quad (2.9.28)$$

$$d_d^{\tilde{\chi}^+(E)}/e = \frac{-\alpha_{em}}{4\pi \sin^2 \theta_W} \sum_{i,k=1}^2 \text{Im}(\Gamma_{dik}) \frac{m_{\tilde{\chi}_i^+}}{M_{\tilde{u}_k}^2} \left[Q_{\tilde{u}} B \left(\frac{m_{\tilde{\chi}_i^+}^2}{M_{\tilde{u}_k}^2} \right) + (Q_d - Q_{\tilde{u}}) A \left(\frac{m_{\tilde{\chi}_i^+}^2}{M_{\tilde{u}_k}^2} \right) \right], \quad (2.9.29)$$

where the chargino vertex Γ_{fik} is defined as

$$\Gamma_{uik} = \kappa_u V_{i2}^* R_{1k}^{\tilde{d}} (U_{i1}^* R_{1k}^{\tilde{d}*} - \kappa_d U_{i2}^* R_{2k}^{\tilde{d}*}), \quad (2.9.30)$$

$$\Gamma_{dik} = \kappa_d U_{i2}^* R_{1k}^{\tilde{u}} (V_{i1}^* R_{1k}^{\tilde{u}*} - \kappa_u V_{i2}^* R_{2k}^{\tilde{u}*}). \quad (2.9.31)$$

The quantities κ_f are the Yukawa couplings

$$\kappa_u = \frac{m_u}{\sqrt{2}m_W \sin \beta}, \quad \kappa_{d,e} = \frac{m_{d,e}}{\sqrt{2}m_W \cos \beta}. \quad (2.9.32)$$

Also, the SUSY contributions to the quark chromoelectric dipole moment are given by [93]

$$d_q^{\tilde{g}(C)} = \frac{g_s \alpha_s}{4\pi} \sum_{k=1}^2 \text{Im} [R_{2k}^{\tilde{q}} R_{1k}^{\tilde{q}*}] \frac{M_{\tilde{g}}}{M_{\tilde{q}_k}^2} C \left(\frac{M_g^2}{M_{\tilde{q}_k}^2} \right), \quad (2.9.33)$$

$$d_q^{\tilde{X}^+(C)} = \frac{-g^2 g_s}{16\pi^2} \sum_{k=1}^2 \sum_{i=1}^2 \text{Im}(\Gamma_{qik}) \frac{m_{\tilde{X}_i^+}}{M_{\tilde{q}_k}^2} B \left(\frac{m_{\tilde{X}_i^+}^2}{M_{\tilde{q}_k}^2} \right), \quad (2.9.34)$$

$$d_q^{\tilde{X}^0(C)} = \frac{g_s g^2}{16\pi^2} \sum_{k=1}^2 \sum_{i=1}^4 \text{Im}(\eta_{qik}) \frac{m_{\tilde{X}_i^0}}{M_{\tilde{q}_k}^2} B \left(\frac{m_{\tilde{X}_i^0}^2}{M_{\tilde{q}_k}^2} \right), \quad (2.9.35)$$

where $C(r)$ is defined by

$$C(r) = \frac{1}{6(r-1)^2} \left(10r - 26 + \frac{2r \ln r}{1-r} - \frac{18 \ln r}{1-r} \right). \quad (2.9.36)$$

Finally, the contribution to the Weinberg operator from the two-loop gluino-top-stop and gluino-bottom-sbottom diagrams reads [93]

$$d^G = -3\alpha_s m_t \left(\frac{g_s}{4\pi} \right)^3 \text{Im}(\Gamma_t^{12}) \frac{z_1 - z_2}{(M_3)^3} H(z_1, z_2, z_t) + (t \rightarrow b), \quad (2.9.37)$$

where $z_i = \left(\frac{M_{\tilde{t}_i}}{M_3} \right)^2$, $z_t = \left(\frac{m_t}{M_3} \right)^2$. The two-loop function $H(z_1, z_2, z_t)$ is given by

$$H(z_1, z_2, z_t) = \frac{1}{2} \int_0^1 dx \int_0^1 du \int_0^1 dy x(1-x)u \frac{N_1 N_2}{D^4}, \quad (2.9.38)$$

where

$$N_1 = u(1-x) + z_t x(1-x)(1-u) - 2ux[z_1 y + z_2(1-y)], \quad (2.9.39)$$

$$N_2 = (1-x)^2(1-u)^2 + u^2 - \frac{1}{9}x^2(1-u)^2, \quad (2.9.40)$$

$$D = u(1 - x) + z_t x(1 - x)(1 - u) + ux[z_1 y + z_2(1 - y)] . \quad (2.9.41)$$

In addition to the Weinberg two-loop diagram, there is another (so-called Barr-Zee) two-loop contribution, which originates from the CP-odd Higgs exchange. Its numerical effect is, however, negligible.

3

Prediction for $B \rightarrow \tau\bar{\mu} + \mu\bar{\tau}$ in the MSSM and 2HDM

In this chapter we will study the process $B \rightarrow \tau\bar{\mu} + \mu\bar{\tau}$ which involves two FCNC interactions, one in the quark and one in the lepton sector. We consider the decay $B \rightarrow \tau\bar{\mu} + \mu\bar{\tau}$ in two extensions of the SM, extended Higgs sector model (2HDM) and the MSSM.

3.1 Looking for New Physics in $B_{s,d} \rightarrow \ell_i \ell_j$

The FCNC are a natural place to look for NP because the loop-level effects of NP can be pronounced against the SM predictions. We now motivate our interest in the rare decay $B_{s,d} \rightarrow \ell_i \ell_j$. Typically one believes that this process will be very small even with new physics and in many NP models this is indeed true. This is because in these NP models this FCNC process arises in loops and is suppressed. In this chapter we will focus on the decay $B \rightarrow \tau\bar{\mu} + \mu\bar{\tau}$ and make predictions for its rate. This

decay was considered in earlier papers in various extensions of the SM [94–97]. In this context we will calculate the process in two scenarios representing two extension of the SM.

In the first scenario we will consider this process mediated by a scalar, which we will call X , in a model with an extended higgs sector. In this model we will assume that there are scalar particles that have FCNC couplings and can mediate the decay $B \rightarrow \tau\mu$. We will identify the lowest mass such state with the X particle. It is true that in this model there can be other heavier neutral states that can also mediate the decay $B \rightarrow \tau\mu$. When multiple scalar states contribute to the decay there will be more parameters that cannot be all constrained from the data resulting in loss of predictive power. To regain some predictive power, our assumption will be that the dominant contribution to the decay comes from the lowest lying state X . We expect that the contributions to the decay from the higher mass states will in general not lower our estimated branching ratios significantly unless there are strong cancellations among the various amplitudes. We will assume that there are no strong cancellations among the various contributions. In simple terms, the contributions of higher order mass state X' to the branching ratio is of order $M_X^2/M_{X'}^2$ smaller than the leading order term. Therefore, if $M_{X'} \gg M_X$, then this effect is quite negligible.

We will identify X with the newly discovered particle, with mass ~ 125 GeV, observed at LHC [98, 99] with supporting evidence for its existence from Fermilab [100]. In this case $B \rightarrow \tau\bar{\mu} + \mu\bar{\tau}$ is generated at tree level due to the presence of the bqX and $\tau\mu X$ couplings. The coupling bqX can be constrained by $B_{d,s}$ mixing while the $\tau\mu X$ couplings can be constrained from rare τ decays. Recently in Ref. [101, 102] these coupling were considered and it was noticed that the $\tau\mu X$ coupling could be of similar size as the SM Higgs to $\tau\tau$ coupling. Since $B_{s,d}$ are pseudoscalars, the flavor changing coupling of X to the $B_{s,d}$ mesons must contain a pseudoscalar component of the form $(\bar{q}\gamma_5 b)X$ where $q = s, d$.

The spin parity of the X particle is now somewhat known through various measurements [103–105] (Please check the new LHC reference with ref [111]). In this scenario the branching ratio for $B \rightarrow \tau \bar{\mu} + \mu \bar{\tau}$ can be significant and can potentially be observed at the LHCb. Generally, final states with τ leptons are difficult to measure at LHCb, however because of an associated μ in the final state this process may be somewhat easier to observe than $B_{d,s} \rightarrow \tau^+ \tau^-$. However, a detailed knowledge of the detector is essential to draw a realistic estimate about the sensitivity of the process.

We would like to point that the decay can also be mediated through the exchange of a spin one particle at tree level. Since we will be identifying the mediator with the newly discovered state at LHC we will not consider the mediator to be a spin one particle. Furthermore, as can be seen from Eq. (3.3.7) the matrix elements with scalar currents, associated with a scalar mediator, are enhanced relative to vector currents associated with a spin one mediator.

In the second scenarios we will consider a popular extension of the SM- the minimal supersymmetric SM (MSSM). Here the FCNC couplings arise via loops and so our expectation is that the branching ratio for $B \rightarrow \tau \bar{\mu} + \mu \bar{\tau}$ will be tiny. The main point here is that if this process were to be observed at the LHCb in the near future this would indicate the presence of a rather non-traditional NP with appreciable tree level FCNC couplings in the lepton sector and possibly a light spin zero mediator.

3.2 Weak Effective Hamiltonian

In order to describe weak processes with characteristic energies $\ll M_W$, such as B decays, we can use an Effective field theory (EFT) without $W^\pm$, Z^0 , Higgs. The flavor-changing hadron transitions are multi-scale processes conveniently studied using the operator product expansion (OPE) [106, 107] where at least there are two relevant energy scales: the electroweak scale, given for instance by the W boson mass $M_W \sim 100$ GeV, relevant for the flavor-changing weak transition, and the scale of

strong interactions $\Lambda_{QCD} \sim 0.2 - 1$ GeV, related to the hadron formation [108]. All possible operators produced by these processes, using the OPE, can be described by effective weak Hamiltonians. Coefficients of local interaction operators in this Hamiltonian are obtained by matching at the renormalization scale $\mu \sim M_W$. In order to calculate B decays one needs to know these coefficients at a much lower $\mu \sim m_b$: these are usual called the Wilson coefficients, namely the effective couplings multiplying the operators of the Hamiltonian. They are obtained by solving renormalization group equations (RGE). One needs to calculate the matrix of anomalous dimensions of the operators entering the effective Hamiltonian.

The effective weak Hamiltonian has the following generalized form [109]

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} \sum_i V_{CKM}^i C_i(\mu) Q_i(\mu), \quad (3.2.1)$$

where G_F is the Fermi constant $G_F/\sqrt{2} = g^2/8m_W^2$ and Q_i are the relevant local operators. C_i are the Wilson coefficients and V_{CKM}^i are The Cabibbo-Kobayashi-Maskawa (CKM) matrix elements.

The amplitude can be calculated through projection of the H_{eff} onto the initial and final states

$$\mathcal{M}(i \rightarrow f) = \langle f | H_{eff} | i \rangle = \frac{G_F}{\sqrt{2}} \sum_i V_{CKM}^i C_i(\mu) \langle f | Q_i(\mu) | i \rangle, \quad (3.2.2)$$

where $\langle f | Q_i(\mu) | i \rangle$ are the hadronic matrix elements of Q_i between i and f .

Since the Wilson coefficients $C_i(\mu)$ summarize the physics contributions from scales higher than μ and due to asymptotic freedom of QCD they are calculable at any order of perturbation theory as long as μ is not too small. The effect of particles heavier than M_W enters only through these coefficients such as top quark, charged Higgs or supersymmetric particles. On the other hand, the hadronic matrix elements of Q_i depend on the renormalization scale μ similarly to $C_i(\mu)$.

The calculation of the Wilson coefficients at a given order in perturbation theory can be obtained by: (i) the matching between the full theory and the effective Hamiltonian at a scale $M \sim \mathcal{O}(M_W)$ and (ii) the RGE evolution from the matching scale M down to the renormalization scale μ (for more details see Refs [64, 108–110]). This implies

- Calculation of the amplitude in the full theory at highest energy scale.
- Calculation of the operator matrix elements Q_i at the lowest energy scale.
- Extraction of the Wilson coefficients C_i from $M_{full} = M_{eff}$.

The effective Hamiltonian for $|\Delta B| = 1$ and $|\Delta S| = 1$ is given by [109, 110]

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} \left[\lambda_u (C_1(\mu_b) Q_1^u + C_2(\mu_b) Q_2^u) + \lambda_c (C_1(\mu_b) Q_1^c + C_2(\mu_b) Q_2^c) + \lambda_t \sum_{i=3}^6 C_i(\mu) Q_i \right], \quad (3.2.3)$$

where $\lambda_q = V_{qb} V_{qs}^*$. The list of the operators that play the role in weak B decays can be extracted from the typical diagrams in the full theory which are shown in Fig. 3.3. The six classes of operators are given as follows (α and β are color indices): $Q_{1,2}$ refer for current-current operators (fig. 3.3 (a)), Q_{3-6} for QCD penguins operators (fig. 3.3 (b)), Q_{7-10} for electroweak penguins operators (fig. 3.3 (c)) and $Q_{7\gamma, 8G}$ for magnetic penguins (fig. 3.3 (d)). $Q(\Delta S = \Delta B = 2)$ are $\Delta S = \Delta B = 2$ operators (fig. 3.3 (e)), $Q_{9V, 10A}$ and $Q_{\nu\bar{\nu}, \mu\bar{\mu}}$ refer for semi-leptonic operators (fig. 3.3 (f)). They are given as [110]

$$Q_1 = (\bar{s}_i u_j)_{V-A} (\bar{u}_j d_i)_{V-A}, \quad Q_2 = (\bar{s} u)_{V-A} (\bar{u} d)_{V-A}, \quad (3.2.4)$$

$$Q_3 = (\bar{s} d)_{V-A} \sum_q (\bar{q} q)_{V-A}, \quad Q_4 = (\bar{s}_i d_j)_{V-A} \sum_q (\bar{q}_j q_i)_{V-A}, \quad (3.2.5)$$

$$Q_5 = (\bar{s} d)_{V-A} \sum_q (\bar{q} q)_{V+A}, \quad Q_6 = (\bar{s}_i d_j)_{V-A} \sum_q (\bar{q}_j q_i)_{V+A}, \quad (3.2.6)$$

$$Q_7 = \frac{3}{2} (\bar{s}d)_{V-A} \sum_q e_q (\bar{q}q)_{V+A}, \quad Q_8 = \frac{3}{2} (\bar{s}_i d_j)_{V-A} \sum_q e_q (\bar{q}_j q_i)_{V+A} \quad (3.2.7)$$

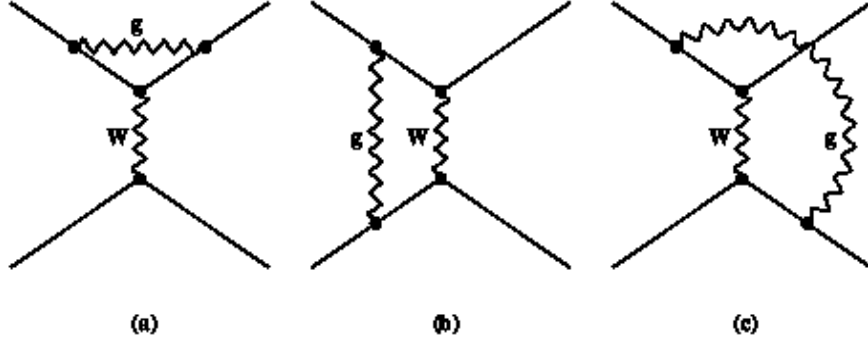


Figure 3.1: One-loop current-current diagrams in the full theory.

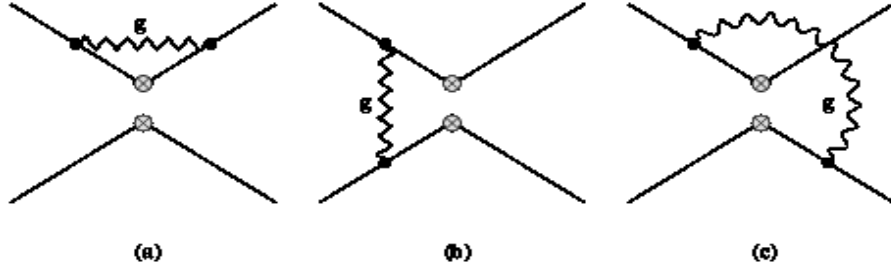


Figure 3.2: One loop current-current diagrams in the effective theory.

$$Q_9 = \frac{3}{2} (\bar{s}d)_{V-A} \sum_q e_q (\bar{q}q)_{V-A}, \quad Q_{10} = \frac{3}{2} (\bar{s}_i d_j)_{V-A} \sum_q e_q (\bar{q}_j q_i)_{V-A}, \quad (3.2.8)$$

$$Q_{7\gamma} = \frac{e}{8\pi^2} m_b \bar{s}_i \sigma^{\mu\nu} (1 + \gamma_5) b_i F_{\mu\nu}, \quad Q_{8G} = \frac{g}{8\pi^2} m_b \bar{s}_i \sigma^{\mu\nu} (1 + \gamma_5) T_{ij}^a b_j G_{\mu\nu}^a, \quad (3.2.9)$$

$$Q(\Delta S = 2) = (\bar{s}d)_{V-A} (\bar{s}d)_{V-A}, \quad Q(\Delta B = 2) = (\bar{b}d)_{V-A} (\bar{b}d)_{V-A}, \quad (3.2.10)$$

$$Q_{7V} = (\bar{s}d)_{V-A} (\bar{e}e)_V, \quad Q_{7A} = (\bar{s}d)_{V-A} (\bar{e}e)_A, \quad (3.2.11)$$

$$Q_{9V} = (\bar{b}s)_{V-A}(\bar{e}e)_V, \quad Q_{10A} = (\bar{b}s)_{V-A}(\bar{e}e)_A, \quad (3.2.12)$$

$$Q(\bar{\nu}\nu) = (\bar{s}d)_{V-A}(\bar{\nu}\nu)_{V-A}, \quad Q(\bar{\mu}\mu) = (\bar{s}d)_{V-A}(\bar{\mu}\mu)_{V-A}, \quad (3.2.13)$$

where $\sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu]$, T_{ij}^a are the SU(3) color matrices and $(\bar{q}q)_{V\pm A} = \bar{q}\gamma_\mu(1 \pm \gamma_5)q$ where $q = u, c, d, s$ while e_q are quark electric charges. Now due to the fact that the Wilson coefficients should be computed at the scale $\mu = \mu_b$, i.e. $\mu \ll M_{W,Z}, m_t$, large logarithms $\alpha_s^n(\log(M_W^2/\mu^2))^n$, $\alpha_s(\alpha_s \log(M_W^2/\mu^2))^n$, etc, that appear in the perturbative expansion of the Wilson coefficients become much large and spoil the convergence of the perturbative series. This situation can be treated by employing the method of the Renormalization Group (RG). For more clarification, let us evaluate the current-current diagrams in Figs 3.1 and 3.2:

- Calculation of the full amplitude

The full amplitudes M from Fig 3.1(a-c) can be written as follows

$$\begin{aligned} \mathcal{M}_{full}^a &= i\mu^{2\varepsilon} \int \frac{d^d k}{(2\pi)^d} [\bar{s}_i(-ig_s\gamma_\mu T_{il}^a) \frac{i(\not{p} + \not{k})}{(p+k)^2} \frac{ig_2}{2\sqrt{2}} \gamma_\rho(1 - \gamma_5) V_{cs} \frac{i(\not{p} + \not{k})}{(p+k)^2} (-ig_s\gamma_\nu T_{lm}^a) c_m] \\ &\quad \times [\bar{u}_k \frac{ig_2}{2\sqrt{2}} \gamma_\sigma(1 - \gamma_5) V_{ud}^* d_k] \frac{-ig^{\rho\sigma}}{k^2 - m_W^2} \frac{-ig^{\mu\nu}}{k^2} \\ &= -i \frac{g_s^2 g_2^2}{32m_W^2} V_{cs} V_{ud}^* \mu^{2\varepsilon} \int \frac{d^d k}{(2\pi)^d} \frac{1}{(p+k)^2 k^2} [\bar{s}_i \gamma^\mu \gamma^\lambda \gamma^\rho \gamma_\lambda \gamma_\mu (1 - \gamma_5) c_i] [\bar{u}_k \gamma_\rho (1 - \gamma_5) d_k] \\ &= C_F \frac{G_F}{\sqrt{2}} \frac{\alpha_s}{4\pi} V_{cs} V_{ud}^* (4\pi)^\varepsilon \left(\frac{\mu^2}{-p^2}\right)^\varepsilon \Gamma(\varepsilon) S_2, \end{aligned} \quad (3.2.14)$$

$$\begin{aligned} \mathcal{M}_{full}^b &= i\mu^{2\varepsilon} \int \frac{d^d k}{(2\pi)^d} [\bar{s}_i(-ig_s\gamma_\mu T_{ij}^a) \frac{i(\not{p} - \not{k})}{(p-k)^2} \frac{ig_2}{2\sqrt{2}} \gamma_\rho(1 - \gamma_5) V_{cs} c_j] \\ &\quad \times [\bar{u}_k(-ig_s\gamma_\nu T_{kl}^a) \frac{i(\not{p} + \not{k})}{(p+k)^2} \frac{ig_2}{2\sqrt{2}} \gamma_\sigma(1 - \gamma_5) V_{ud}^* d_l] \frac{-ig^{\rho\sigma}}{k^2 - m_W^2} \frac{-ig^{\mu\nu}}{k^2} \\ &= -i \frac{G_F}{4\sqrt{2}} g_s^2 V_{cs} V_{ud}^* \mu^{2\varepsilon} \int \frac{d^d k}{(2\pi)^d} \frac{1}{(p^2 - k^2) k^2} T_{ij}^a T_{kl}^a [\bar{s}_i \gamma^\mu T_{ij}^a \gamma^\lambda \gamma^\rho (1 - \gamma_5) c_j] \\ &\quad \times [\bar{u}_k \gamma_\mu T_{kl}^a \gamma_\lambda \gamma_\rho (1 - \gamma_5) d_l] \end{aligned}$$

$$= -\frac{4G_F}{\sqrt{2}} \frac{g_s}{4\pi} V_{cs} V_{ud}^* \left(\frac{1}{\varepsilon} + \ln \frac{\mu^2}{-p^2} \right) \left[\frac{1}{2} (S_1 - \frac{1}{N} S_2) \right], \quad (3.2.15)$$

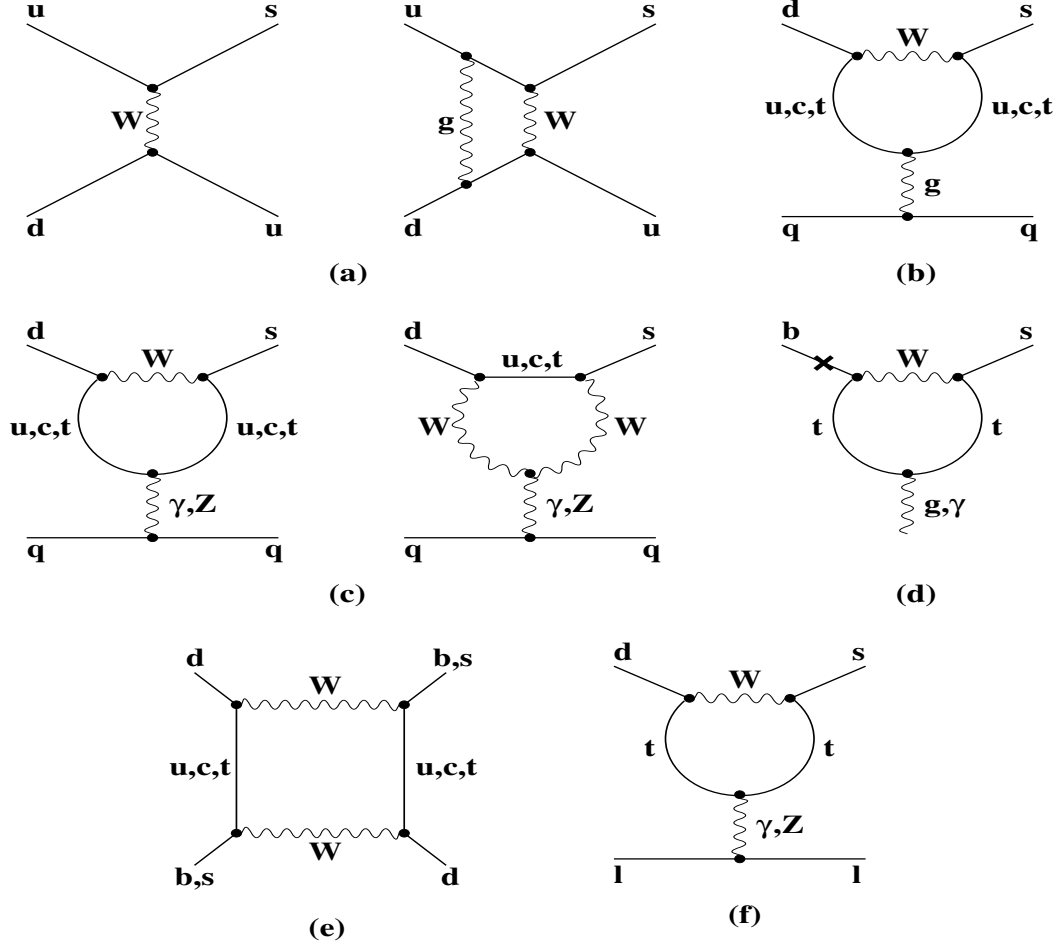


Figure 3.3: Typical diagrams in the full theory, tree, penguin and box in the SM.

$$\begin{aligned} \mathcal{M}_{full}^c &= i\mu^{2\varepsilon} \int \frac{d^d k}{(2\pi)^d} [\bar{s}_i (-ig_s \gamma_\mu T_{ij}^a) \frac{i(\not{p} - \not{k})}{(p-k)^2} \frac{ig_2}{2\sqrt{2}} \gamma_\rho (1 - \gamma_5) V_{cs} c_j] \\ &\quad \times [\bar{u}_k \frac{ig_2}{2\sqrt{2}} \gamma_\sigma (1 - \gamma_5) V_{ud}^* \frac{i(\not{p} - \not{k})}{(p-k)^2} (-ig_s \gamma_\nu T_{kl}^a) d_l] \frac{-ig^{\rho\sigma}}{k^2 - m_W^2} \frac{-ig^{\mu\nu}}{k^2} \\ &= -i \frac{g_s^2 g_2^2}{32m_W^2} V_{cs} V_{ud}^* \int \frac{d^d k}{(2\pi)^d} \frac{1}{(p-k)^2 k^2} [\bar{s}_i \gamma^\mu T_{ij}^a \gamma^\lambda \gamma^\rho (1 - \gamma_5) c_j] \end{aligned}$$

$$\begin{aligned}
 & \times [\gamma_\rho \bar{u}_k T_{kl}^a \gamma_\lambda \gamma_\mu (1 - \gamma_5) d_l] \\
 = & \frac{G_F}{\sqrt{2}} \frac{\alpha_s}{4\pi} V_{cs} V_{ud}^* \left[\frac{1}{\varepsilon} + \ln \frac{\mu^2}{-p^2} \right] \left[\frac{1}{2} (S_1 - \frac{1}{N} S_2) \right].
 \end{aligned} \tag{3.2.16}$$

Therefore, the sum of the above amplitudes is

$$\begin{aligned}
 \mathcal{M}_{full} = & \frac{G_F}{\sqrt{2}} V_{cs} V_{ud}^* \left[(1 + 2C_F \frac{\alpha_s}{4\pi} (\frac{1}{\varepsilon} + \ln \frac{\mu^2}{-p^2})) s_2 + \frac{3}{N} \frac{\alpha_s}{4\pi} \ln \frac{M_W^2}{-p^2} s_2 \right. \\
 & \left. - 3 \frac{\alpha_s}{4\pi} \ln \frac{M_W^2}{-p^2} s_1 \right],
 \end{aligned} \tag{3.2.17}$$

where N denotes the number of colours (N = 3 in QCD), and

$$T_{il}^a T_{lm}^a = (T^a T^a)_{im} = \frac{N^2 - 1}{N} \delta_{im} = C_F \delta_{im}, \tag{3.2.18}$$

$$T_{ij}^a T_{kl}^a = \frac{1}{2} (\delta_{il} \delta_{kj} - \frac{1}{N} \delta_{ij} \delta_{kl}), \tag{3.2.19}$$

$$k^\mu k^\nu \rightarrow \frac{k^2}{d} g^{\mu\nu}, \quad \gamma^\mu \gamma^\lambda \gamma^\rho \gamma_\lambda \gamma_\mu = 4\gamma^\rho, \tag{3.2.20}$$

$$[\gamma^\mu \gamma^\lambda \gamma^\rho (1 - \gamma_5)] [\gamma_\mu \gamma_\lambda \gamma_\rho (1 - \gamma_5)] = 16 [\gamma^\mu (1 - \gamma_5)] [\gamma_\mu (1 - \gamma_5)], \tag{3.2.21}$$

$$S_1 = [\bar{s}_i \gamma_L^\mu c_j] [\bar{u}_j \gamma_{\mu L} d_i], \quad S_2 = [\bar{s}_i \gamma_L^\mu c_i] [\bar{u}_k \gamma_{\mu L} d_k]. \tag{3.2.22}$$

To cancel the divergence term in the first term in Eq (3.2.17) we should renormalize the quark field and thus we need to include the renormalization constant $Z_q^{1/2}$ for each of the four external quark fields

$$q^0 = Z_q^{1/2} q \Rightarrow S_2^0 = Z_q^2 S_2, \tag{3.2.23}$$

to this end, we have

$$\mathcal{M}_{full} = \frac{G_F}{\sqrt{2}} V_{cs} V_{ud}^* \left[(1 + 2C_F \frac{\alpha_s}{4\pi} \ln \frac{\mu^2}{-p^2}) s_2 + \frac{3}{N} \frac{\alpha_s}{4\pi} \ln \frac{M_W^2}{-p^2} s_2 - 3 \frac{\alpha_s}{4\pi} \ln \frac{M_W^2}{-p^2} s_1 \right]. \tag{3.2.24}$$

- Calculation of hadronic matrix elements

The unrenormalized current-current matrix elements of the operators Q_1 and Q_2 can be found by calculating the diagrams in Fig 3.1(a-c), one finds

$$\begin{aligned}\langle Q_1 \rangle^0 &= \left(1 + 2C_F \frac{\alpha_s}{4\pi} \left[\frac{1}{\varepsilon} + \ln \frac{\mu^2}{-p^2} \right] \right) S_1 + \frac{3}{N} \frac{\alpha_s}{4\pi} \left(\frac{1}{\varepsilon} + \ln \frac{\mu^2}{-p^2} \right) S_1 \\ &\quad - 3 \frac{\alpha_s}{4\pi} \left(\frac{1}{\varepsilon} + \ln \frac{\mu^2}{-p^2} \right) S_2,\end{aligned}\tag{3.2.25}$$

$$\begin{aligned}\langle Q_2 \rangle^0 &= \left(1 + 2C_F \frac{\alpha_s}{4\pi} \left[\frac{1}{\varepsilon} + \ln \frac{\mu^2}{-p^2} \right] \right) S_2 + \frac{3}{N} \frac{\alpha_s}{4\pi} \left(\frac{1}{\varepsilon} + \ln \frac{\mu^2}{-p^2} \right) S_2 \\ &\quad - 3 \frac{\alpha_s}{4\pi} \left(\frac{1}{\varepsilon} + \ln \frac{\mu^2}{-p^2} \right) S_1.\end{aligned}\tag{3.2.26}$$

The divergences in the first terms can again be eliminated through the quark field renormalization. In the other hand, to remove the additional divergences, an additional operator renormalization is necessary

$$Q_i^0 = Z_{ij} Q_j,\tag{3.2.27}$$

where Q_i^0 denotes the unrenormalized operator and $Z_{2 \times 2}$ is the renormalization constant matrix. It can be found from Eqs. (3.2.25) and (3.2.26), we have

$$Z = 1 + \frac{\alpha_s}{4\pi} \frac{1}{\varepsilon} \begin{pmatrix} 3/N & -3 \\ -3 & 3/N \end{pmatrix}.\tag{3.2.28}$$

The relation between the unrenormalized $\langle Q \rangle^0$ and the renormalized amputated Green functions $\langle Q \rangle_i$ is then

$$\langle Q_i \rangle^0 = Z_q^{-2} Z_{ij} \langle Q_j \rangle.\tag{3.2.29}$$

Finally, the renormalized matrix elements $\langle Q_i \rangle$ are given by

$$\langle Q_1 \rangle = \left(1 + 2C_F \frac{\alpha_s}{4\pi} \ln \frac{\mu^2}{-p^2} \right) s_1 + \frac{3}{N} \frac{\alpha_s}{4\pi} \ln \frac{\mu^2}{-p^2} s_1 - 3 \frac{\alpha_s}{4\pi} \ln \frac{\mu^2}{-p^2} s_2,\tag{3.2.30}$$

$$\langle Q_2 \rangle = \left(1 + 2C_F \frac{\alpha_s}{4\pi} \ln \frac{\mu^2}{-p^2} \right) s_2 + \frac{3}{N} \frac{\alpha_s}{4\pi} \ln \frac{\mu^2}{-p^2} s_2 - 3 \frac{\alpha_s}{4\pi} \ln \frac{\mu^2}{-p^2} s_1.\tag{3.2.31}$$

- **Extraction of the Wilson coefficients**

From the requirement that

$$\mathcal{M}_{full} = \mathcal{M}_{eff} = \frac{G_F}{\sqrt{2}} V_{cs} V_{ud}^* (C_1 \langle Q_1 \rangle + C_2 \langle Q_2 \rangle). \quad (3.2.32)$$

Substituting $\langle Q_i \rangle$ into Eq. (3.2.32), we find

$$C_1 = -3 \frac{\alpha_s}{4\pi} \ln \frac{M_W^2}{\mu^2}, \quad C_2 = 1 + \frac{3}{N} \frac{\alpha_s}{4\pi} \ln \frac{M_W^2}{\mu^2}. \quad (3.2.33)$$

Without QCD corrections, the above Wilson coefficients become

$$C_1 = 0, \quad C_2 = 1. \quad (3.2.34)$$

The RG equations which give the μ independence of the amplitudes is given by

$$\frac{dC_i(\mu)}{d \ln \mu} = \gamma_{ij}(\alpha_s, \alpha) C_j(\mu), \quad (3.2.35)$$

where γ is the anomalous dimension of the operator Q_i and given by

$$\gamma(g) = \frac{1}{Z} \frac{dZ}{d \ln \mu}. \quad (3.2.36)$$

The low energy coefficients $C_i(\mu_b)$ can be extrapolated from the high energy ones $C_i(\mu_W)$ by solving the RGs for QCD and QED. The solution of the Eq. (3.2.35) is generally expressed as follows

$$C(\mu) = \sum_j U_{ij}(\mu, \mu_W) C_j(\mu_W), \quad (3.2.37)$$

where $U_{ij}(\mu, \mu_W)$ is the evolution matrix which takes into account the re-summation of terms proportional to large logs $(\alpha_s(\mu_W) \log(\mu_W/\mu_b))^n$ (leading) and $(\alpha_s^2(\mu_W) \log(\mu_W/\mu_b))^n$ (next-to-leading), etc., in QCD.

3.3 Effective Hamiltonian for $b \rightarrow q(=d, s)l_i^- l_j^+$

In this section we discuss the effective Hamiltonian for $b \rightarrow q(=d, s)l_i^- l_j^+$ transitions. We will focus on the $b \rightarrow sl_i^- l_j^+$ Hamiltonian as the $b \rightarrow dl_i^- l_j^+$ Hamiltonian can be obtained from it with the obvious replacements.

The effective Hamiltonian for the quark-level transition $b \rightarrow sl_i^- l_j^+$ ($l = e, \mu, \tau$) in the SM is mainly given by

$$\begin{aligned} \mathcal{H}_{\text{eff}}^{SM} = & -\frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \left\{ \sum_{i=1}^6 C_i(\mu) \mathcal{O}_i(\mu) + C_7 \frac{e}{16\pi^2} [\bar{s} \sigma_{\mu\nu} (m_s P_L + m_b P_R) b] F^{\mu\nu} \right. \\ & \left. + C_9 \frac{\alpha_{em}}{4\pi} (\bar{s} \gamma^\mu P_L b) L_{ij}^\mu + C_{10} \frac{\alpha_{em}}{4\pi} (\bar{s} \gamma^\mu P_L b) L_{ij}^{5\mu} \right\}, \end{aligned} \quad (3.3.1)$$

where $L_{ij}^\mu = \bar{l}_i \gamma_\mu l_j$, $L_{ij}^{5\mu} = \bar{l}_i \gamma_\mu \gamma_5 l_j$, and $P_{L,R} = (1 \mp \gamma_5)/2$. The operators $\mathcal{O}_i$ ($i = 1, \dots, 6$) correspond to the P_i in [112], and $m_b = m_b(\mu)$ is the running b -quark mass in the $\overline{\text{MS}}$ scheme. We use the SM Wilson coefficients as given in [113, 114].

The total effective Hamiltonian for $b \rightarrow sl_i^- l_j^+$ in the presence of new physics operators with all the possible Lorentz structure excluding tensor once can be expressed as [115]

$$\mathcal{H}_{\text{eff}}(b \rightarrow sl_i^- l_j^+) = \mathcal{H}_{\text{eff}}^{SM} + \mathcal{H}_{\text{eff}}^{VA} + \mathcal{H}_{\text{eff}}^{SP}, \quad (3.3.2)$$

where $\mathcal{H}_{\text{eff}}^{SM}$ is given by Eq. (3.3.1), and the NP contributions are

$$\begin{aligned} \mathcal{H}_{\text{eff}}^{VA} = & -N_F \left\{ R_V (\bar{s} \gamma^\mu P_L b) L_{ij}^\mu + R_A (\bar{s} \gamma^\mu P_L b) L_{ij}^{5\mu} + R'_V (\bar{s} \gamma^\mu P_R b) L_{ij}^\mu \right. \\ & \left. + R'_A (\bar{s} \gamma^\mu P_R b) L_{ij}^{5\mu} \right\}, \end{aligned} \quad (3.3.3)$$

$$\begin{aligned} \mathcal{H}_{\text{eff}}^{SP} = & -N_F \left\{ R_S (\bar{s} P_R b) L_{ij} + R_P (\bar{s} P_R b) L_{ij}^5 + R'_S (\bar{s} P_L b) L_{ij} \right. \\ & \left. + R'_P (\bar{s} P_L b) L_{ij}^5 \right\}, \end{aligned} \quad (3.3.4)$$

where $N_F = \frac{4G_F}{\sqrt{2}} \frac{\alpha_{em}}{4\pi} V_{ts}^* V_{tb}$, $L_{ij} = \bar{l}_i l_j$, and $L_{ij}^5 = \bar{l}_i \gamma_5 l_j$. In the above expressions, R_i , R'_i ($i = V, A, S, P$) are the NP effective couplings which are in general complex.

In terms of all the Wilson coefficients, the branching ratio for the decay $B_s \rightarrow l_i^- l_j^+$ can be obtained from

$$BR(B_s \rightarrow l_i^- l_j^+) = \frac{\Gamma(B_s \rightarrow l_i^- l_j^+)}{\Gamma_{\text{total}}}, \quad (3.3.5)$$

where $\Gamma(B_s \rightarrow l_i^- l_j^+)$ is the decay width and $\Gamma_{\text{total}} = \hbar/\tau_{B_s}$.

On the other, the decay width for the process $B_s \rightarrow l_i^- l_j^+$ can be given by

$$\Gamma(B_s \rightarrow l_i^- l_j^+) = \frac{1}{16\pi m_{B_s}} \sqrt{\left[1 - \left(\frac{m_i + m_j}{m_{B_s}}\right)^2\right] \left[1 - \left(\frac{m_i - m_j}{m_{B_s}}\right)^2\right]} |\mathcal{M}(B_s \rightarrow l_i^- l_j^+)|^2. \quad (3.3.6)$$

Therefore, one finds

$$\begin{aligned} BR(B_s \rightarrow l_i^- l_j^+) &= \frac{\tau_{B_s}}{\hbar} \frac{f_{B_s}^2 G_F^2 m_{B_s} \alpha_{em}^2 |V_{tb} V_{ts}^*|^2}{64\pi^3} \sqrt{\left[1 - \left(\frac{m_i + m_j}{m_{B_s}}\right)^2\right] \left[1 - \left(\frac{m_i - m_j}{m_{B_s}}\right)^2\right]} \\ &\times \left\{ \left[1 - \left(\frac{m_i + m_j}{m_{B_s}}\right)^2\right] |F_V(m_i - m_j) + F_S|^2 \right. \\ &\left. + \left[1 - \left(\frac{m_i - m_j}{m_{B_s}}\right)^2\right] |F_A(m_i + m_j) + F_P|^2 \right\}, \end{aligned} \quad (3.3.7)$$

where

$$\begin{aligned} F_V &= C_9 + R_V - R'_V, \\ F_A &= C_{10} + R_A - R'_A, \\ F_S &= r_\chi (R_S - R'_S), \\ F_P &= r_\chi (R_P - R'_P), \end{aligned} \quad (3.3.8)$$

with $r_\chi = \frac{m_{B_s}^2}{m_b(\mu) + m_s(\mu)}$. The Wilson coefficient F_V does not contribute in the lepton flavor conserving decays $B_s \rightarrow \mu^- \mu^+$ and $B_s \rightarrow \tau^- \tau^+$. Within the SM, $F_A = C_{10}$ only contributes to $B_s \rightarrow \mu^- \mu^+$ and $B_s \rightarrow \tau^- \tau^+$. The lepton flavor violating decay $B_s \rightarrow \tau \mu$ is not allowed in the SM. Thus, the Wilson coefficients C_9 and C_{10} do not contribute to this decay. Next, we look at two models of NP that contribute to $B_{s,d} \rightarrow l_i^- l_j^+$ decays.

3.4 $B_{s,d} \rightarrow l_i^- l_j^+$ in 2HDM

In this section we consider the model where a neutral scalar boson X with mass ~ 125 GeV and flavor-violating couplings [101, 102] mediates the $B_{s,d} \rightarrow l_i^- l_j^+$ decays, as shown in Fig. 3.4. Figure should have l_i and l_j .

In this model, the effective Lagrangian which describes the possible flavor-violating couplings of X to the SM fermion pairs in the mass basis is, [101, 102]

$$\mathcal{L}_Y = -m_i \bar{f}_L^i f_R^i - Y_{ij} (\bar{f}_L^i f_R^j) X + h.c + \dots, \quad (3.4.1)$$

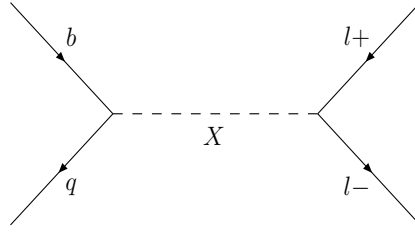


Figure 3.4: $b \rightarrow ql_+l_-$ through the scalar X mediation.

where the ellipses denote nonrenormalizable couplings involving more than one Higgs field operator and $f_L = q_L, l_L$ are the $SU(2)_L$ doublets while $f_R = u_R d_R, \nu_R, l_R$ are the weak singlets. The indices run over generations and fermion flavors with summation implicitly understood. In the SM the Higgs couplings are diagonal, $Y_{ij} = (m_i/v)\delta_{ij}$ with $v = 246$ GeV, but in a general NP models the structure of the Y_{ij} can be different. The couplings in Eq. (3.4.1) will also lead to the decay $b \rightarrow sl_i^- l_j^+$ at tree level, mediated by a virtual X . The lagrangian gives

$$\begin{aligned} R_{S(P)} &= \frac{1}{N_F} \frac{Y_{sb}(Y_{\mu\tau} + Y_{\tau\mu}^*)}{4m_X^2}, \\ R'_{S(P)} &= \frac{1}{N_F} \frac{Y_{bs}^*(Y_{\mu\tau} - Y_{\tau\mu}^*)}{4m_X^2}, \\ R_{V(A)} &= R'_{V(A)} = 0. \end{aligned} \quad (3.4.2)$$

The branching ratio for the $B_s \rightarrow \mu^- \tau^+$ decay can be obtained from Eq. (3.3.7) as,

$$\begin{aligned}
 BR(B_s \rightarrow \mu^- \tau^+) &= \frac{\tau_{B_s}}{\hbar} \frac{f_{B_s}^2 m_{B_s}^5}{512\pi(m_b(\mu) + m_s(\mu))^2} \\
 &\times \sqrt{\left[1 - \left(\frac{m_\tau + m_\mu}{m_{B_s}}\right)^2\right] \left[1 - \left(\frac{m_\tau - m_\mu}{m_{B_s}}\right)^2\right]} \\
 &\times \frac{|Y_{sb} - Y_{bs}^*|^2}{m_X^4} \left\{ \left[1 - \left(\frac{m_\tau + m_\mu}{m_{B_s}}\right)^2\right] |Y_{\mu\tau} + Y_{\tau\mu}^*|^2 \right. \\
 &\quad \left. + \left[1 - \left(\frac{m_\tau - m_\mu}{m_{B_s}}\right)^2\right] |Y_{\mu\tau} - Y_{\tau\mu}^*|^2 \right\}. \quad (3.4.3)
 \end{aligned}$$

Also, the branching ratio for the $B_s \rightarrow l^- l^+$ ($l = \mu, \tau$) decays is

$$\begin{aligned}
 BR(B_s \rightarrow l^- l^+) &= \frac{\tau_{B_s}}{\hbar} \frac{f_{B_s}^2 G_F^2 m_{B_s} \alpha_{em}^2 |V_{tb} V_{ts}^*|^2}{64\pi^3} \sqrt{1 - \frac{4m_l^2}{m_{B_s}^2}} \left[\left(1 - \frac{4m_l^2}{m_{B_s}^2}\right) \right. \\
 &\quad \left. \left| \frac{Re[Y_{ll}](Y_{sb} - Y_{bs}^*)}{2m_X^2 N_F(m_b(\mu) + m_s(\mu))} \right|^2 + \left(\frac{2m_l}{m_{B_s}}\right)^2 |C_{10}|^2 \right], \quad (3.4.4)
 \end{aligned}$$

where we assume Y_{ll} is real and set it to its SM value $Y_{ll} = m_l/v$. Flavor violating Higgs coupling in Eq. (3.4.1) can generate flavor changing neutral currents at tree level. The couplings Y_{sb} and Y_{bs} are constrained by the mass difference ΔM_s in the $B_s - \bar{B}_s$ mixing. The $\Delta B = 2$ weak Hamiltonian for this process can be found in the Ref. [102].

The theoretical expression for ΔM_s ($\Delta M_s = \frac{1}{m_{B_s}} \langle B_s^0 | H_{\text{eff}}^{\Delta B=2} | \bar{B}_s^0 \rangle$) is given in Ref. [116]. It is found that one can reproduce the measured value of $\Delta M_s = (17.719 \pm 0.043) ps^{-1}$ [117] if the upper bound on the flavor changing couplings is

$$|Y_{sb} - Y_{bs}^*| \sim 10^{-3}. \quad (3.4.5)$$

We perform a fit to constrain the leptonic couplings $Y_{\mu\tau}$ and $Y_{\tau\mu}$. The bound on these couplings are obtained from the decay rates of $\tau \rightarrow \mu\gamma$, $\tau \rightarrow 3\mu$, and the magnetic (δa_μ) and the electric dipole (d_μ) moments. The theoretical expressions for $\Gamma(\tau \rightarrow \mu\gamma)$, $\Gamma(\tau \rightarrow 3\mu)$, $\Delta a_\mu \equiv a_\mu^{\text{exp}} - a_\mu^{\text{SM}}$, and d_μ are given by [102]

$$\Gamma(\tau \rightarrow \mu\gamma) = \frac{\alpha m_\tau^5}{64\pi^4} (|c_L|^2 + |c_R|^2), \quad (3.4.6)$$

$$\Gamma(\tau \rightarrow 3\mu) \simeq \frac{\alpha^2 m_\tau^5}{6(2\pi)^5} \left| \log \frac{m_\mu^2}{m_\tau^2} - \frac{11}{4} \right| (|c_L|^2 + |c_R|^2), \quad (3.4.7)$$

$$a_\mu \equiv \frac{g_\mu - 2}{2} \simeq \frac{\Re(Y_{\mu\tau} Y_{\tau\mu})}{8\pi^2} \frac{m_\mu m_\tau}{2m_h^2} \left(2 \log \frac{m_h^2}{m_\tau^2} - 3 \right), \quad (3.4.8)$$

$$d_\mu \simeq -\frac{\Im(Y_{\mu\tau} Y_{\tau\mu})}{16\pi^2} \frac{e m_\tau}{2m_h^2} \left(2 \log \frac{m_h^2}{m_\tau^2} - 3 \right), \quad (3.4.9)$$

where c_L and c_R are the corresponding Wilson coefficients, and their expressions are

$$c_L \simeq \frac{1}{12m_h^2} Y_{\tau\tau} Y_{\tau\mu}^* \left(-4 + 3 \log \frac{m_h^2}{m_\tau^2} \right), \quad (3.4.10)$$

$$c_R \simeq \frac{1}{12m_h^2} Y_{\mu\tau} Y_{\tau\tau} \left(-4 + 3 \log \frac{m_h^2}{m_\tau^2} \right). \quad (3.4.11)$$

The experimental bounds for these observables are taken from [118–120] :

$$\begin{aligned} BR(\tau \rightarrow \mu\gamma) &< 4.4 \times 10^{-8} \\ BR(\tau \rightarrow 3\mu) &< 2.1 \times 10^{-8} \\ \Delta a_\mu &= (2.87 \pm 0.63 \pm 0.49) 10^{-9} \\ -10 \times 10^{-20} e\,cm &< d_\mu < 8 \times 10^{-20} e\,cm. \end{aligned} \quad (3.4.12)$$

In the fit, $BR(\tau \rightarrow \mu\gamma)$, $BR(\tau \rightarrow 3\mu)$, Δa_μ and d_μ are constrained by their experimental results in Eq. (3.4.12) and Δa_μ is varied within 1σ errors. As shown in Fig. 3.5 (right panel), Δa_μ provides a strong bound on the couplings $Y_{\mu\tau}$ and $Y_{\tau\mu}$, and in particular requires $|Y_{\mu\tau} + Y_{\tau\mu}^*| \gtrsim 0.09$. It is found that one can satisfy the above four experimental bounds if $|Y_{\mu\tau}| < 0.064$, $|Y_{\tau\mu}| < 0.061$ for $m_X = 125$ GeV. The fit results allow us to estimate the decay rate $BR(B_s \rightarrow \mu\tau) = BR(B_s \rightarrow \mu^- \tau^+) + BR(B_s \rightarrow \tau^- \mu^+)$. The variations of the decay rates $BR(B_s \rightarrow \mu\tau)$ with $|Y_{sb} - Y_{bs}^*|$ and $|Y_{\mu\tau} + Y_{\tau\mu}^*|$ are shown in Fig. 3.5. Our result predict that the $BR(B_s \rightarrow \mu\tau)$ can be as large as $\sim 6 \times 10^{-6}$ for $m_X = 125$ GeV and $f_{B_s} = 0.229 \pm 0.006$ GeV [121].

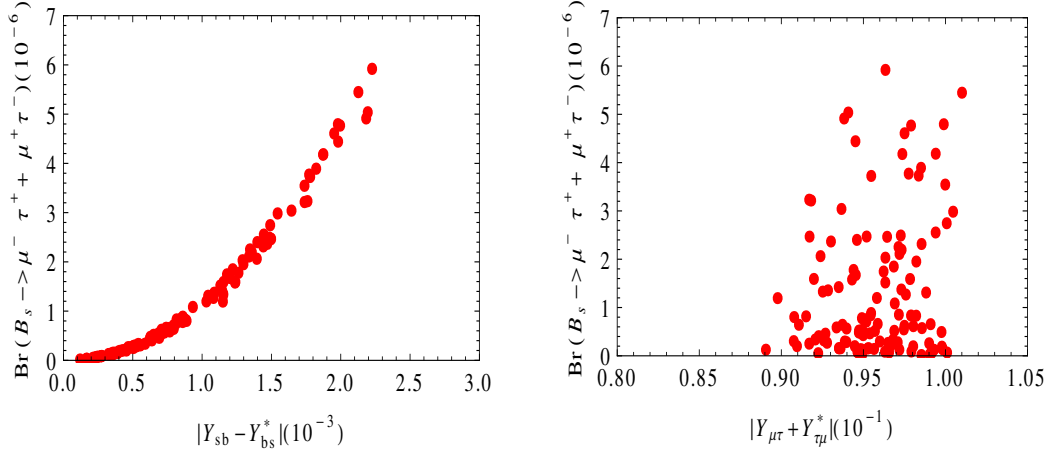


Figure 3.5: The variation of $BR(B_s \rightarrow \mu^- \tau^+ + \mu^+ \tau^-)$ with the couplings $|Y_{sb} - Y_{bs}^*|$ and $|Y_{\mu\tau} - Y_{\tau\mu}^*|$ for $m_X = 125 \text{ GeV}$ and $f_{B_s} = .229 \pm 0.006 \text{ GeV}$. Scatter points are allowed by ΔM_s^{exp} and the experimental bounds in Eq. (3.4.12).

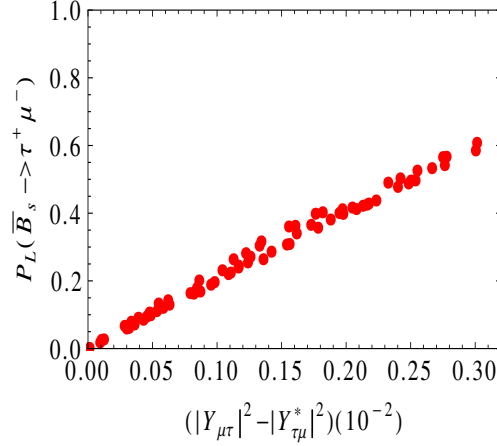


Figure 3.6: Figure shows the $(|Y_{\mu\tau}|^2 - |Y_{\tau\mu}^*|^2)$ dependence of $P_L(B_s \rightarrow \mu^- \tau^+)$. Scatter points are allowed by ΔM_s^{exp} and the experimental bounds in Eq. (3.4.12).

In this model, we obtain the branching ratio $BR(B_s \rightarrow \mu^- \mu^+) < 4.3 \times 10^{-9}$ using $Y_{\mu\mu} = m_\mu/v$ with $v = 246 \text{ GeV}$, $m_X = 125 \text{ GeV}$ and $f_{B_s} = 0.229 \pm 0.006 \text{ GeV}$. The corresponding SM prediction is $\approx 3.17 \times 10^{-9}$. Our results for $BR(B_s \rightarrow \mu^- \mu^+)$ is

consistent with the current upper limit $BR(B_s \rightarrow \mu^- \mu^+) \equiv [1.1 - 6.4] \times 10^{-9}$ at 95% C.L. in Ref. [122]. Also, we obtain the branching ratio $BR(B_s \rightarrow \tau^- \tau^+) < 8.1 \times 10^{-7}$ using $Y_{\tau\tau} = m_\tau/v$ and $m_X = 125$ GeV. The values of this branching ratio in the SM is $\approx 6.81 \times 10^{-7}$. The latest LHCb-measurement of Γ_d/Γ_s implies a limit of $BR(B_s \rightarrow \tau^- \tau^+) < 3\%$ [3]. In passing we note that, the large width difference in the B_s meson system can changes these results by $\sim \mathcal{O}(10\%)$ as pointed out in [124–126].

Finally, in this model we obtain the tau longitudinal polarization fraction for the decays $B_s \rightarrow \mu^- \tau^+$:

$$\begin{aligned}
 P_L &= \frac{Br_{B_s \rightarrow \mu^- \tau^+}[\lambda_\tau = 1/2] - Br_{B_s \rightarrow \mu^- \tau^+}[\lambda_\tau = -1/2]}{Br_{B_s \rightarrow \mu^- \tau^+}[\lambda_\tau = 1/2] + Br_{B_s \rightarrow \mu^- \tau^+}[\lambda_\tau = -1/2]} \\
 &= \sqrt{\left[1 - \left(\frac{m_\tau + m_\mu}{m_{B_s}}\right)^2\right] \left[1 - \left(\frac{m_\tau - m_\mu}{m_{B_s}}\right)^2\right]} \times \left(2(|Y_{\mu\tau}|^2 - |Y_{\tau\mu}^*|^2)\right) \\
 &\quad / \left\{ \left[1 - \left(\frac{m_\tau + m_\mu}{m_{B_s}}\right)^2\right] |Y_{\mu\tau} + Y_{\tau\mu}^*|^2 + \left[1 - \left(\frac{m_\tau - m_\mu}{m_{B_s}}\right)^2\right] |Y_{\mu\tau} - Y_{\tau\mu}^*|^2 \right\}. \quad (3.4.13)
 \end{aligned}$$

The result indicates P_L does not depend on the $b \rightarrow s$ couplings. Fig. 3.6 shows the dependency of P_L on the quantity $(|Y_{\mu\tau}|^2 - |Y_{\tau\mu}^*|^2)$. The couplings Y_{db} and Y_{bd} for the decay $BR(B \rightarrow \mu\tau) = BR(B \rightarrow \mu^- \tau^+) + BR(B \rightarrow \tau^- \mu^+)$ are similarly constrained by the mass difference $\Delta M_d = (0.507 \pm 0.004) ps^{-1}$ [123]. The measured ΔM_d can be reproduced if the upper bound on the flavor changing couplings is $|Y_{db} - Y_{bd}^*| \sim 10^{-4}$. Using this result, we find $BR(B \rightarrow \mu\tau) < 2 \times 10^{-7}$.

3.5 $B_{s,d} \rightarrow l_i^- l_j^+$ in the MSSM

The supersymmetry contributions to the process $B_s \rightarrow \mu^+ \tau^-$ are given by the one-loop box diagrams, shown in Fig 3.7, where charginos and neutralinos are exchanged. The interaction of charginos, neutralinos can be written as

$$\mathcal{L} = \sum_{j=1}^2 \bar{\tilde{\chi}}_j \left[\tilde{u}_i^\dagger (G_{IJij}^{U_L} P_L + G_{IJij}^{U_R} P_R) d^I + \tilde{\nu}_J^\dagger (G_{IJj}^{L_L} P_L + G_{IJj}^{L_R} P_R) l^I \right] + h.c.$$

$$+ \sum_{k=1}^4 \bar{\tilde{\chi}}_k^0 \left[\tilde{d}_i^\dagger (Z_{Ik}^{D_L} P_L + Z_{Ik}^{D_R} P_R) d^I + \tilde{l}_i^\dagger (Z_{Ik}^{L_L} P_L + Z_{Ik}^{L_R} P_R) l^I \right] + h.c., \quad (3.5.1)$$

where the mixing matrices in the super-CKM basis are given by [127]

$$G_{IJij}^{U_L} = g \left[-V_{j1}^* (\Gamma^{U_L})_{Ji} + V_{j2}^* (\Gamma^{U_R})_{Ji} \frac{m_{u^J}}{\sqrt{2} M_W \sin \beta} \right] V_{JI}^{CKM}, \quad (3.5.2)$$

$$G_{IJij}^{U_R} = g U_{j2} (\Gamma^{U_L})_{Ji} \frac{m_{d^I}}{\sqrt{2} m_W \cos \beta} V_{JI}^{CKM}, \quad (3.5.3)$$

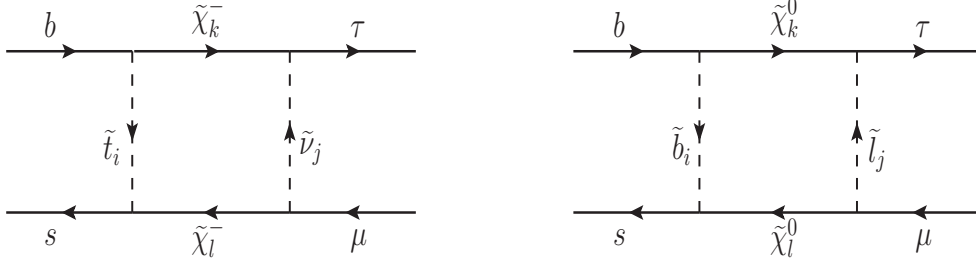


Figure 3.7: One loop SUSY contributions to $B_s \rightarrow \tau^- \mu^+$.

$$G_{IJj}^{L_L} = -g V_{j1}^* (\Gamma^{\tilde{\nu}})_{IJ}, \quad G_{IJj}^{L_R} = g U_{j2} (\Gamma^{\tilde{\nu}})_{IJ} \frac{m_{l^I}}{\sqrt{2} m_W \cos \beta}, \quad (3.5.4)$$

$$Z_{Ik}^{L_L} = -\frac{g}{\sqrt{2}} \left[(N_{k2}^* + \tan \theta_W N_{k1}^*) (\Gamma^{L_L})_{Ii} - N_{k3}^* (\Gamma^{L_R})_{Ii} \frac{m_l}{m_W \cos \beta} \right], \quad (3.5.5)$$

$$Z_{Ik}^{L_R} = -\frac{g}{\sqrt{2}} \left[2 \tan \theta_W N_{k1} (\Gamma^{L_R})_{Ii} + N_{k3} (\Gamma^{L_L})_{Ii} \frac{m_l}{m_W \cos \beta} \right], \quad (3.5.6)$$

$$Z_{Ik}^{D_L} = -\frac{g}{\sqrt{2}} \left[(N_{k2}^* + \frac{1}{2} \tan \theta_W N_{k1}^*) (\Gamma^{D_L})_{Ii} + N_{k3}^* (\Gamma^{D_R})_{Ii} \frac{m_d}{M_W \cos \beta} \right], \quad (3.5.7)$$

$$Z_{Ik}^{D_R} = -\frac{g}{\sqrt{2}} \left[\frac{2}{3} \tan \theta_W N_{k1} (\Gamma^{D_R})_{Ii} + N_{k3} (\Gamma^{D_L})_{Ii} \frac{m_d}{m_W \cos \beta} \right], \quad (3.5.8)$$

where $I, J = 1..3$ and $i = 1..6$. The matrices Γ 's are the matrices that diagonalize the squark and slepton mass matrices, while U and V are the unitary matrices that diagonalize the charged mass matrices. Finally, N is the matrix that diagonalizes the neutralino mass matrix.

3.5.1 Chargino Contribution

The general expression for the amplitude of chargino contribution to $b \rightarrow sl^+l^-$ is given by

$$\begin{aligned}
 \mathcal{M}_{\tilde{\chi}^\pm} = & -\frac{i}{16\pi^2} \left(m_{\tilde{\chi}_l^+} m_{\tilde{\chi}_k^+} B(m_{\tilde{\chi}_k^+}, m_{\tilde{\chi}_l^+}, m_{\tilde{t}_i}, m_{\tilde{\nu}_j}) \left[\frac{1}{2} G_{2Nl}^{LL} G_{3Nk}^{LR*} G_{3Hik}^{UL} G_{2Jil}^{*UR} \right. \right. \\
 & \times (\bar{s} P_L b)(\bar{\tau} P_L \mu) + \frac{1}{2} G_{2Nl}^{LR} G_{3Nk}^{LL*} G_{3Hik}^{UR} G_{2Jil}^{*UL} (\bar{s} P_R b)(\bar{\tau} P_R \mu) \\
 & + \frac{1}{4} G_{3Nk}^{LL*} G_{2Nl}^{LL} G_{3Hik}^{UR} G_{2Jil}^{*UR} (\bar{s} \gamma^\mu P_L b)(\bar{\tau} \gamma_\mu P_R \mu) \\
 & + \frac{1}{4} G_{3Nk}^{LR*} G_{2Nl}^{LR} G_{3Hik}^{UL} G_{2Jil}^{*UL} (\bar{s} \gamma^\mu P_R b)(\bar{\tau} \gamma_\mu P_L \mu) \Big] \\
 & + \frac{1}{4} S(m_{\tilde{\chi}_k^+}, m_{\tilde{\chi}_l^+}, m_{\tilde{t}_i}, m_{\tilde{\nu}_j}) \left[G_{2Nl}^{LL} G_{3Nk}^{LR*} G_{3Hik}^{UL} G_{2Jil}^{*UR} (\bar{s} \gamma^\mu P_L b)(\bar{\tau} \gamma_\mu P_L \mu) \right. \\
 & + G_{2Nl}^{LR} G_{3Nk}^{LL*} G_{3Hik}^{UR} G_{2Jil}^{*UL} (\bar{s} \gamma^\mu P_R b)(\bar{\tau} \gamma_\mu P_R \mu) + 2 G_{3Nk}^{LL*} G_{2Nl}^{LL} G_{3Hik}^{UR} G_{2Jil}^{*UR} \\
 & \times (\bar{s} P_L b)(\bar{\tau} P_R \mu) + 2 G_{3Nk}^{LR*} G_{2Nl}^{LR} G_{3Hik}^{UL} G_{2Jil}^{*UL} (\bar{s} P_R b)(\bar{\tau} P_L \mu) \Big] \Big). \quad (3.5.9)
 \end{aligned}$$

Therefore, one can write this amplitude as

$$\mathcal{M}_{\tilde{\chi}^\pm} = \mathcal{M}_{\tilde{\chi}^\pm}^{S-P} + \mathcal{M}_{\tilde{\chi}^\pm}^{V-A}, \quad (3.5.10)$$

with

$$\begin{aligned}
 \mathcal{M}_{\tilde{\chi}^\pm}^{S-P} = & -\frac{i}{16\pi^2} \left((C_{SRR} + C_{SRL})(\bar{s} P_R b) \bar{\tau} \mu + (C_{SLL} + C_{SLR})(\bar{s} P_L b) \bar{\tau} \mu \right. \\
 & + (C_{SRR} - C_{SRL})(\bar{s} P_R b) \bar{\tau} \gamma^5 \mu \\
 & \left. + (C_{SLR} - C_{SLL})(\bar{s} P_L b) \bar{\tau} \gamma^5 \mu \right), \quad (3.5.11)
 \end{aligned}$$

$$\quad (3.5.12)$$

and

$$\begin{aligned}
 \mathcal{M}_{\tilde{\chi}^\pm}^{V-A} = & -\frac{i}{16\pi^2} \left((C_{VLL} + C_{VLR})(\bar{s} \gamma^\mu P_L b)(\bar{\tau} \gamma_\mu \mu) + (C_{VRR} + C_{VRL})(\bar{s} \gamma^\mu P_R b) \right. \\
 & \times (\bar{\tau} \gamma_\mu \mu) + (C_{VLR} - C_{VLL})(\bar{s} \gamma^\mu P_L b)(\bar{\tau} \gamma_\mu \gamma^5 \mu) + (C_{VRR} - C_{VRL})(\bar{s} \gamma^\mu P_R b) \\
 & \left. \times (\bar{\tau} \gamma_\mu \gamma^5 \mu) \right), \quad (3.5.13)
 \end{aligned}$$

where

$$C_{SLL} = \frac{g^4}{2m_W^2} V_{tb}^{CKM} V_{ts}^{CKM*} V_{l1}^* U_{l2}^* \Gamma_{33}^{\tilde{\nu}} \Gamma_{23}^{\tilde{\nu}} \left[U_{k2}^* V_{k1}^* \Gamma_{3i}^{UL} \Gamma_{3i}^{UL*} - \frac{m_t}{\sqrt{2}m_W \sin \beta} U_{k2}^* V_{k2}^* \Gamma_{3i}^{UR} \Gamma_{3i}^{UL*} \right]$$

$$\times \frac{m_s m_\tau m_{\tilde{\chi}_l^+} m_{\tilde{\chi}_k^+}}{4 \cos^2 \beta} B(m_{\tilde{\chi}_k^+}, m_{\tilde{\chi}_l^+}, m_{\tilde{t}_i}, m_{\tilde{\nu}_j}), \quad (3.5.14)$$

$$\begin{aligned} C_{SRR} = & \frac{g^4}{2m_W^2} V_{tb}^{CKM} V_{ts}^{CKM*} V_{k1} U_{k2} \Gamma_{33}^{\tilde{\nu}*} \Gamma_{23}^{\tilde{\nu}} \left[U_{l2} V_{l1} \Gamma_{3i}^{U_L} \Gamma_{3i}^{U_L*} - \frac{m_t}{\sqrt{2} m_W \sin \beta} U_{l2} V_{l2} \Gamma_{3i}^{U_R*} \Gamma_{3i}^{U_L} \right] \\ & \times \frac{m_b m_\mu m_{\tilde{\chi}_l^+} m_{\tilde{\chi}_k^+}}{4 \cos^2 \beta} B(m_{\tilde{\chi}_k^+}, m_{\tilde{\chi}_l^+}, m_{\tilde{t}_i}, m_{\tilde{\nu}_j}), \end{aligned} \quad (3.5.15)$$

$$\begin{aligned} C_{SLR} = & \frac{g^4}{2m_W^2} V_{tb}^{CKM} V_{ts}^{CKM*} \Gamma_{33}^{\tilde{\nu}*} \Gamma_{23}^{\tilde{\nu}} (U_{l2}^* V_{l1}^*) (U_{k2} V_{k1}) \Gamma_{3i}^{U_L} \Gamma_{3i}^{U_L*} \\ & \times \frac{m_b m_s}{4 \cos^2 \beta} S(m_{\tilde{\chi}_k^+}, m_{\tilde{\chi}_l^+}, m_{\tilde{t}_i}, m_{\tilde{\nu}_j}), \end{aligned} \quad (3.5.16)$$

$$\begin{aligned} C_{SRL} = & \frac{g^4}{2m_W^2} V_{tb}^{CKM} V_{ts}^{CKM*} \Gamma_{33}^{\tilde{\nu}*} \Gamma_{23}^{\tilde{\nu}} \left[(U_{k2}^* V_{k1}^*) (U_{l2}^* V_{l1}) \Gamma_{3i}^{U_L} \Gamma_{3i}^{U_L*} \right. \\ & + (U_{k2}^* V_{k2}^*) (U_{l2}^* V_{l2}) \Gamma_{3i}^{U_R} \Gamma_{3i}^{U_R*} \frac{m_t^2}{2m_W^2 \sin^2 \beta} \\ & \left. - \frac{m_t}{\sqrt{2} m_W \sin \beta} \left((U_{k2}^* V_{k1}^*) (U_{l2}^* V_{l2}) \Gamma_{3i}^{U_L} \Gamma_{3i}^{U_R*} + (U_{k2}^* V_{k2}^*) (U_{l2}^* V_{l1}) \Gamma_{3i}^{U_L*} \Gamma_{3i}^{U_R} \right) \right] \\ & \times \frac{m_\mu m_\tau}{4 \cos^2 \beta} S(m_{\tilde{\chi}_k^+}, m_{\tilde{\chi}_l^+}, m_{\tilde{t}_i}, m_{\tilde{\nu}_j}), \end{aligned} \quad (3.5.17)$$

and

$$\begin{aligned} C_{VLL} = & \frac{g^4}{2m_W^2} V_{tb}^{CKM} V_{ts}^{CKM*} V_{l1}^* U_{l2}^* \Gamma_{33}^{\tilde{\nu}*} \Gamma_{23}^{\tilde{\nu}} \left(U_{k2}^* V_{k1}^* \Gamma_{3i}^{U_L} \Gamma_{3i}^{U_L*} \right. \\ & \left. - \frac{m_t}{\sqrt{2} m_W \sin \beta} U_{k2}^* V_{k2}^* \Gamma_{3i}^{U_R} \Gamma_{3i}^{U_L*} \right) \frac{m_s m_\tau}{8 \cos^2 \beta} S(m_{\tilde{\chi}_k^+}, m_{\tilde{\chi}_l^+}, m_{\tilde{t}_i}, m_{\tilde{\nu}_j}), \end{aligned} \quad (3.5.18)$$

$$\begin{aligned} C_{VRR} = & \frac{g^4}{2m_W^2} V_{tb}^{CKM} V_{ts}^{CKM*} V_{k1} U_{k2} \Gamma_{33}^{\tilde{\nu}*} \Gamma_{23}^{\tilde{\nu}} \left(U_{l2} V_{l1} \Gamma_{3i}^{U_L} \Gamma_{3i}^{U_L*} \right. \\ & \left. - \frac{m_t}{\sqrt{2} m_W \sin \beta} U_{l2} V_{l2} \Gamma_{3i}^{U_R*} \Gamma_{3i}^{U_L} \right) \frac{m_b m_\mu}{8 \cos^2 \beta} S(m_{\tilde{\chi}_k^+}, m_{\tilde{\chi}_l^+}, m_{\tilde{t}_i}, m_{\tilde{\nu}_j}), \end{aligned} \quad (3.5.19)$$

$$\begin{aligned} C_{VLR} = & \frac{g^4}{2m_W^2} V_{tb}^{CKM} V_{ts}^{CKM*} \Gamma_{33}^{\tilde{\nu}*} \Gamma_{23}^{\tilde{\nu}} (U_{l2}^* V_{l1}^*) (U_{k2} V_{k1}) \Gamma_{3i}^{U_L} \Gamma_{3i}^{U_L*} \frac{m_b m_s m_{\tilde{\chi}_l^+} m_{\tilde{\chi}_k^+}}{8 \cos^2 \beta} \\ & \times B(m_{\tilde{\chi}_k^+}, m_{\tilde{\chi}_l^+}, m_{\tilde{t}_i}, m_{\tilde{\nu}_j}), \end{aligned} \quad (3.5.20)$$

$$\begin{aligned} C_{VRL} = & \frac{g^4}{2m_W^2} V_{tb}^{CKM} V_{ts}^{CKM*} \Gamma_{33}^{\tilde{\nu}*} \Gamma_{23}^{\tilde{\nu}} \left[(U_{k2}^* V_{k1}^*) (U_{l2}^* V_{l1}) \Gamma_{3i}^{U_L} \Gamma_{3i}^{U_L*} \right. \\ & + (U_{k2}^* V_{k2}^*) (U_{l2}^* V_{l2}) \Gamma_{3i}^{U_R} \Gamma_{3i}^{U_R*} \frac{m_t^2}{2m_W^2 \sin^2 \beta} \\ & \left. - \frac{m_t}{\sqrt{2} m_W \sin \beta} \left((U_{k2}^* V_{k1}^*) (U_{l2}^* V_{l2}) \Gamma_{3i}^{U_L} \Gamma_{3i}^{U_R*} + (U_{k2}^* V_{k2}^*) (U_{l2}^* V_{l1}) \Gamma_{3i}^{U_L*} \Gamma_{3i}^{U_R} \right) \right] \end{aligned}$$

$$\times \frac{m_\mu m_\tau m_{\tilde{\chi}_l^+} m_{\tilde{\chi}_k^+}}{8 \cos^2 \beta} S(m_{\tilde{\chi}_k^+}, m_{\tilde{\chi}_l^+}, m_{\tilde{t}_i}, m_{\tilde{\nu}_j}). \quad (3.5.21)$$

Therefore, from Eqs.(3.3.3) and (3.3.4) one finds

$$R_{V,A} = \frac{i}{16\pi^2} \frac{1}{N_F} (C_{VLR} \pm C_{VLL}), \quad R'_{V,A} = \frac{i}{16\pi^2} \frac{1}{N_F} (C_{VRR} \pm C_{VRL}) \quad (3.5.22)$$

$$R_{S,P} = \frac{i}{16\pi^2} \frac{1}{N_F} (C_{SRR} \pm C_{SRL}), \quad R'_{S,P} = \frac{i}{16\pi^2} \frac{1}{N_F} (C_{SLR} \pm C_{SLL}). \quad (3.5.23)$$

These coefficients can be used in Eq. 3.3.7 to calculate the branching ratio for $B_s \rightarrow \mu^+ \tau^-$. The loop functions $B(m_{\tilde{\chi}_k^+}, m_{\tilde{\chi}_l^+}, m_{\tilde{t}_i}, m_{\tilde{\nu}_j})$ and $S(m_{\tilde{\chi}_k^+}, m_{\tilde{\chi}_l^+}, m_{\tilde{t}_i}, m_{\tilde{\nu}_j})$ are given by

$$\begin{aligned} B(m_{\tilde{\chi}_k^+}, m_{\tilde{\chi}_l^+}, m_{\tilde{t}_i}, m_{\tilde{\nu}_j}) &= \frac{m_{\tilde{t}_i}^2}{(m_{\tilde{t}_i}^2 - m_{\tilde{\nu}_j}^2)(m_{\tilde{t}_i}^2 - m_{\tilde{\chi}_k^+}^2)(m_{\tilde{t}_i}^2 - m_{\tilde{\chi}_l^+}^2)} \ln \frac{m_{\tilde{t}_i}^2}{m_{\tilde{\chi}_l^+}^2} \\ &+ \frac{m_{\tilde{\nu}_j}^2}{(m_{\tilde{\nu}_j}^2 - m_{\tilde{t}_i}^2)(m_{\tilde{\nu}_j}^2 - m_{\tilde{\chi}_k^+}^2)(m_{\tilde{\nu}_j}^2 - m_{\tilde{\chi}_l^+}^2)} \ln \frac{m_{\tilde{\nu}_j}^2}{m_{\tilde{\chi}_l^+}^2} \\ &+ \frac{m_{\tilde{\chi}_k^+}^2}{(m_{\tilde{\chi}_k^+}^2 - m_{\tilde{t}_i}^2)(m_{\tilde{\chi}_k^+}^2 - m_{\tilde{\nu}_j}^2)(m_{\tilde{\chi}_k^+}^2 - m_{\tilde{\chi}_l^+}^2)} \\ &\times \ln \frac{m_{\tilde{\chi}_k^+}^2}{m_{\tilde{\chi}_l^+}^2}, \end{aligned} \quad (3.5.24)$$

and

$$\begin{aligned} S(m_{\tilde{\chi}_k^+}, m_{\tilde{\chi}_l^+}, m_{\tilde{t}_i}, m_{\tilde{\nu}_j}) &= \frac{m_{\tilde{t}_i}^4}{(m_{\tilde{t}_i}^2 - m_{\tilde{\nu}_j}^2)(m_{\tilde{t}_i}^2 - m_{\tilde{\chi}_k^+}^2)(m_{\tilde{t}_i}^2 - m_{\tilde{\chi}_l^+}^2)} \ln \frac{m_{\tilde{t}_i}^2}{m_{\tilde{\chi}_l^+}^2} \\ &+ \frac{m_{\tilde{\nu}_j}^4}{(m_{\tilde{\nu}_j}^2 - m_{\tilde{t}_i}^2)(m_{\tilde{\nu}_j}^2 - m_{\tilde{\chi}_k^+}^2)(m_{\tilde{\nu}_j}^2 - m_{\tilde{\chi}_l^+}^2)} \ln \frac{m_{\tilde{\nu}_j}^2}{m_{\tilde{\chi}_l^+}^2} \\ &+ \frac{m_{\tilde{\chi}_k^+}^4}{(m_{\tilde{\chi}_k^+}^2 - m_{\tilde{t}_i}^2)(m_{\tilde{\chi}_k^+}^2 - m_{\tilde{\nu}_j}^2)(m_{\tilde{\chi}_k^+}^2 - m_{\tilde{\chi}_l^+}^2)} \\ &\times \ln \frac{m_{\tilde{\chi}_k^+}^2}{m_{\tilde{\chi}_l^+}^2}. \end{aligned} \quad (3.5.25)$$

where $i, j, k, l = 1..2$ denote the heavy and light particles (stop-quark, sneutrino and chargino). It is worth mentioning that the above expression for the chargino

amplitude is consistent with the amplitude reported in Ref. [128] and also with the expressions given in Ref. [95]. We have expressed it in the above form to be close to the the new physics amplitudes discussed in Ref. [115] and adopted in the previous section. Finally, in deriving these expression, the following hadronic matrix elements have been used

$$\langle 0 | \bar{s} \gamma^5 b | \bar{B}_s(p) \rangle = -\frac{i}{2} f_{B_s} \frac{m_{B_s}^2}{m_b + m_s}, \quad \langle 0 | \bar{s} b | \bar{B}_s(p) \rangle = 0, \quad (3.5.26)$$

$$\langle 0 | \bar{s} \gamma_\mu \gamma^5 b | \bar{B}_s(p) \rangle = \frac{i}{2} p_\mu f_{B_s}, \quad \langle 0 | \bar{s} \gamma_\mu b | \bar{B}_s(p) \rangle = 0, \quad (3.5.27)$$

where p_μ is the momentum of the B_s meson with mass m_{B_s} .

3.5.2 Neutralino Contribution

Similar to the chargino contribution, we can write the amplitude of $b \rightarrow s \mu^+ \tau^-$ through the neutralino exchange as follows

$$\begin{aligned} \mathcal{M}_{\tilde{\chi}^0} = & -\frac{i}{16\pi^2} \left(m_{\tilde{\chi}_l^0} m_{\tilde{\chi}_k^0} B(m_{\tilde{\chi}_k^0}, m_{\tilde{\chi}_l^0}, m_{\tilde{b}_i}, m_{\tilde{\tau}_j}) \left[\frac{1}{2} Z_{2jl}^{L_L} Z_{3jk}^{L_R*} Z_{3ik}^{D_L} Z_{2il}^{*D_R} (\bar{s} P_L b) (\bar{\tau} P_L \mu) \right. \right. \\ & + \frac{1}{2} Z_{3jk}^{L_L*} Z_{2jl}^{L_R} Z_{3ik}^{D_R} Z_{2il}^{*D_L} (\bar{s} P_R b) (\bar{\tau} P_R \mu) + \frac{1}{4} Z_{2jl}^{L_L} Z_{3jk}^{*L_L} Z_{3ik}^{D_R} Z_{2il}^{*D_R} (\bar{s} \gamma^\mu P_L b) (\bar{\tau} \gamma_\mu P_R \mu) \\ & + \frac{1}{4} Z_{2jl}^{L_R} Z_{3jk}^{*L_R} Z_{3ik}^{D_L} Z_{2il}^{*D_L} (\bar{s} \gamma^\mu P_R b) (\bar{\tau} \gamma_\mu P_L \mu) \Big] \\ & + \frac{1}{4} S(m_{\tilde{\chi}_k^+}, m_{\tilde{\chi}_l^+}, m_{\tilde{b}_i}, m_{\tilde{\nu}_j}) \left[Z_{2jl}^{L_L} Z_{3jk}^{L_R*} Z_{3ik}^{D_L} Z_{2il}^{*D_R} (\bar{s} \gamma^\mu P_L b) (\bar{\tau} \gamma_\mu P_L \mu) \right. \\ & + Z_{3jk}^{L_L*} Z_{2jl}^{L_R} Z_{3ik}^{D_R} Z_{2il}^{*D_L} (\bar{s} \gamma^\mu P_R b) (\bar{\tau} \gamma_\mu P_R \mu) + 2 Z_{2jl}^{L_L} Z_{3jk}^{*L_L} Z_{3ik}^{D_R} Z_{2il}^{*D_R} (\bar{s} P_L b) (\bar{\tau} P_R \mu) \\ & \left. \left. + 2 Z_{2jl}^{L_R} Z_{3jk}^{*L_R} Z_{3ik}^{D_L} Z_{2il}^{*D_L} (\bar{s} P_R b) (\bar{\tau} P_L \mu) \right] \right). \end{aligned} \quad (3.5.28)$$

Therefore, one can write the neutralino amplitude as

$$\mathcal{M}_{\tilde{\chi}^0} = \mathcal{M}_{\tilde{\chi}^0}^{S-P} + \mathcal{M}_{\tilde{\chi}^0}^{V-A}, \quad (3.5.29)$$

with

$$\mathcal{M}_{\tilde{\chi}^0}^{S-P} = -\frac{i}{16\pi^2} \left((N_{SRR} + N_{SRL}) (\bar{s} P_R b) \bar{\tau} \mu + (N_{SLL} + N_{SLR}) (\bar{s} P_L b) \bar{\tau} \mu \right)$$

$$+(N_{SRR} - N_{SRL})(\bar{s}P_R b)\bar{\tau}\gamma^5\mu + (N_{SLR} - N_{SLL})(\bar{s}P_L b)\bar{\tau}\gamma^5\mu \Big) \quad (3.5.30)$$

and

$$\begin{aligned} \mathcal{M}_{\tilde{\chi}^0}^{V-A} = & -\frac{i}{16\pi^2} \Big((N_{VLL} + N_{VLR})(\bar{s}\gamma^\mu P_L b)(\bar{\tau}\gamma_\mu\mu) + (N_{VRR} + N_{VRL})(\bar{s}\gamma^\mu P_R b) \\ & \times (\bar{\tau}\gamma_\mu\mu) + (N_{VLR} - N_{VLL})(\bar{s}\gamma^\mu P_L b)(\bar{\tau}\gamma_\mu\gamma^5\mu) + (N_{VRR} - N_{VRL})(\bar{s}\gamma^\mu P_R b) \\ & \times (\bar{\tau}\gamma_\mu\gamma^5\mu) \Big). \end{aligned} \quad (3.5.31)$$

where

$$\begin{aligned} N_{SLL} = & \frac{g^4}{2m_W^2} \Big[\left((N_{l2}^* + \tan\theta_W N_{l1})\Gamma_{2j}^{LL} - N_{l3}^*\Gamma_{2j}^{LR} \frac{m_\mu}{m_W \cos\beta} \right) \left((2\tan\theta_W N_{k1}^*\Gamma_{3j}^{LR*} \right. \\ & \left. + N_{k3}^*\Gamma_{3j}^{LL*} \frac{m_\tau}{m_W \cos\beta} \right) \left((N_{k2}^* + \frac{1}{2}\tan\theta_W N_{k1}^*)\Gamma_{3i}^{DL} + N_{k3}^*\Gamma_{3i}^{DR} \frac{m_b}{m_W \cos\beta} \right) \\ & \times \left(\frac{2}{3}\tan\theta_W N_{l1}^*\Gamma_{2i}^{DR*} + N_{l3}^*\Gamma_{2i}^{DL*} \frac{m_s}{m_W \cos\beta} \right) \Big] \\ & \times \frac{m_W^2 m_{\tilde{\chi}_l^0} m_{\tilde{\chi}_k^0}}{8} B(m_{\tilde{\chi}_k^0}, m_{\tilde{\chi}_l^0}, m_{\tilde{b}_i}, m_{\tilde{l}_j}), \end{aligned} \quad (3.5.32)$$

$$\begin{aligned} N_{SRR} = & \frac{g^4}{2m_W^2} \Big[\left((N_{k2} + \tan\theta_W N_{k1})\Gamma_{3j}^{LR*} - N_{k3}\Gamma_{3j}^{LR} \frac{m_\tau}{m_W \cos\beta} \right) \left((2\tan\theta_W N_{l1}\Gamma_{2j}^{LR} \right. \\ & \left. + N_{l3}\Gamma_{2j}^{LL} \frac{m_\mu}{m_W \cos\beta} \right) \left(\frac{2}{3}\tan\theta_W N_{k1}\Gamma_{3i}^{DR} + N_{l3}\Gamma_{3i}^{DL} \frac{m_b}{m_W \cos\beta} \right) \\ & \times \left((N_{l2} + \frac{1}{2}\tan\theta_W N_{l1})\Gamma_{2i}^{DL*} + N_{l3}\Gamma_{2i}^{DR*} \frac{m_s}{m_W \cos\beta} \right) \Big] \\ & \times \frac{m_W^2 m_{\tilde{\chi}_l^0} m_{\tilde{\chi}_k^0}}{8} B(m_{\tilde{\chi}_k^0}, m_{\tilde{\chi}_l^0}, m_{\tilde{b}_i}, m_{\tilde{l}_j}), \end{aligned} \quad (3.5.33)$$

$$\begin{aligned} N_{SLR} = & \frac{g^4}{2m_W^2} \Big[\left((N_{l2}^* + \tan\theta_W N_{l1}^*)\Gamma_{2j}^{LL} - N_{l3}^*\Gamma_{2j}^{LR} \frac{m_\mu}{m_W \cos\beta} \right) \left((N_{k2} + \tan\theta_W N_{k1})\Gamma_{3j}^{LL*} \right. \\ & \left. - N_{k3}\Gamma_{3j}^{LR*} \frac{m_\tau}{m_W \cos\beta} \right) \left(\frac{2}{3}\tan\theta_W N_{k1}\Gamma_{3i}^{DR} + N_{l3}\Gamma_{3i}^{DL} \frac{m_b}{m_W \cos\beta} \right) \\ & \times \left(\frac{2}{3}\tan\theta_W N_{l1}^*\Gamma_{2i}^{DR*} + N_{l3}^*\Gamma_{2i}^{DL*} \frac{m_s}{m_W \cos\beta} \right) \Big] \frac{m_W^2}{8} S(m_{\tilde{\chi}_k^0}, m_{\tilde{\chi}_l^0}, m_{\tilde{b}_i}, m_{\tilde{l}_j}) \end{aligned} \quad (3.5.34)$$

$$N_{SRL} = \frac{g^4}{2m_W^2} \Big[\left((2\tan\theta_W N_{l1}\Gamma_{2j}^{LR} + N_{l3}\Gamma_{2j}^{LL} \frac{m_\mu}{m_W \cos\beta} \right)$$

$$\begin{aligned}
 & \times \left((2 \tan \theta_W N_{k1}^* \Gamma_{3j}^{LR*} + N_{k3}^* \Gamma_{3j}^{LL*} \frac{m_\tau}{m_W \cos \beta}) \right. \\
 & \times \left((N_{k2}^* + \frac{1}{2} \tan \theta_W N_{k1}^*) \Gamma_{3i}^{DL} + N_{k3}^* \Gamma_{3i}^{DR} \frac{m_b}{m_W \cos \beta} \right) \\
 & \times \left. \left((N_{l2} + \frac{1}{2} \tan \theta_W N_{l1}) \Gamma_{2i}^{DL*} + N_{l3} \Gamma_{2i}^{DR*} \frac{m_s}{m_W \cos \beta} \right) \right] \\
 & \times \frac{m_W^2}{8} S(m_{\tilde{\chi}_k^0}, m_{\tilde{\chi}_l^0}, m_{\tilde{b}_i}, m_{\tilde{l}_j}), \tag{3.5.35}
 \end{aligned}$$

and

$$\begin{aligned}
 N_{VLL} &= \frac{g^4}{2m_W^2} \left[\left((N_{l2}^* + \tan \theta_W N_{l1}) \Gamma_{2j}^{LL} - N_{l3}^* \Gamma_{2j}^{LR} \frac{m_\mu}{m_W \cos \beta} \right) \left((2 \tan \theta_W N_{k1}^* \Gamma_{3j}^{LR*} \right. \right. \\
 & \left. \left. + N_{k3}^* \Gamma_{3j}^{LL*} \frac{m_\tau}{m_W \cos \beta} \right) \left((N_{k2}^* + \frac{1}{2} \tan \theta_W N_{k1}^*) \Gamma_{3i}^{DL} + N_{k3}^* \Gamma_{3i}^{DR} \frac{m_b}{m_W \cos \beta} \right) \right. \\
 & \times \left. \left(\frac{2}{3} \tan \theta_W N_{l1}^* \Gamma_{2i}^{DR*} + N_{l3}^* \Gamma_{2i}^{DL*} \frac{m_s}{m_W \cos \beta} \right) \right] \\
 & \times \frac{m_W^2}{16} S(m_{\tilde{\chi}_k^0}, m_{\tilde{\chi}_l^0}, m_{\tilde{b}_i}, m_{\tilde{l}_j}), \tag{3.5.36}
 \end{aligned}$$

$$\begin{aligned}
 N_{VRR} &= \frac{g^4}{2m_W^2} \left[\left((N_{k2} + \tan \theta_W N_{k1}) \Gamma_{3j}^{LL*} - N_{k3} \Gamma_{3j}^{LR*} \frac{m_\tau}{m_W \cos \beta} \right) \left((2 \tan \theta_W N_{l1} \Gamma_{2j}^{LR} \right. \right. \\
 & \left. \left. + N_{l3} \Gamma_{2j}^{LL} \frac{m_\mu}{m_W \cos \beta} \right) \left(\frac{2}{3} \tan \theta_W N_{k1} \Gamma_{3i}^{DR} + N_{l3} \Gamma_{3i}^{DL} \frac{m_b}{m_W \cos \beta} \right) \right. \\
 & \times \left. \left((N_{l2} + \frac{1}{2} \tan \theta_W N_{l1}) \Gamma_{2i}^{DL*} + N_{l3} \Gamma_{2i}^{DR*} \frac{m_s}{m_W \cos \beta} \right) \right] \\
 & \times \frac{m_W^2}{16} S(m_{\tilde{\chi}_k^0}, m_{\tilde{\chi}_l^0}, m_{\tilde{b}_i}, m_{\tilde{l}_j}), \tag{3.5.37}
 \end{aligned}$$

$$\begin{aligned}
 N_{VLR} &= \frac{g^4}{2m_W^2} \left[\left((N_{l2}^* + \tan \theta_W N_{l1}^*) \Gamma_{2j}^{LL} - N_{l3}^* \Gamma_{2j}^{LR} \frac{m_\mu}{m_W \cos \beta} \right) \left((N_{k2} + \tan \theta_W N_{k1}) \Gamma_{3j}^{LL*} \right. \right. \\
 & \left. \left. - N_{k3} \Gamma_{3j}^{LR*} \frac{m_\tau}{m_W \cos \beta} \right) \left(\frac{2}{3} \tan \theta_W N_{k1} \Gamma_{3i}^{DR} + N_{l3} \Gamma_{3i}^{DL} \frac{m_b}{m_W \cos \beta} \right) \right. \\
 & \times \left. \left(\frac{2}{3} \tan \theta_W N_{l1}^* \Gamma_{2i}^{DR*} + N_{l3}^* \Gamma_{2i}^{DL*} \frac{m_s}{m_W \cos \beta} \right) \right] \\
 & \times \frac{m_W^2 m_{\tilde{\chi}_l^0} m_{\tilde{\chi}_k^0}}{16} B(m_{\tilde{\chi}_k^0}, m_{\tilde{\chi}_l^0}, m_{\tilde{b}_i}, m_{\tilde{l}_j}), \tag{3.5.38}
 \end{aligned}$$

$$N_{VRL} = \frac{g^4}{2m_W^2} \left[\left((2 \tan \theta_W N_{l1} \Gamma_{2j}^{LR} + N_{l3} \Gamma_{2j}^{LL} \frac{m_\mu}{m_W \cos \beta}) \right) \right.$$

$$\begin{aligned}
 & \times \left((2 \tan \theta_W N_{k1}^* \Gamma_{3j}^{LR*} + N_{k3}^* \Gamma_{3j}^{LL*} \frac{m_\tau}{m_W \cos \beta}) \right. \\
 & \left((N_{k2}^* + \frac{1}{2} \tan \theta_W N_{k1}^*) \Gamma_{3i}^{DL} + N_{k3}^* \Gamma_{3i}^{DR} \frac{m_b}{m_W \cos \beta} \right) \\
 & \times \left((N_{l2} + \frac{1}{2} \tan \theta_W N_{l1}) \Gamma_{2i}^{DL*} + N_{l3} \Gamma_{2i}^{DR*} \frac{m_s}{m_W \cos \beta} \right) \Big] \\
 & \times \frac{m_W^2 m_{\tilde{\chi}_l^0} m_{\tilde{\chi}_k^0}}{16} B(m_{\tilde{\chi}_k^0}, m_{\tilde{\chi}_l^0}, m_{\tilde{b}_i}, m_{\tilde{l}_j}). \tag{3.5.39}
 \end{aligned}$$

Thus, one finds

$$\begin{aligned}
 \mathcal{M}_{\tilde{\chi}^0}^{S-P} &= i \frac{f_{B_s} N_F}{2 |V_{tb}^{CKM} V_{ts}^{CKM*}|} \left(\frac{m_{B_s}^2}{m_b + m_s} (R_{0S} - R'_{0S}) \bar{\tau} \mu \right. \\
 & \left. + \frac{m_{B_s}^2}{m_b + m_s} (R_{0P} - R'_{0P}) \bar{\tau} \gamma^5 \mu \right), \tag{3.5.40}
 \end{aligned}$$

and

$$\mathcal{M}_{\tilde{\chi}^0}^{V-A} = i \frac{f_{B_s} N_F}{2 |V_{tb}^{CKM} V_{ts}^{CKM*}|} \left((R_{0V} - R'_{0V}) (\bar{\tau} \gamma_\mu \mu) + (R_{0A} - R'_{0A}) (\bar{\tau} \gamma_\mu \gamma^5 \mu) \right) \tag{3.5.41}$$

where

$$R_{0V,A} = \frac{i}{16\pi^2} \frac{|V_{tb}^{CKM} V_{ts}^{CKM*}|}{N_F} (N_{VLR} \pm N_{VLL}), \tag{3.5.42}$$

$$R'_{0V,A} = \frac{i}{16\pi^2} \frac{|V_{tb}^{CKM} V_{ts}^{CKM*}|}{N_F} (N_{VRR} \pm N_{VRL}), \tag{3.5.43}$$

$$R_{0S,P} = \frac{i}{16\pi^2} \frac{|V_{tb}^{CKM} V_{ts}^{CKM*}|}{N_F} (N_{SRR} \pm N_{SRL}), \tag{3.5.44}$$

$$R'_{0S,P} = \frac{i}{16\pi^2} \frac{|V_{tb}^{CKM} V_{ts}^{CKM*}|}{N_F} (N_{SLR} \pm N_{SLL}). \tag{3.5.45}$$

In this case, these coefficients can be used in Eq. (3.3.7) to calculate the branching ratio for $B_s \rightarrow \mu^+ \tau^-$.

3.5.3 Result and Discussion

We can estimate the typical prediction of SUSY models to the branching ratio of the $B_s \rightarrow \tau \bar{\mu} + \mu \bar{\tau}$ process. As usual one can easily show that the neutralino contribution is

typically one or two order of magnitude less than the light chargino effect. Therefore, here we will neglect the neutralino diagrams and focus on the dominant chargino contribution.

As explicitly shown above, the chargino amplitude depends on stops and tau-sneutrinos masses and mixing. Therefore a light stop and light tau-sneutrinos may significantly enhance the chargino contribution to the $BR(B_s \rightarrow \tau \bar{\mu} + \mu \bar{\tau})$. In the MSSM, the stop mass matrix is given by

$$M_t^2 = \begin{pmatrix} m_Q^2 + m_t^2 + \frac{1}{6} M_Z^2 \cos 2\beta (3 - 4 \sin^2 \theta_W) & m_t (A_t - \mu \cot \beta) \\ m_t (A_t - \mu \cot \beta) & m_U^2 + m_t^2 + \frac{2}{3} M_Z^2 \cos 2\beta \sin^2 \theta_W \end{pmatrix}, \quad (3.5.46)$$

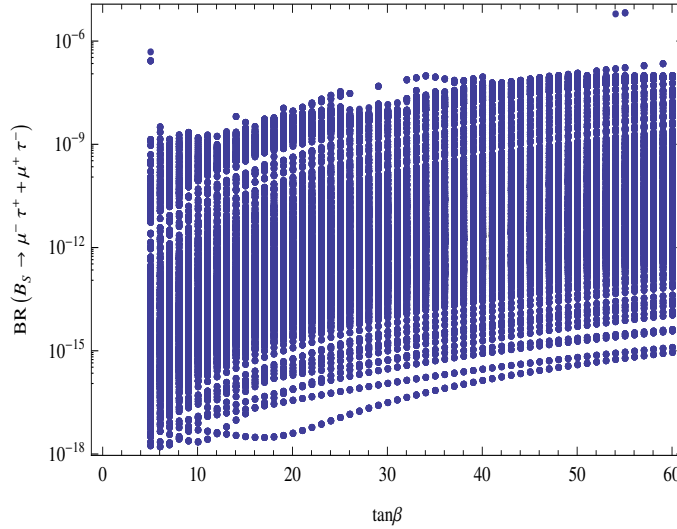


Figure 3.8: The chargino contribution for $BR(B_s \rightarrow \tau \bar{\mu} + \mu \bar{\tau})$ as function of $\tan \beta$ for stop masses of order 200 and 800 GeV and $M_2 \in [100, 1000]$ GeV and $\mu \in [250, 1000]$ GeV.

where m_Q^2 and m_U^2 are the low energy values of the soft SUSY breaking scalar masses. Due to large off-diagonal elements, the diagonalization of the stop mass-squared matrix leads to the physical eigenstates with one light and one heavy stops. In addition, with no right-handed neutrinos in the MSSM the sneutrino mass matrix

is given by

$$M_{\tilde{\nu}}^2 = M_L^2 + \frac{1}{2} M_Z^2 \cos^2 \beta, \quad (3.5.47)$$

where M_L is the soft SUSY scalar masses of the sleptons. It is also a part of the left-left sector of the charged slepton mass-squared matrix. However, it is worth noting that the experimental limits of lepton flavor violations, like $\mu \rightarrow e\gamma$ impose stingiest constraints on the off-diagonal elements of M_L^2 between the first and second generations, while the mixing between first or the second and third generations are much less constrained [129–131].

In Fig. 3.8 we present the $BR(B_s \rightarrow \tau\bar{\mu} + \mu\bar{\tau})$ as function of $\tan\beta$. We assume that the lightest and heaviest stop masses are of order 200 GeV and 800 GeV respectively. We consider $100 \text{ GeV} \leq M_2 \leq 1 \text{ TeV}$ and take μ between 250 GeV and 1 TeV. The lightest sneutrino mass is assumed to be of order 200 GeV and the sneutrino mixing parameters are of order $\mathcal{O}(0.1)$. As can be seen from this figure, the branching ratio $BR(B_s \rightarrow \tau\bar{\mu} + \mu\bar{\tau})$ can be enhanced up to 10^{-7} for large $\tan\beta$ and light charginos and sneutrinos. Also if we have semi-degenerate masses for the charginos, stops, and sneutrino, then a kind of a resonance in the branching ratio may be obtained (as can be seen from the loop functions: $B(m_{\tilde{\chi}_k^+}, m_{\tilde{\chi}_l^+}, m_{\tilde{t}_i}, m_{\tilde{\nu}_j})$ and $S(m_{\tilde{\chi}_k^+}, m_{\tilde{\chi}_l^+}, m_{\tilde{t}_i}, m_{\tilde{\nu}_j})$). In this case the branching ratio can reach 10^{-6} , however, we think this is a fine tuning and therefore we assume that the mass difference between these particles $> 50 \text{ GeV}$ at least. From this analysis one can conclude that the typical SUSY predictions for this branching ratio of $B_s \rightarrow \tau\bar{\mu} + \mu\bar{\tau}$ is of order $10^{-8} - 10^{-9}$. Finally, it is worth noting that in Ref. [95], it was emphasized that in the supersymmetric seesaw extensions of the SM, the two loop double penguin diagrams may be relevant at a very large $\tan\beta$ (of order 60) even with flavor universal slepton masses. In this case, it was shown that the double penguins mediated by Higgs bosons may induce a branching ratio of order $B_s \rightarrow \mu^+\tau^- \sim 10^{-7} \times (\tan\beta/60)^8 \times (100 \text{ GeV}/m_A)^4$, which may give the dominant results at very large $\tan\beta$ and low m_A .

4

$R(D)$ and $R(D^*)$ in the MSSM

In this chapter we argue that SUSY contributions may explain the discrepancy between the experimental results for $\bar{B} \rightarrow D\tau\nu_\tau$ and $\bar{B} \rightarrow D^*\tau\nu_\tau$ and the corresponding SM expectations. For the first time in literature, to our knowledge, we consider here all contributions up to Next-to-Leading Order (NLO) within the MSSM: tree-level ones due to charged gauge boson and Higgs exchange as well as one-loop ones due to bubbles, triangles (penguins) and boxes onset by the exchanges of 2HDM states (i.e., γ , Z , $W^\pm$, $H^\pm$, h^0 , H^0 and A^0) alongside the SUSY ones due to gauginos (charginos and neutralinos) and sfermions (squarks, sleptons and sneutrinos) taking into account both CP-conserving and CP-violating cases.

4.1 Recent Experimental Result for $\mathcal{R}(D)$ and $\mathcal{R}(D^*)$

Rare B -decays provide a good opportunity for probing NP Beyond the Standard Model. In fact, experimental studies of flavour at (Super) B -factories (BaBar and Belle) and LHCb are complementary to the direct search for NP at LHC. The origin

4.1 Recent Experimental Result for $\mathcal{R}(D)$ and $\mathcal{R}(D^*)$

of flavour and CP violation is one of the most profound open questions in particle physics. Most extensions of the SM, wherein the latter is embedded as a low energy effective theory, include new sources of flavour and CP violation. SUSY is one promising candidate for BSM physics which has these characteristics, particularly if the soft SUSY-breaking terms are non-universal.

It has been recently reported a deviation from the SM expectations in the ratios

$$\mathcal{R}(D) = \frac{\text{BR}(\bar{B} \rightarrow D\tau\bar{\nu}_\tau)}{\text{BR}(\bar{B} \rightarrow Dl\bar{\nu}_l)}, \quad \mathcal{R}(D^*) = \frac{\text{BR}(\bar{B} \rightarrow D^*\tau\bar{\nu}_\tau)}{\text{BR}(\bar{B} \rightarrow D^*l\bar{\nu}_l)}, \quad (4.1.1)$$

where, here, l refers to either electron or muon. In particular, the Belle collaboration measured [23, 25]

$$\mathcal{R}(D)^{\text{Belle}} = 0.375 \pm 0.064, \quad (4.1.2)$$

$$\mathcal{R}(D^*)^{\text{Belle}} = 0.302 \pm 0.030 \pm 0.011, \quad (4.1.3)$$

whereas the results by the BaBar collaboration are [26]

$$\mathcal{R}(D)^{\text{BaBar}} = 0.440 \pm 0.072, \quad (4.1.4)$$

$$\mathcal{R}(D^*)^{\text{BaBar}} = 0.332 \pm 0.030. \quad (4.1.5)$$

In addition, the LHCb collaboration has found $\mathcal{R}(D^*)^{\text{LHCb}} = 0.336 \pm 0.027 \pm 0.030$ [27].

The SM predictions for $\mathcal{R}(D)$ and $\mathcal{R}(D^*)$ are [28, 29]:

$$\mathcal{R}(D)^{\text{SM}} = 0.305 \pm 0.012, \quad (4.1.6)$$

$$\mathcal{R}(D^*)^{\text{SM}} = 0.252 \pm 0.004, \quad (4.1.7)$$

which deviate by $\sim 1.7\sigma$ from the combined experimental result for $\mathcal{R}(D)$ and $\sim 3\sigma$ from the one for $\mathcal{R}(D^*)$. In addition, the combined four results disagree with the SM expectations at the $\sim 3.9\sigma$ level. These deviations, if confirmed, could be important hints for NP, especially because the SM results for $\mathcal{R}(D)$ and $\mathcal{R}(D^*)$ are essentially independent of the parameterisation of the hadronic matrix elements.

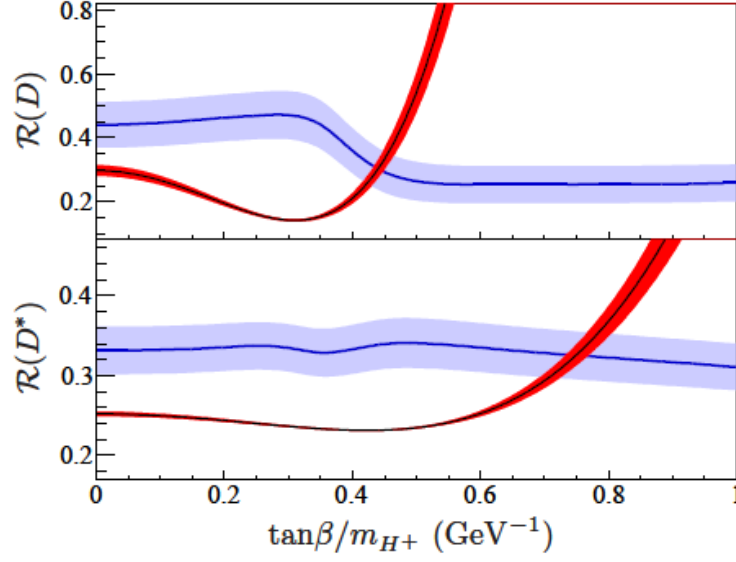


Figure 4.1: Comparison of the BaBar results (light band, blue) with predictions that include a charged Higgs boson of type II 2HDM (red band, dark). The SM corresponds to $\tan\beta/m_{H^+} = 0$ [26, 132].

Since the semileptonic decay $b \rightarrow c\tau\nu_\tau$ takes place in the SM at tree level, it is naively expected that any BSM contribution would be subdominant, even those embedding a charged Higgs boson entering at tree-level, since $M_{H^\pm} \geq M_{W^\pm}$. Indeed, it is notoriously challenging to account for large deviations from the SM rates. This has been shown explicitly in some SM extensions [22, 133, 134]. In particular, it was emphasized that in 2-Higgs Doublet Models (2HDMs) the above experimental results for $R(D)$ and $R(D^*)$ cannot be simultaneously explained as shown in Fig. (4.1).

4.2 Helicity Amplitudes and Related Observables

In the presence of NP, the effective Hamiltonian for $b \rightarrow cl\bar{\nu}_l$ is

$$\begin{aligned} \mathcal{H}_{\text{eff}} = & \frac{4G_F V_{cb}}{\sqrt{2}} \left[(1 + g_{VL}) [\bar{c}\gamma_\mu P_L b] [\bar{l}\gamma_\mu P_L \nu_l] + g_{VR} [\bar{c}\gamma_\mu P_R b] [\bar{l}\gamma_\mu P_L \nu_l] + g_{SL} [\bar{c}P_L b] [\bar{l}P_L \nu_l] \right. \\ & \left. + g_{SR} [\bar{c}P_R b] [\bar{l}P_L \nu_l] + g_T [\bar{c}\sigma^{\mu\nu} P_L b] [\bar{l}\sigma_{\mu\nu} P_L \nu_l] \right], \end{aligned} \quad (4.2.1)$$

where G_F is the Fermi coupling constant, V_{cb} is the Cabibbo-Kobayashi-Maskawa (CKM) matrix element between charm and bottom quarks while $P_{L/R} = (1 -/+ \gamma_5)/2$. Finally, g_i is defined in terms of the Wilson coefficients (see [135] for prospects of extracting these using optimal observables) C_i as $g_i = C_i^{SUSY}/C^{\text{SM}}$, with $C^{\text{SM}} = \frac{4G_F V_{cb}}{\sqrt{2}}$. The amplitudes of possible NP contributions to $\bar{B} \rightarrow D^{(*)} l \bar{\nu}_l$, $\mathcal{M} \equiv \langle D^{(*)} l \bar{\nu}_l | \mathcal{H}_{\text{eff}} | \bar{B} \rangle$, can be written in the form [136, 137]

$$\mathcal{M}_{S(L,R)}^{\lambda_{D^{(*)}}, \lambda_l} = -\frac{G_F}{\sqrt{2}} V_{cb} g_{S(L,R)} H_{S(L,R)}^{\lambda_{D^{(*)}}} L^{\lambda_l}, \quad (4.2.2)$$

$$\mathcal{M}_{V(L,R)}^{\lambda_{D^{(*)}}, \lambda_l} = \frac{G_F}{\sqrt{2}} V_{cb} g_{V(L,R)} \sum_{\lambda} \eta_{\lambda} H_{V(L,R), \lambda}^{\lambda_{D^{(*)}}} L_{\lambda}^{\lambda_l}, \quad (4.2.3)$$

$$\mathcal{M}_T^{\lambda_{D^{(*)}}, \lambda_l} = -\frac{G_F}{\sqrt{2}} V_{cb} g_T \sum_{\lambda, \lambda'} \eta_{\lambda'} \eta_{\lambda} H_{\lambda \lambda'}^{\lambda_{D^{(*)}}} L_{\lambda \lambda'}^{\lambda_l}. \quad (4.2.4)$$

The SM amplitude is given by

$$\mathcal{M}_{\text{SM}}^{\lambda_{D^{(*)}}, \lambda_l} = \frac{G_F}{\sqrt{2}} V_{cb} \sum_{\lambda} \eta_{\lambda} H_{V L, \lambda}^{\lambda_{D^{(*)}}} L_{\lambda}^{\lambda_l}, \quad (4.2.5)$$

where λ_l is the helicity of the lepton l and $\lambda, \lambda' = \pm, 0$ or s are the helicity of virtual vector bosons. The $D^{(*)}$ -meson is taken to be either a spin-0 D -meson, with $\lambda_D = s$, or a spin-1 D^* -meson, with $\lambda_{D^{(*)}} = \pm, 0$. The summation is over the virtual vector boson helicities with the metric $\eta_{\pm} = \eta_0 = -\eta_s = 1$, H 's and L 's are the hadronic and leptonic amplitudes which are defined as [28, 111, 138]

- Hadronic amplitudes

$$H_{S(L,R)}^{\lambda_M} = \langle M(p_M, \epsilon(\lambda_M)) | \bar{c}(1 \mp \gamma_5) b | \bar{B}(p_B) \rangle, \quad (4.2.6)$$

$$H_{V(L,R), \lambda}^{\lambda_M} = \epsilon_{\mu}^*(\lambda) \langle M(p_M, \epsilon(\lambda_M)) | \bar{c} \gamma^{\mu} (1 \mp \gamma_5) b | \bar{B}(p_B) \rangle, \quad (4.2.7)$$

$$H_{\lambda \lambda'}^{\lambda_M} = i \epsilon_{\mu}^*(\lambda) \epsilon_{\nu}^*(\lambda') \langle M(p_M, \epsilon(\lambda_M)) | \bar{c} \sigma^{\mu\nu} (1 - \gamma_5) b | \bar{B}(p_B) \rangle, \quad (4.2.8)$$

- Leptonic amplitudes

$$L^{\lambda_\tau}(q^2, \cos \theta_\tau) = \langle \tau(p_\tau, \lambda_\tau) \bar{\nu}_\tau(p_\nu) | \bar{\tau}(1 - \gamma_5) \nu_\tau | 0 \rangle \quad (4.2.9)$$

$$L_\lambda^{\lambda_\tau}(q^2, \cos \theta_\tau) = \epsilon_\mu(\lambda) \langle \tau(p_\tau, \lambda_\tau) \bar{\nu}_\tau(p_\nu) | \bar{\tau} \gamma^\mu (1 - \gamma_5) \nu_\tau | 0 \rangle, \quad (4.2.10)$$

$$L_{\lambda\lambda'}^{\lambda_\tau}(q^2, \cos \theta_\tau) = -i\epsilon_\mu(\lambda)\epsilon_\nu(\lambda') \langle \tau(p_\tau, \lambda_\tau) \bar{\nu}_\tau(p_\nu) | \bar{\tau} \sigma^{\mu\nu} (1 - \gamma_5) \nu_\tau | 0 \rangle, \quad (4.2.11)$$

where $\epsilon_\mu(\lambda)$ and $\epsilon_\mu(\lambda_M)$ are the polarization vectors of the virtual vector boson and M -meson respectively. Furthermore, one can also define the differential rate for the process $\bar{B} \rightarrow D^{(*)} l \bar{\nu}_l$ as [21, 139]

$$d\Gamma^{\lambda_l, \lambda_M} = \frac{1}{2m_B} \sum_{\lambda_l, \lambda_M} |\mathcal{M}^{\lambda_l, \lambda_{D^{(*)}}}|^2 d\Phi_3, \quad (4.2.12)$$

where $d\Phi_3$ is the phase space factor can be written as

$$d\Phi_3 = \frac{\sqrt{Q_+ Q_-} v_l}{256\pi^3 m_B^2} dq^2 d\cos \theta_l, \quad (4.2.13)$$

and thus

$$\frac{d\Gamma}{dq^2 d\cos \theta_l} = \frac{\sqrt{Q_+ Q_-} v_l}{256\pi^3 m_B^3} |\mathcal{M}(\bar{B} \rightarrow D^{(*)} l \nu_l)|^2, \quad (4.2.14)$$

where $v_l = 1 - \frac{m_l^2}{q^2}$, q^2 varies in the range $m_l^2 \leq q^2 \leq (m_B - m_{D^{(*)}})^2$, $Q_\pm = (m_B \pm m_{D^{(*)}})^2 - q^2$ with $q^\mu = p_B^\mu - p_{D^{(*)}}^\mu = p_l^\mu + p_{\nu_l}^\mu$ and $-1 \leq \cos \theta_l \leq 1$. Therefore, the full amplitude takes the form

$$\mathcal{M} = \mathcal{M}_{SM}^{\lambda_{D^{(*)}}, \lambda_l} + \mathcal{M}_{S(L,R)}^{\lambda_{D^{(*)}}, \lambda_l} + \mathcal{M}_{V(L,R)}^{\lambda_{D^{(*)}}, \lambda_l} + \mathcal{M}_T^{\lambda_{D^{(*)}}, \lambda_l}. \quad (4.2.15)$$

For the τ helicities $\lambda_\tau = \pm 1$, the differential decay rate for the process $\bar{B} \rightarrow D^* \tau \bar{\nu}_\tau$ can be written in the form

$$\begin{aligned} \frac{d\Gamma^{D^*}[\lambda_\tau = 1/2]}{dq^2 d\cos \theta_\tau} &= \frac{G_F^2 |V_{cb}|^2 q^2}{256\pi^3 m_B^3} \sqrt{Q_+ Q_-} \left(1 - \frac{m_\tau^2}{q^2}\right)^2 \left(\frac{m_\tau^2}{q^2} [(|1 + g_{VL}|^2 + |g_{VR}|^2) \right. \\ &\quad \times (\cos^2 \theta H_{VL,0}^2 + H_{VL,s}^2) - 2 \operatorname{Re}[(1 + g_{VL}|^2 g_{VR}^*](\cos^2 \theta H_{VL,0}^2 + H_{VL,s}^2)] \\ &\quad + \frac{m_\tau^2}{2q^2} \sin^2 \theta [(|1 + g_{VL}|^2 + g_{VR}^2)(H_{VL,+}^2 + H_{VL,-}^2) \\ &\quad \left. - 4 \operatorname{Re}[(1 + g_{VL})g_{VR}^*]H_{VL,+}H_{VL,-}] + |g_{SR} - g_{SL}|^2 H_S^2 \right) \end{aligned}$$

$$\begin{aligned}
 & + 8|g_T|^2[2\cos^2\theta H_{+-}^2 + \sin^2\theta(H_{+0}^2 + H_{-0}^2)] \\
 & + \frac{2m_\tau}{\sqrt{q^2}} \text{Re}[(1 + g_{VL} - g_{VR})(g_{SR}^* - g_{SL}^*)]H_S H_{VL,s} \\
 & - \frac{4m_\tau}{\sqrt{q^2}} \text{Re}[(1 + g_{VL})g_T^*][2\cos^2\theta H_{+-}H_{VL,0} + \sin^2\theta(H_{+0}H_{VL,+} - H_{-0}H_{VL,-})] \\
 & + \frac{4m_\tau}{\sqrt{q^2}} \text{Re}[g_{VR}g_T^*][2\cos^2\theta H_{+-}H_{VL,0} + \sin^2\theta(H_{+0}H_{VL,-} - H_{-0}H_{VL,+})] \\
 & + 2\frac{m_\tau^2}{q^2} \cos\theta(1 + g_{VL} - g_{VR})^2 H_{VL,0}H_{VL,s} \\
 & + \frac{2m_\tau}{\sqrt{q^2}} \cos\theta[\text{Re}[(1 + g_{VL} - g_{VR})(g_{SR}^* - g_{SL}^*)]]H_S H_{VL,0} \\
 & - \frac{8m_\tau}{\sqrt{q^2}} \cos\theta \text{Re}[(1 + g_{VL} - g_{VR})g_T^*]H_{+-}H_{VL,s} - 8\cos\theta \text{Re}[(g_{SR} - g_{SL})g_T^*]H_S H_{+-}),
 \end{aligned} \tag{4.2.16}$$

and

$$\begin{aligned}
 \frac{d\Gamma^{D^*}[\lambda_\tau = -1/2]}{dq^2 d\cos\theta\tau} &= \frac{G_F^2 |V_{cb}|^2 q^2}{256\pi^3 m_B^3} \sqrt{Q_+ Q_-} \left(1 - \frac{m_\tau^2}{q^2}\right)^2 \left(\sin^2\theta[|1 + g_{VL}|^2 + |g_{VR}|^2]H_{VL,0}^2 \right. \\
 & - 2\text{Re}[(1 + g_{VL})g_{VR}^*]H_{VL,0}^2 + \frac{1}{2}(1 + \cos\theta)^2[|1 + g_{VL}|^2 H_{VL,+}^2 \\
 & + |g_{VR}^2 H_{VL,-}^2 - 2\text{Re}[(1 + g_{VL})g_{VR}^*]H_{VL,+}H_{VL,-}] \\
 & + \frac{1}{2}(1 - \cos\theta)^2[|1 + g_{VL}|^2 H_{VL,-}^2 + |g_{VR}|^2 H_{VL,+}^2 \\
 & - 2\text{Re}[(1 + g_{VL})g_{VR}^*]H_{VL,-}H_{VL,+}] \\
 & + \frac{8m_\tau^2}{q^2} |g_T|^2 [2\sin^2\theta H_{+-}^2 + (1 + \cos\theta)^2 H_{+0}^2 + (1 - \cos\theta)^2 H_{-0}^2] \\
 & - \frac{8m_\tau}{\sqrt{q^2}} \sin^2\theta (\text{Re}[(1 + g_{VL} - g_{VR})g_T^*]H_{VL,0}H_{+-}) \\
 & - \frac{4m_\tau}{\sqrt{q^2}} (1 + \cos\theta)^2 (\text{Re}[(1 + g_{VL})g_T^*]H_{VL,+} - \text{Re}[g_{VR}g_T^*]H_{VL,-})H_{+0} \\
 & \left. + \frac{4m_\tau}{\sqrt{q^2}} (1 - \cos\theta)^2 (\text{Re}[(1 + g_{VL})g_T^*]H_{VL,-} - \text{Re}[g_{VR}g_T^*]H_{VL,+})H_{-0} \right),
 \end{aligned} \tag{4.2.17}$$

Summing Eqs. (4.2.16) and (4.2.17), and after performing integration over $d \cos \theta \tau$ we can obtain the total differential decay rate,

$$\begin{aligned}
 \frac{d\Gamma^{D^*}}{dq^2} = & \frac{G_F^2 |V_{cb}|^2 q^2}{192\pi^3 m_B^3} \sqrt{Q_+ Q_-} \left(1 - \frac{m_\tau^2}{q^2}\right)^2 \left((|1 + g_{VL}|^2 + |g_{VR}|^2) \right. \\
 & \times \left[\left(1 + \frac{m_\tau^2}{2q^2}\right) (H_{VL,+}^2 + H_{VL,-}^2 + H_{VL,0}^2) + \frac{3}{2} \frac{m_\tau^2}{q^2} H_{VL,s}^2 \right] \\
 & - 2 \operatorname{Re}[(1 + g_{VL})g_{VR}^*] \left[\left(1 + \frac{m_\tau^2}{2q^2}\right) (H_{VL,0}^2 + 2H_{VL,+}H_{VL,-}) + \frac{3}{2} \frac{m_\tau^2}{q^2} H_{VL,s}^2 \right] \\
 & + \frac{3}{2} |g_{SR} - g_{SL}|^2 H_S^2 + 8|g_T|^2 \left(1 + \frac{2m_\tau^2}{q^2}\right) (H_{+0}^2 + H_{-0}^2 + H_{+-}^2) \\
 & + 3 \operatorname{Re}[(1 + g_{VL} - g_{VR})(g_{SR}^* - g_{SL}^*)] H_S H_{VL,s} \\
 & - 12 \operatorname{Re}[(1 + g_{VL})g_T^*] \frac{m_\tau}{\sqrt{q^2}} (H_{+-}H_{VL,0}^2 + H_{+0}H_{VL,+} - H_{-0}H_{VL,-}) \\
 & \left. + 12 \operatorname{Re}[g_{VR}g_T^*] \frac{m_\tau}{\sqrt{q^2}} (H_{+-}H_{VL,0}^2 + H_{+0}H_{VL,-} - H_{-0}H_{VL,+}) \right), \quad (4.2.18)
 \end{aligned}$$

In a similar manner, for the process $\bar{B} \rightarrow D\tau\bar{\nu}_\tau$, one finds

$$\begin{aligned}
 \frac{d\Gamma^D[\lambda_\tau = 1/2]}{dq^2 d \cos \theta \tau} = & \frac{G_F^2 |V_{cb}|^2 q^2}{256\pi^3 m_B^3} \sqrt{Q_+ Q_-} \left(1 - \frac{m_\tau^2}{q^2}\right)^2 \left(|1 + g_{VL} + g_{VR}|^2 \right. \\
 & \times \frac{m_\tau^2}{q^2} (\cos \theta H_{V,0}^s + H_{V,s}^s)^2 + |g_{SL} + g_{SR}|^2 H_S^{s2} + 16 \cos^2 \theta |g_T|^2 H_{+-}^{s2} \\
 & + 2 \frac{m_\tau}{\sqrt{q^2}} \operatorname{Re}[(1 + g_{VL} + g_{VR})(g_{SL}^* + g_{SR}^*)] (\cos \theta H_{V,0}^s + H_{V,s}^s) H_S^s \\
 & - 8 \frac{m_\tau}{\sqrt{q^2}} \cos \theta \operatorname{Re}[(1 + g_{VL} + g_{VR})g_T^*] (\cos \theta H_{V,0}^s + H_{V,s}^s) H_{+-}^s \\
 & \left. - 8 \cos \theta \operatorname{Re}[(g_{SL} + g_{SR})g_T^*] H_S^s H_{+-}^s \right), \quad (4.2.19)
 \end{aligned}$$

and

$$\begin{aligned}
 \frac{d\Gamma^D[\lambda_\tau = -1/2]}{dq^2 d \cos \theta \tau} = & \frac{G_F^2 |V_{cb}|^2 q^2}{256\pi^3 m_B^3} \sqrt{Q_+ Q_-} \left(1 - \frac{m_\tau^2}{q^2}\right)^2 \left(\sin^2 \theta [|1 + g_{VL} + g_{VR}|^2 H_{V,0}^{s2} \right. \\
 & \left. + 16 \frac{m_\tau^2}{q^2} |g_T|^2 H_{+-}^{s2} - 8 \frac{m_\tau}{\sqrt{q^2}} \operatorname{Re}[(1 + g_{VL} + g_{VR})g_T^*] H_{V,0}^s H_{+-}^s \right], \quad (4.2.20)
 \end{aligned}$$

and thus, the total differential decay rate is

$$\begin{aligned}
 \frac{d\Gamma^D}{dq^2} = & \frac{G_F^2 |V_{cb}|^2 q^2}{256\pi^3 m_B^3} \sqrt{Q_+ Q_-} \left(1 - \frac{m_\tau^2}{q^2}\right)^2 \left(|1 + g_{VL} + g_{VR}|^2 \left[\left(1 + \frac{m_\tau^2}{2q^2}\right) H_{V,0}^{s2} + \frac{3}{2} \frac{m_\tau^2}{q^2} H_{V,s}^s \right] \right. \\
 & + \frac{3}{2} |g_{SL} + g_{SR}|^2 H_S^{s2} + 8|g_T|^2 \left(1 + \frac{2m_\tau^2}{q^2}\right) H_{+-}^{s2} + 3 \operatorname{Re}[(1 + g_{VL} + g_{VR})(g_{SL}^* + g_{SR}^*)] \\
 & \times \frac{m_\tau}{\sqrt{q^2}} H_S^s H_{V,s}^s - 12 \operatorname{Re}[(1 + g_{VL} + g_{VR})g_T^*] \frac{m_\tau}{\sqrt{q^2}} H_{V,0}^s H_{+-}^s \Big), \quad (4.2.21)
 \end{aligned}$$

Eventually, one can define both observables $\mathcal{R}(D)$ and $\mathcal{R}(D^*)$ as follows

$$\mathcal{R}(D) = \frac{\Gamma(\bar{B} \rightarrow D\tau\nu_\tau)}{\Gamma(\bar{B} \rightarrow Dl\nu_l)}, \quad \mathcal{R}(D^*) = \frac{\Gamma(\bar{B} \rightarrow D^*\tau\nu_\tau)}{\Gamma(\bar{B} \rightarrow D^*l\nu_l)}. \quad (4.2.22)$$

Using the explicit formulae of the hadronic and leptonic amplitudes¹ in Refs. [28, 29] (when the l contribution is assumed to be described by the SM) and upon fixing the SM parameters and the form factors involved in the definition of the matrix elements to their central values as in Ref. [26], we can cast the explicit dependence of $\mathcal{R}(D)$ and $\mathcal{R}(D^*)$ upon the Wilson coefficients in the MSSM as follows:

$$\begin{aligned}
 \mathcal{R}(D) = & \mathcal{R}(D)^{\text{SM}} \left[0.981 |g_{SR} + g_{SL}|^2 + |1 + g_{VL} + g_{VR}|^2 + 0.811 |g_T|^2 \right. \\
 & \left. + 1.465 \operatorname{Re}[(1 + g_{VL} + g_{VR})(g_{SR} + g_{SL})^*] + 1.074 \operatorname{Re}[(1 + g_{VL} + g_{VR})g_T^*] \right], \quad (4.2.23)
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{R}(D^*) = & \mathcal{R}(D^*)^{\text{SM}} \left[0.025 |g_{SR} - g_{SL}|^2 + |1 + g_{VL}|^2 + |g_{VR}|^2 + 16.739 |g_T|^2 \right. \\
 & + 0.094 \operatorname{Re}[(1 + g_{VL} + g_{VR})(g_{SR} - g_{SL})^*] \\
 & \left. + 6.513 \operatorname{Re}[g_{VR}g_T^*] - 4.457 \operatorname{Re}[(1 + g_{VL})g_T^*] - 1.748 \operatorname{Re}[(1 + g_{VL})g_{VR}^*] \right]. \quad (4.2.24)
 \end{aligned}$$

From the above expressions one can easily see that, since $g_i < 1$, the leading contributions to $\mathcal{R}(D)$ and $\mathcal{R}(D^*)$ are ²

$$\mathcal{R}(D) = \mathcal{R}(D)^{\text{SM}} (1 + 2\operatorname{Re}[g_{VL} + g_{VR}] + 1.465 \operatorname{Re}[(g_{SR} + g_{SL})^*] + 1.074 \operatorname{Re}[g_T^*]),$$

¹For more details see appendix B.

²However, notice that we have retained in the numerical calculation the full quadratic dependence upon the g_i 's.

$$\begin{aligned} \mathcal{R}(D^*) = & \mathcal{R}(D^*)^{\text{SM}}(1 + 2\text{Re}[g_{VL}] + 0.094\text{Re}[(g_{SR} - g_{SL})^*] - 4.457\text{Re}[g_T^*] \\ & - 1.748\text{Re}[g_{VR}^*]). \end{aligned} \quad (4.2.25)$$

Thus, in case of a dominant scalar contribution (and negligible vector and tensor ones), it is clear that $\mathcal{R}(D^*)$ cannot be significantly larger than the SM expectation unless $g_{SR} - g_{SL}$ is larger than one (*i.e.*, $C_S^{\text{SUSY}} > C^{\text{SM}}$), which is not possible. Recall that g_{SR} is larger than g_{SL} and it receives a contribution at the tree-level via charged Higgs exchange that yields

$$\mathcal{H}_{eff} = -\frac{4G_F V_{cb}}{\sqrt{2}} \left(m_b m_\tau \left[\frac{\tan \beta}{m_{H^\pm}} \right]^2 [\bar{c} P_R b][\bar{\tau} P_L \nu_\tau] + \frac{m_c m_\tau}{m_{H^\pm}^2} [\bar{c} P_L b][\bar{\tau} P_L \nu_\tau] \right) \quad (4.2.26)$$

therefore

$$g_{SR} = \frac{-m_b m_\tau \tan \beta^2}{m_{H^\pm}^2}, \quad g_{SL} = \frac{-m_c m_\tau}{m_{H^\pm}^2} \quad (4.2.27)$$

where $m_{H^\pm}$ is the mass of the charged Higgs boson and $\tan \beta$ the ratio of the two vacuum expectation values.

In Fig. 4.2 we display the regions in the (g_{SL}, g_{SR}) and (g_{VL}, g_{VR}) planes that can accommodate the experimental results of $\mathcal{R}(D)$ and $\mathcal{R}(D^*)$ within 1σ and 2σ Confidence Level (CL) for, e.g., Belle, the experiment with predictions closer to the SM.

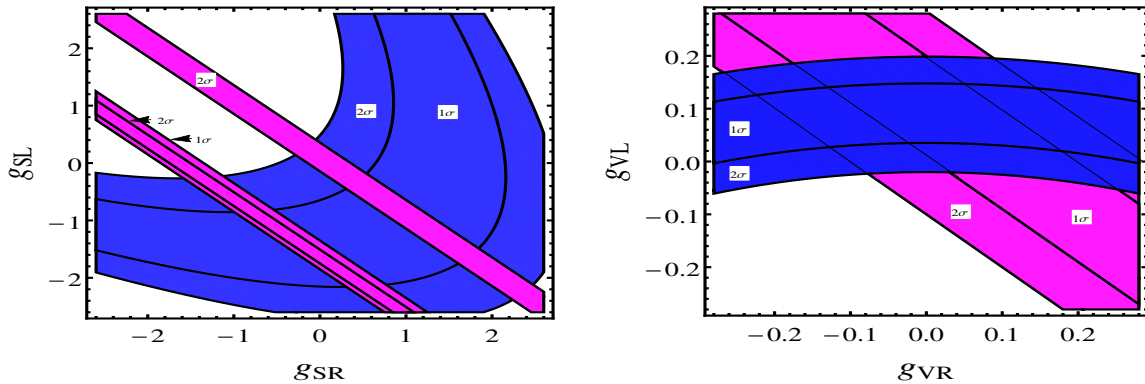


Figure 4.2: The allowed regions in the (g_{SL}, g_{SR}) (left) and (g_{VL}, g_{VR}) (right) planes by the 1σ and 2σ experimental results on $\mathcal{R}(D)$ (magenta) and $\mathcal{R}(D^*)$ (blue) of Belle.

From this figure, it is clear that the scalar contribution alone cannot account for both $\mathcal{R}(D)$ and $\mathcal{R}(D^*)$ simultaneously. In order to get $\mathcal{R}(D)$ and $\mathcal{R}(D^*)$ within 2σ of the Belle results, (g_{SL}, g_{SR}) should lie between $(-2.79, -1.59)$ and $(0.41, 2.47)$, respectively. In this respect, either g_{SL} or g_{SR} is larger than one, which is not possible. In case of a dominant vector contribution, one gets $\mathcal{R}(D)$ and $\mathcal{R}(D^*)$ inside the 2σ region of Belle if (g_{VL}, g_{VR}) varies between $(-0.023, -0.26)$ and $(0.19, 0.38)$, respectively. Furthermore, it is remarkable that, unlike the scalar contribution, a small vector contribution, $g_{VL} \lesssim \mathcal{O}(10\%)$ and $g_{VR} \lesssim \mathcal{O}(1\%)$, can induce significant enhancement for both $\mathcal{R}(D)$ and $\mathcal{R}(D^*)$: e.g., $\mathcal{R}(D) \sim 0.37$ and $\mathcal{R}(D^*) \sim 0.31$ if $g_{VL} \sim 0.1$ and $g_{VR} \sim 0$, which, as we will see, are quite plausible values. Finally, the tensor contribution, which is typically quite small, may affect only $\mathcal{R}(D^*)$.

4.3 New Physics Wilson Coefficients for $b \rightarrow c\tau\nu_\tau$

Apart from corrections to the external legs, the SUSY contribution to the quark-level transition $b \rightarrow c\tau\nu_\tau$, is represented by the tree-level, penguin and box diagrams Fig.4.2, where the total number of all Feynman diagrams is more than 150.

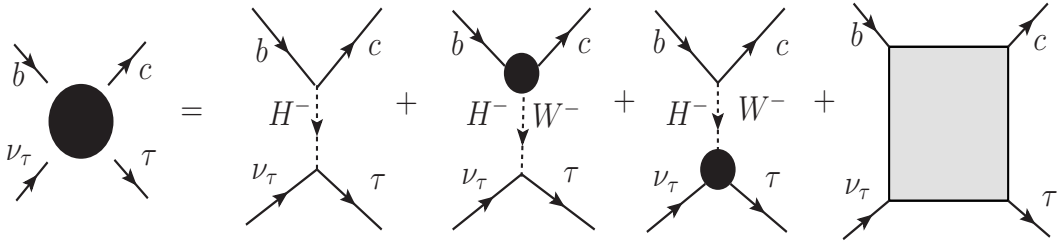


Figure 4.3: Diagrams contributing to $b \rightarrow c\tau\nu_\tau$ transitions

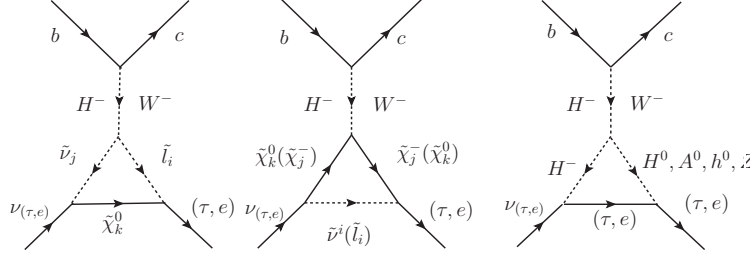


Figure 4.4: Triangle diagrams (penguins) contributing to, *e.g.*, $b \rightarrow c(\tau, e)\nu_{(\tau, e)}$ affecting the leptonic vertex.

We are primarily interested in SUSY contributions to g_{VL} are generated from the penguin corrections to the vertex $W^\pm l \nu_l$ ($l = e, \mu, \tau$) through the exchange of charginos and neutralinos alongside sleptons and sneutrinos, respectively, as displayed in Fig. 4.4, since they are the dominant ones for enhancing $\mathcal{R}(D)$ and $\mathcal{R}(D^*)$. Furthermore, the charged Higgs H^- and W^- boson are responsible for the momentum transfer squared q^2 . Due to the huge topologies for the transition $b \rightarrow c\tau\nu_\tau$, it is difficult to calculate the Wilson coefficients by the usual methods. Therefore, Our calculation is based on FlavorKit [140], SARAH [141] and SPheno [142], although the dominant penguin corrections were also derived analytically. Renormalization is performed at one loop using the $\overline{\text{DR}}$ scheme (following SARAH and SPheno) including the full momentum dependence for any SUSY and Higgs state. As a cross-check of its implementation, we have explicitly verified that, while our loop integrals for the two and three point functions depend upon the renormalization scale, such a dependence drops out in the computation of physical observables. In fact, it can be extracted from our equations that the loop corrections scale with v^2/M_{SUSY}^2 , so that in the limit of very large M_{SUSY} , the loop effects go to zero, hence $\mathcal{R}(D)$ and $\mathcal{R}(D^*)$ approach their SM values. In order to have sizable loop functions, we will enforce on our scans the condition $m_{\chi_1^0} \approx m_{\chi_1^\pm} \lesssim 300$ GeV.

Let us now try to decode our results, by concentrating on the Wilson coefficient

C_{VL} , which sees contributions induced by the penguin topologies in Fig. 4.4. Firstly, we can confirm that the graph with neutral Higgs bosons is small (yet not negligible) while the other two are roughly comparable. Thus, the emerging C_{VL}^{SUSY} term is essentially ($h^0 \equiv H_{SM}$)

$$C_{VL}^{\text{SUSY}} = C_{VL}^{\tilde{\tau}} + C_{VL}^{\tilde{\nu}} + C_{VL}^{(A^0, H^0, h^0)}. \quad (4.3.1)$$

One can show that the leading contributions to C_{VL} are

$$\begin{aligned} C_{VL}^{\tilde{\tau}} = & \frac{\Gamma_{\tilde{\chi}_j^- \nu_{l_I} \tilde{\tau}_i^*}^L \Gamma_{\tilde{l}_I \tilde{\chi}_k^0 \tilde{\tau}_i}^R \Gamma_{\tilde{c} b W^-}^L}{16\pi^2 m_W^2} \left[\Gamma_{\tilde{\chi}_j^+ \tilde{\chi}_k^0 W^-}^R m_{\tilde{\chi}_j^-} m_{\tilde{\chi}_k^0} C_0(0, 0, 0, m_{\tilde{\chi}_k^0}^2, m_{\tilde{\chi}_j^-}^2, m_{\tilde{\tau}_i}^2) \right. \\ & - \Gamma_{\tilde{\chi}_j^+ \tilde{\chi}_k^0 W^-}^L (B_0(0, m_{\tilde{\chi}_j^-}^2, m_{\tilde{\chi}_k^0}^2) - 2C_{00}(0, 0, 0, m_{\tilde{\chi}_k^0}^2, m_{\tilde{\chi}_j^-}^2, m_{\tilde{\tau}_i}^2) \\ & \left. + m_{\tilde{\tau}_i}^2 C_0(0, 0, 0, m_{\tilde{\chi}_k^0}^2, m_{\tilde{\chi}_j^-}^2, m_{\tilde{\tau}_i}^2)) \right], \end{aligned} \quad (4.3.2)$$

$$C_{VL}^{A^0} = \frac{2\Gamma_{\tilde{l}_I H^-}^L \Gamma_{\tilde{l}_I A^0}^R \Gamma_{A^0 H^+ W^-} \Gamma_{\tilde{c} b W^-}^L}{16\pi^2 m_W^2} C_{00}(0, 0, 0, m_l^2, m_{H^-}^2, m_{A^0}^2). \quad (4.3.3)$$

The Wilson coefficient $C_{VL}^{\tilde{\nu}}$ can be obtained from $C_{VL}^{\tilde{\tau}}$ by exchanging $\Gamma_{\tilde{\chi}_j^- \nu_{l_I} \tilde{\tau}_i^*}^L \leftrightarrow \Gamma_{\tilde{\chi}_j^- \tilde{l}_I \tilde{\nu}_i}^R$, $\Gamma_{\tilde{l}_I \tilde{\chi}_k^0 \tilde{\tau}_i}^R \leftrightarrow \Gamma_{\nu_{l_I} \tilde{\chi}_k^0 \tilde{\nu}_i^*}^L$, $\Gamma_{\tilde{\chi}_j^+ \tilde{\chi}_k^0 W^-}^R \leftrightarrow -\Gamma_{\tilde{\chi}_j^+ \tilde{\chi}_k^0 W^-}^L$ and $m_{\tilde{\chi}_k^0} \leftrightarrow m_{\tilde{\chi}_j^-}$ in the loop functions. Also $C_{VL}^{(H^0, h^0)}$ is obtained from $C_{VL}^{A^0}$ by exchanging $A^0 \leftrightarrow (H^0, h^0)$. The corresponding couplings are given by

$$\Gamma_{\tilde{\chi}_j^- \nu_{l_I} \tilde{\tau}_i^*}^L = g(-Z_L^{iI*} Z_-^{j1*} + \frac{m_{l_I}}{\sqrt{2}m_W \cos \beta} Z_L^{i(I+3)*} Z_-^{j2}), \quad (4.3.4)$$

$$\Gamma_{\tilde{l}_I \tilde{\chi}_k^0 \tilde{\tau}_i}^R = \frac{g}{\sqrt{2}} (Z_L^{iI*} (\tan \theta_W Z_N^{k1*} + Z_N^{k2*}) - \frac{m_{l_I}}{m_W \cos \beta} Z_L^{i(I+3)*} Z_N^{j3*}), \quad (4.3.5)$$

$$\Gamma_{\tilde{\chi}_k^0 \nu_{l_I} \tilde{\nu}_i}^L = \frac{g}{\sqrt{2}} Z_\nu^{iI*} (\tan \theta_W Z_N^{k1} - Z_N^{k2}), \quad (4.3.6)$$

$$\Gamma_{\tilde{l}_I \tilde{\chi}_j^+ \tilde{\nu}_i}^R = -g Z_+^{j1*} Z_\nu^{iI}, \quad \Gamma_{\tilde{c} b W^+}^L = -\frac{g}{\sqrt{2}} V_{cb}, \quad (4.3.7)$$

$$\Gamma_{\tilde{\chi}_j^+ \tilde{\chi}_k^0 W^-}^L = -g(Z_-^{j1} Z_N^{k2*} + \frac{1}{\sqrt{2}} Z_-^{j2} Z_N^{k3*}), \quad (4.3.8)$$

$$\Gamma_{\tilde{\chi}_j^+ \tilde{\chi}_k^0 W^-}^R = -g(Z_+^{j1} Z_N^{k2*} - \frac{1}{\sqrt{2}} Z_+^{j2} Z_N^{k4*}), \quad (4.3.9)$$

$$\Gamma_{\tilde{\tau}\nu H^-}^L = \frac{gm_\tau}{\sqrt{2}m_W \cos \beta} Z_{H^-}^{21}, \quad \Gamma_{A^0 H^+ W^-} = \frac{g}{2}, \quad (4.3.10)$$

$$\Gamma_{\tilde{\tau}\tau A^0}^R = -\frac{1}{\sqrt{2}} \frac{gm_\tau}{\sqrt{2}m_W \cos \beta} Z_A^{21}, \quad (4.3.11)$$

$$\Gamma_{\tilde{\tau}\tau H^0}^R = \frac{1}{\sqrt{2}} \frac{gm_\tau}{\sqrt{2}m_W \cos \beta} Z_H^{21}, \quad (4.3.12)$$

$$\Gamma_{H^0 H^+ W^-} = \frac{g}{2} (Z_H^{22} Z_{H^-}^{22} - Z_H^{21} Z_{H^-}^{21}), \quad (4.3.13)$$

where Z_L , Z_ν , $Z_\pm$, Z_N and $Z_{(H,A,H^-)}$ are the diagonalizing matrices for slepton, sneutrino, chargino, neutralino and Higgs masses, respectively. In addition, the loop functions are given by [143]

$$B_0(0, x, y) = \eta_\epsilon - 1 + \log \frac{x}{\tilde{\mu}^2} - \frac{y \log \frac{y}{x}}{x - y}, \quad (4.3.14)$$

$$C_0(0, 0, 0, x, y, z) = \frac{1}{y - z} \left(\frac{y \log \frac{y}{x}}{y - x} + \frac{z \log \frac{z}{x}}{x - z} \right), \quad (4.3.15)$$

$$C_{00}(0, 0, 0, x, y, z) = \frac{1}{4} \left(\eta_\epsilon - \log \frac{x}{\tilde{\mu}^2} \right) + \frac{3}{8} + \frac{1}{y - z} \left(\frac{y^2 \log \frac{y}{x}}{4(x - y)} - \frac{z^2 \log \frac{z}{x}}{4(x - z)} \right), \quad (4.3.16)$$

with $\eta_\epsilon = \frac{2}{d-4} + \log 4\pi\gamma_E$ and $\tilde{\mu}$ is the renormalization scale with the dimensions of mass. It is worth noting that the suppression loop factor, $1/16\pi^2$, in $C_{VL}^{\tilde{\tau}}$ can be compensated by degenerate and light neutralino-chargino masses as well as large values of the stau mass. This can be seen from noticing that, although $C_0 \sim 10^{-6}$ as shown in Fig. 4.5 (right), the last term in Eq. (4.3.2) gives the largest contribution to $C_{VL}^{\tilde{\tau}}$ if $m_{\tilde{\tau}} \gtrsim \mathcal{O}(1 \text{ TeV})$.

Specifically, for $m_{\tilde{\chi}_k^0} \simeq m_{\tilde{\chi}_j^-}$, the loop function takes the form

$$C_0(m_{\tilde{\chi}_i^0}^2, m_{\tilde{\tau}_j}^2) = \frac{1}{m_{\tilde{\chi}_i^0}^2 - m_{\tilde{\tau}_j}^2} \left[m_{\tilde{\chi}_i^0}^2 - m_{\tilde{\tau}_j}^2 + m_{\tilde{\tau}_j}^2 \log \left(\frac{m_{\tilde{\tau}_j}^2}{m_{\tilde{\chi}_i^0}^2} \right) \right]. \quad (4.3.17)$$

In Fig. 4.5 we show the behavior of the first term in Eq. (4.3.2), $C_0(m_{\tilde{\chi}_0^0}^2, m_{\tilde{\chi}^-}^2, m_{\tilde{\tau}}^2)$ and $m_{\tilde{\chi}^-} m_{\tilde{\chi}_0^0} C_0(m_{\tilde{\chi}_0^0}^2, m_{\tilde{\chi}^-}^2, m_{\tilde{\tau}}^2)$ respectively, as function of $m_{\tilde{\tau}}$ (right panel) and $m_{\tilde{\chi}}$ (left panel) for degenerate chargino/neutralino masses. As it can be seen from this figure, the sensitivity of the first term in Eq. (4.3.2) to chargino/neutralino and

stau masses through the loop function C_0 . The largest corrections are obtained if chargino/neutralino masses are more than 200 GeV and the stau mass is less than 1 TeV, this leads to $m_{\tilde{\chi}^-}m_{\tilde{\chi}^0}C_0(m_{\tilde{\chi}^0}^2, m_{\tilde{\chi}^-}^2, m_{\tilde{\tau}}^2) \sim 0.15$, however, the last term in Eq. (4.3.2) requires light chargino/neutralino and heavy stau which gives $m_{\tilde{\chi}^-}m_{\tilde{\chi}^0}C_0(m_{\tilde{\chi}^0}^2, m_{\tilde{\chi}^-}^2, m_{\tilde{\tau}}^2) \sim 1$. It is further clear that, in the light gaugino mass regime, chargino/neutralino mass degeneracy is not a pre-condition for enhancing the aforementioned loop contribution, it so happens that there can be spectrum configurations in the scan when they are close in mass, as allowed by experimental constraints. We stress again that, even if the stau is very heavy, we are not in the decoupling limit, where SUSY effects must diminish, since charginos/neutralinos are kept quite light.

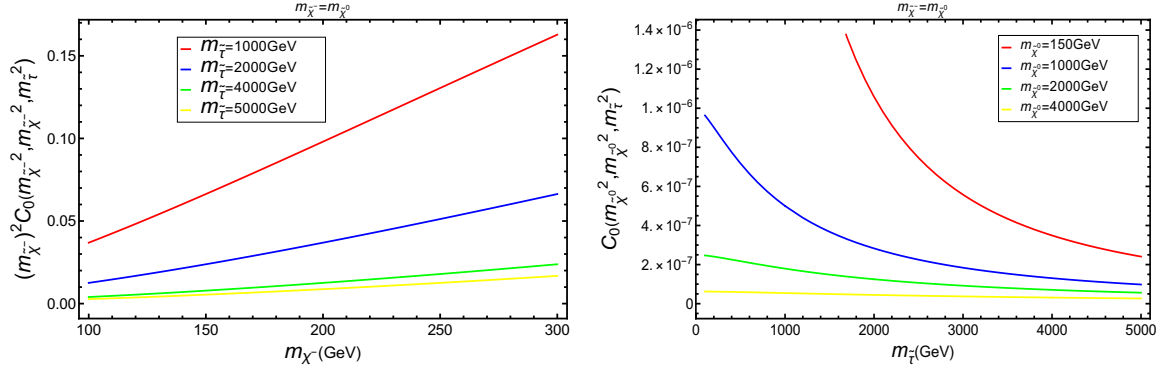


Figure 4.5: Behaviour of the first term in Eq (4.3.2), $m_{\tilde{\chi}^0}m_{\tilde{\chi}^-}C_0(m_{\tilde{\chi}^0}^2, m_{\tilde{\chi}^-}^2, m_{\tilde{\tau}}^2)$ (left) and $C_0(m_{\tilde{\chi}^0}^2, m_{\tilde{\chi}^-}^2, m_{\tilde{\tau}}^2)$ (right), respectively, with $m_{\tilde{\chi}^-}$ and $m_{\tilde{\tau}}$ for degenerate chargino/neutralino masses.

Finally, the Wilson coefficients $C^{\text{SUSY}}(W \rightarrow l\nu)$ and $C^{\text{SUSY}}(\tau \rightarrow \nu_\tau l\nu_l)$ can be obtained from Eqs. (4.3.1) and (4.3.2) as follows

$$C^{\text{SUSY}}(\tau \rightarrow \nu_\tau l\nu_l) = \frac{\Gamma_{\tilde{\nu}_\tau \tau W^-}^L}{\Gamma_{cbW^-}^L} (C_{VL}^{\tilde{\tau}} + C_{VL}^{\tilde{\nu}} + C_{VL}^{(A^0, H^0, h^0)}), \quad (4.3.18)$$

$$C^{\text{SUSY}}(W \rightarrow l\nu) = \frac{m_W^2}{\Gamma_{cbW^-}^L} (C_{VL}^{\tilde{\tau}} + C_{VL}^{\tilde{\nu}} + C_{VL}^{(A^0, H^0, h^0)}), \quad (4.3.19)$$

where the loop functions in these Wilson coefficients are given by the following changes:

$$\begin{aligned}
C_0(0, 0, 0, m_{\chi_k^0}^2, m_{\chi_j^-}^2, m_{\tilde{\nu}_i}^2) &\rightarrow C_0(m_W^2, 0, 0, m_{\chi_k^0}^2, m_{\chi_j^-}^2, m_{\tilde{\nu}_i}^2), \\
C_{00}(0, 0, 0, m_{\chi_k^0}^2, m_{\chi_j^-}^2, m_{\tilde{\nu}_i}^2) &\rightarrow C_{00}(m_W^2, 0, 0, m_{\chi_k^0}^2, m_{\chi_j^-}^2, m_{\tilde{\nu}_i}^2), \\
C_{00}(0, 0, 0, m_\tau^2, m_{H^-}^2, m_{A^0}^2) &\rightarrow C_{00}(0, m_W^2, 0, m_\tau^2, m_{H^-}^2, m_{A^0}^2), \\
B_0(0, m_{\chi_k^0}^2, m_{\chi_j^-}^2) &\rightarrow B_0(m_W^2, m_{\chi_k^0}^2, m_{\chi_j^-}^2),
\end{aligned} \tag{4.3.20}$$

where the loop functions of non-vanishing m_W can be found in [144, 145].

4.4 Experimental Constraints

Let us now discuss experimental limits coming from other precesses. In this regard, one should consider a possible constraint due to the direct measurement of the W boson decay width that leads to [65]

$$\Gamma(W \rightarrow \tau\nu)/\Gamma(W \rightarrow e\nu) = 1.043 \pm 0.024. \tag{4.4.1}$$

The SM prediction for this ratio is given by ~ 0.999267 , which is consistent with the measured value. Similarly, constraints can also be obtained from [65]

$$\Gamma(W \rightarrow \tau\nu)/\Gamma(W \rightarrow \mu\nu) = 1.07 \pm 0.026, \tag{4.4.2}$$

with which the SM is also compliant. The decay width of $W \rightarrow l\nu$ with SUSY contributions can be parametrized as

$$\Gamma(W \rightarrow l\nu) = \frac{G_F m_W^3}{6\sqrt{2}\pi} \left(1 - \frac{m_l^2}{m_W^2}\right)^2 \left(1 + \frac{1}{2} \frac{m_l^2}{m_W^2}\right) |1 + g'_{VL}|^2, \tag{4.4.3}$$

where $g'_{VL} = C^{\text{SUSY}}(W \rightarrow l\nu)/C^{\text{SM}}(W \rightarrow l\nu)$ and $C^{\text{SM}}(W \rightarrow l\nu) = g/\sqrt{2}$. Another important experimental measurement connected with lepton universality in τ decay

that should be considered here is on $\tau \rightarrow \nu_\tau l \nu_l$ with $l = e, \mu$, which is given by the relation [146]

$$\left(\frac{g_\mu}{g_e}\right)^2 = \frac{BR(\tau \rightarrow \mu \nu_\tau \nu_\mu)}{BR(\tau \rightarrow e \nu_\tau \nu_e)} \frac{f(m_e^2/m_\tau^2)}{f(m_\mu^2/m_\tau^2)}. \quad (4.4.4)$$

In the SM, the universal gauge interaction implies that

$$\frac{\Gamma(\tau \rightarrow \mu \nu_\tau \nu_\mu)}{\Gamma(\tau \rightarrow e \nu_\tau \nu_e)} = \frac{f(m_\mu^2/m_\tau^2)}{f(m_e^2/m_\tau^2)} = 0.9726, \quad (4.4.5)$$

where $f(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \log(x)$. The current experimental result for this ratio is 0.979 ± 0.004 [65]. With SUSY contributions, Eq. (4.4.5) can be written as

$$\frac{\Gamma(\tau \rightarrow \mu \nu_\tau \nu_\mu)}{\Gamma(\tau \rightarrow e \nu_\tau \nu_e)} = 0.9726 \frac{|1 + g_{VL}^\mu|^2}{|1 + g_{VL}^e|^2}, \quad (4.4.6)$$

where $g_{VL}^l = C^{\text{SUSY}}(\tau \rightarrow \nu_\tau l \nu_l)/C^{\text{SM}}(\tau \rightarrow \nu_\tau l \nu_l)$ with $C^{\text{SM}}(\tau \rightarrow \nu_\tau l \nu_l) = 2\sqrt{2}G_F$. (As we will show, this imposes stringent constraints on SUSY contributions to g_{VL}^l .) Furthermore, SUSY loop effects induce a correction to the Fermi coupling via a potential breaking of $\mu - e$ universality. In fact, using Eqs. (4.4.4) and (4.4.6), one can find

$$\frac{g_\mu}{g_e} = \frac{|1 + g_{VL}^\mu|}{|1 + g_{VL}^e|}, \quad (4.4.7)$$

so that the above experimental constraints impose that $0.9983 \leq |1 + g_{VL}^\mu|/|1 + g_{VL}^e| \leq 1.0012$. In our work, we will enforce $g_\mu = g_e = g$, which satisfies Eq. (4.4.7). In summary, in our scans, only points respecting all above limits are retained.

Also the oblique Electro-Weak (EW) parameters S, T and U [147] are useful to constraint NP that enters in self-energy corrections to the gauge boson propagators, denoted by Π_{ij} , which represents the transition ij ($i, j = W, Z, \gamma$), e.g., we have [65]

$$\hat{\alpha}(m_Z)T = \frac{\Pi_{WW}^{\text{new}}(0)}{m_W^2} - \frac{\Pi_{ZZ}^{\text{new}}(0)}{m_Z^2}, \quad (4.4.8)$$

where $\hat{\alpha}(m_Z)$ is the renormalized Electro-Magnetic (EM) coupling constant at the m_Z scale. Here, we are interested in the T parameter. In this respect, a related quantity

known as the ρ parameter is defined as [65]

$$\rho - 1 = \frac{1}{1 - \hat{\alpha}(m_Z)T} \simeq \hat{\alpha}(m_Z)T. \quad (4.4.9)$$

In this work we take $\Delta\rho^{\text{exp}} = \rho - 1 = 0.0006 \pm 0.0009$, which is extracted from the data on the T parameter, (0.08 ± 0.12) [65]. While in the SM $\rho \equiv \rho_0 = m_W^2/m_Z^2 \cos \theta_W = 1$ at tree level, in our scan we obtain $\Delta\rho^{\text{SUSY}} \in [0.0001, 0.0006]$.

4.5 $\mathcal{R}(D)$ and $\mathcal{R}(D^*)$ Result in the MSSM with CP Conservation

We now perform the numerical evaluations in the light of the results in Sects. 4.2 and 4.3 in presence of experimental constraints. Since our focus is on the penguin contributions, let us look at the relevant loop functions entering the numerics. Let us begin with those of $W \rightarrow l\nu$, from the formulae given in Eq. (4.3.20) one can notice that the loop functions of the decay $W \rightarrow l\nu$ are approximately equal to those associated with $b \rightarrow cl\nu$: this is made evident in Tab. 4.1, for the case of the MSSM benchmark of Tab. 4.2, which is one yielding sizable corrections to both $\mathcal{R}(D)$ and $\mathcal{R}(D^*)$.

Loop function	$b \rightarrow c\tau\nu_\tau$	$W \rightarrow \tau\nu_\tau$
C_{00}	-3.3939	1.8622
C_0	1.1094×10^{-6}	2.5985×10^{-6}
B_0	10.5086	0.3371

Table 4.1: Loop functions values which correspond to the benchmark given in Tab. 4.2.

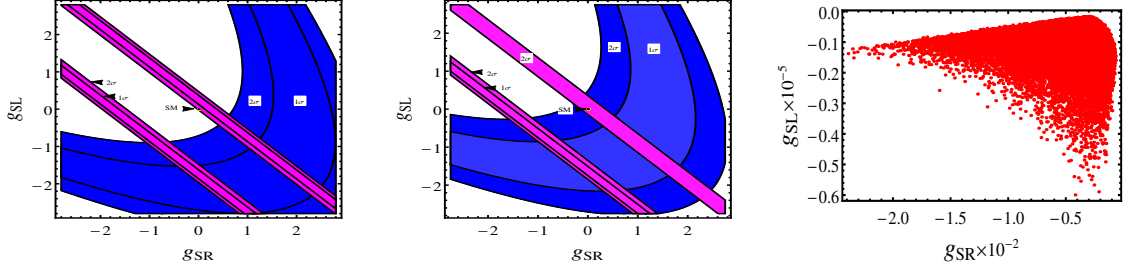


Figure 4.6: The allowed regions in the (g_{SL}, g_{SR}) plane by the 1σ and 2σ experimental results on $\mathcal{R}(D)$ (magenta) and $\mathcal{R}(D^*)$ (blue) of BaBar (left) and Belle (middle). SM and complete SUSY predictions (tree, penguin and box contributions, where tree-level effect is the dominant for the scalar interactions) are also included and they coincide with the black point. The correlation between the SUSY corrected values of g_{SL} and g_{SR} is displayed in the right panel.

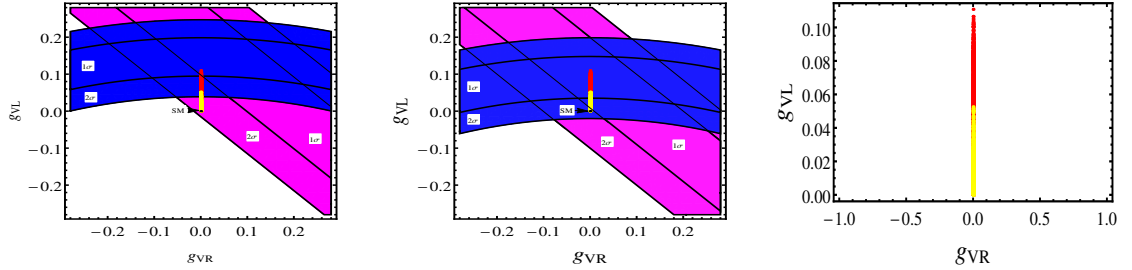


Figure 4.7: The allowed regions in the (g_{VL}, g_{VR}) plane by the 1σ and 2σ experimental results on $\mathcal{R}(D)$ (magenta) and $\mathcal{R}(D^*)$ (blue) of BaBar (left) and Belle (middle). SM (black point) and complete SUSY (red points) predictions (penguin and box contributions, where the penguin is the dominant one) are also included. The correlation between the SUSY corrected values of g_{VL} and g_{VR} is displayed in the right panel, where yellow points represent the configurations that yield $\Gamma(\tau \rightarrow \mu\nu_\tau\nu_\mu)/\Gamma(\tau \rightarrow e\nu_\tau\nu_e)$ within experimental limits while the red ones are the complete sample.

In essence, the one-loop SUSY effects onto the W widths are scaled by the W squared mass while in $\mathcal{R}(D)$ and $\mathcal{R}(D^*)$ only by the meson squared masses. These suppressions are crucial for satisfying the experimental constraints on the ratio of the W decay widths so that the results of $\mathcal{R}(D)$ and $\mathcal{R}(D^*)$ can be accommodated in unexcluded regions of the MSSM parameter space.

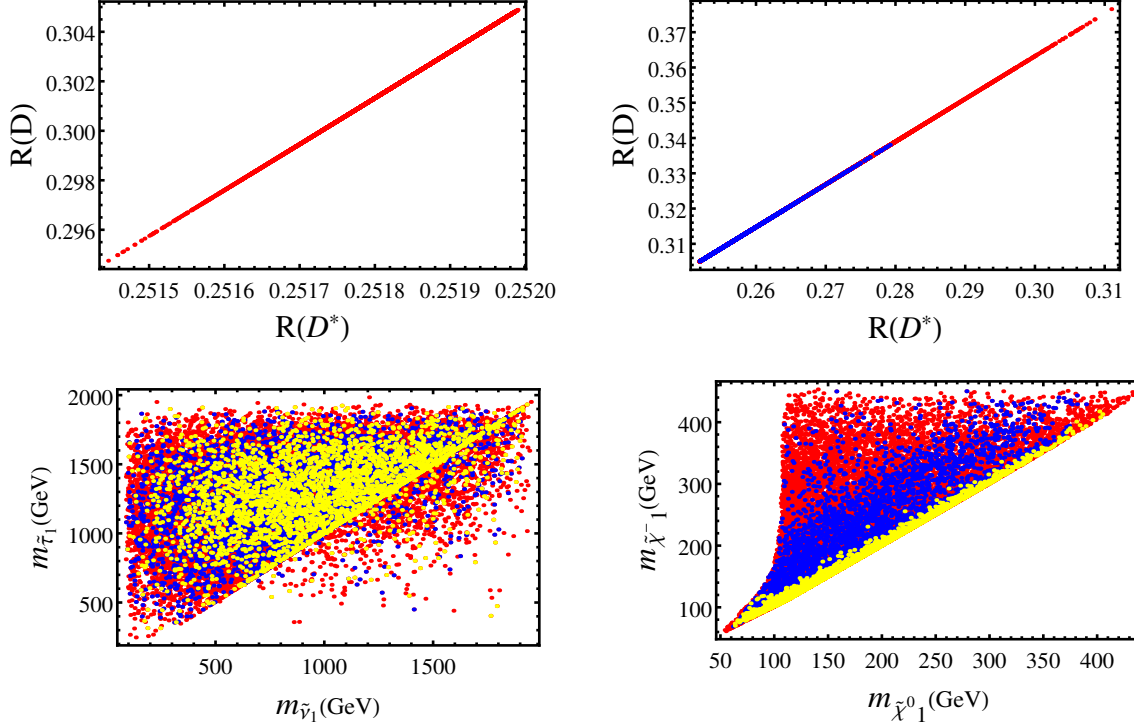


Figure 4.8: The correlation between $\mathcal{R}(D)$ and $\mathcal{R}(D^*)$ at tree level (top-left) and after the one-loop SUSY contributions through the lepton penguins (top-right) where the blue points show the constrained ones by $\Gamma(\tau \rightarrow \mu\nu_\tau\nu_\mu)/\Gamma(\tau \rightarrow e\nu_\tau\nu_e)$. The correlation between sneutrino and stau masses is on the bottom-left (same colour scheme as in the top-right frame with the additional yellow points representing the region with $\mathcal{R}(D) > 0.33$). (Bottom-right) This represents the correlation between chargino and neutralino masses where the colours are the same meaning as in the top and bottom-left panels.

As is mentioned, the enhancement of $C_{VL}^{\tilde{\tau}}$ occurs when the chargino and neutralino masses are (nearly) degenerate, in addition to large $\tan\beta$ and stau mass. Therefore, in our scan, we focus on benchmark points where the gaugino soft masses are given by $M_1, M_2 \in [100, 300]$ GeV and $M_3 = 2$ TeV. Also, we choose the μ parameter $\in [100, 300]$ GeV, $m_{A^0}^2 \in [0, 10^6](\text{GeV})^2$, the A terms $\in [-2000, -100]$ GeV, $M_{\tilde{Q}}, M_{\tilde{U}}$ and $M_{\tilde{D}}$ are fixed in the TeV range while the slepton soft mass terms $m_{\tilde{L}}$ and $m_{\tilde{E}} \in [100, 2000]$ GeV. Since all parameters are real, we are with CP-conserving

case. Finally, we take $\tan \beta \in [5, 60]$. Indeed, the aforementioned Tab. 4.2 shows an example yielding large corrections to our two observables extracted from such a scan.

In Fig. 4.6 we display the regions in the (g_{SL}, g_{SR}) plane that can accommodate the experimental results of $\mathcal{R}(D)$ and $\mathcal{R}(D^*)$ within a 1σ confidence level for BaBar and Belle. We also show how g_{SL} and g_{SR} correlate at one-loop level. It is clear that the scalar contribution alone cannot account for both $\mathcal{R}(D)$ and $\mathcal{R}(D^*)$ simultaneously. While in Fig. 4.7 we display the regions in the (g_{VL}, g_{VR}) plane that can accommodate the BaBar and Belle results on $\mathcal{R}(D)$ and $\mathcal{R}(D^*)$ within a 1σ confidence level and compare these to the MSSM expectations at the one-loop level. It is clear that the contributions that induce vector operators, like the aforementioned triangle diagrams, lead to $\mathcal{R}(D)$ and $\mathcal{R}(D^*)$ close to or within the experimental regions. We can also conclude that g_{VL} must be non-vanishing and of order 0.1 while g_{VR} can be in the range $[-0.1, 0.1]$. Furthermore, it is remarkable that our SUSY model can perfectly account for the Belle results and is within 2σ of the BaBar ones.

In the top-left (top-right) panel of Fig. 4.8 we present the correlation between $\mathcal{R}(D)$ and $\mathcal{R}(D^*)$ at tree-level (at one-loop due to the SUSY contributions to the lepton penguins alone). As can be seen from this plot, in presence of MSSM one-loop corrections, $\mathcal{R}(D)$ can reach 0.37 while $\mathcal{R}(D^*)$ extends to 0.31, which are results rather consistent with the Belle measurements and not that far from the BaBar ones. To our knowledge, these enhancements in both $\mathcal{R}(D)$ and $\mathcal{R}(D^*)$ have never been accounted for before in any NP scenario.

It is also very relevant to extract the typical mass spectra which are responsible for the MSSM configurations yielding $\mathcal{R}(D)$ and $\mathcal{R}(D^*)$ values (potentially) consistent with experimental measurements, as these might be accessible during Run 2 at the LHC. As an indication, this is done in Fig. 4.8 (bottom-left panel) for the case of the lightest stau and sneutrino. The plot shows a predilection of the highest $\mathcal{R}(D)$ and $\mathcal{R}(D^*)$ points for MSSM parameter configurations with $m_{\tilde{\tau}_1} > m_{\tilde{\nu}_1}$ while the absolute

mass scale can cover the entire interval from 200 GeV to 2 TeV. However, the points with $\mathcal{R}(D) > 0.33$ require a rather large $\tilde{\tau}_1$ mass (say above 1 TeV) irrespectively of the $\tilde{\nu}_1$ one as well as large $\tan\beta$. This signals that there occurs an interplay between mass suppressions in the loops and enhancements in the couplings. Finally, the bottom-right frame of Fig. 4.8 illustrates the role of the lightest chargino and neutralino masses with $m_{\tilde{\chi}^-} \approx m_{\tilde{\chi}^0} \lesssim 300$ GeV, wherein it is visible the preference of large $\mathcal{R}(D)$ rates for rather degenerate masses of the two gauginos, as dictated by the structure of the loop functions B_0, C_0 and C_{00} . (In fact, this phenomenon was already visible, albeit with less dramatic effects and limitedly to $\mathcal{O}(1 \text{ TeV})$ mass values, in the case of $m_{\tilde{\tau}_1} \approx m_{\tilde{\nu}_1}$, see bottom-left of Fig. 4.8.)

Parameter	Value
$\tan\beta$	53.93
μ	492.37 GeV
$M_{A^0} \text{ (tree)}$	468.02 GeV
M_1, M_2, M_3	337.59, 186.73, 2000 GeV
$M_{\tilde{U}}, M_{\tilde{D}}, M_{\tilde{Q}}$	1 TeV (all)
$M_{\tilde{E}_1}, M_{\tilde{E}_2}, M_{\tilde{E}_3}$	551.35, 1000.92, 1836.04 GeV
$M_{\tilde{L}_1}, M_{\tilde{L}_2}, M_{\tilde{L}_3}$	1830.02, 1905.48, 1794.61 GeV
A_t, A_b, A_τ	-1708.04, -1186.05, 0 GeV
$m_{\tilde{\chi}_1^0}, m_{\tilde{\chi}_1^-}$	191.19, 191.58 GeV
$m_{\tilde{\nu}_1}, m_{\tilde{\tau}_1}$	1795.19, 1793.10 GeV
$m_{h^0}, m_{H^0}, m_{A^0}, m_{H^\pm}$	124.66, 393.41, 393.49, 356.67 GeV
$g_{VL}(b \rightarrow c\tau\bar{\nu}_\tau)$	0.1014
$g_{VL}(b \rightarrow ce\bar{\nu}_e)$	0
$\mathcal{R}(D)$	0.3702
$\mathcal{R}(D^*)$	0.3057

 Table 4.2: Illustrative benchmark point yielding large $\mathcal{R}(D)$ and $\mathcal{R}(D^*)$ values.

4.6 CP Violation Effect on $\mathcal{R}(D)$ and $\mathcal{R}(D^*)$

So far we have proven that SUSY may provide an explanation to the recent result presented by Belle in CP-conserving case. This implies that the SUSY mass parameters are real. In this section we examine the CP violation effect on the observables $\mathcal{R}(D)$ and $\mathcal{R}(D^*)$, where new sources for CP violation are required. In this respect, SUSY contains many possible sources of CP violation beyond the Kobayashi-Maskawa phase of the SM. As we seen in second chapter, these additional sources arise from the soft supersymmetry-breaking terms in the low-energy effective Lagrangian, and include phases in the sfermion mass matrices, the trilinear scalar couplings and the gaugino masses.

In the presence of CP violating, the mass matrix of charginos can be written as follows

$$M_{\chi^\pm} = \begin{pmatrix} M_2 & \sqrt{2}m_W s_\beta \\ \sqrt{2}m_W c_\beta & |\mu| e^{i\varphi_\mu} \end{pmatrix}, \quad (4.6.1)$$

where M_2 is the soft SUSY-breaking mass of the gaugino partner of the $W^\pm$ gauge boson which also could be complex, μ is the Higgs mixing parameter, φ_μ is the μ complex phase and $s_\beta \equiv \sin \beta, c_\beta \equiv \cos \beta$. The charginos mass matrix can be diagonalized by two unitary matrices Z_+ and Z_- , we have

$$M_{\tilde{\chi}^\pm}^{dig} = Z_+^* M_{\chi^\pm} Z_-^\dagger. \quad (4.6.2)$$

Since the charginos mass matrix is complex, the matrices Z_+ and Z_- must be complex, they are

$$Z_+ = \begin{pmatrix} \cos \theta_U & e^{-i\varphi_U} \sin \theta_U \\ -e^{i\varphi_U} \sin \theta_U & \cos \theta_U \end{pmatrix}, \quad (4.6.3)$$

$$Z_- = \begin{pmatrix} \cos \theta_V e^{-i\phi_1} & \sin \theta_V e^{-i(\varphi_V + \phi_1)} \\ -\sin \theta_V e^{i(\varphi_V - \phi_2)} & \cos \theta_V e^{-i\phi_2} \end{pmatrix}, \quad (4.6.4)$$

where the mixing angles expressions are given by [127]

$$\tan 2\theta_U = \frac{2\sqrt{2}m_W(M_2^2 \cos^2 \beta + |\mu|^2 \sin^2 \beta + |\mu| M_2 \sin 2\beta \cos \varphi_\mu)^{\frac{1}{2}}}{M_2^2 - |\mu|^2 - 2m_W^2 \cos 2\beta}, \quad (4.6.5)$$

$$\tan 2\theta_V = \frac{2\sqrt{2}m_W(M_2^2 \sin^2 \beta + |\mu|^2 \cos^2 \beta + |\mu| M_2 \sin 2\beta \cos \varphi_\mu)^{\frac{1}{2}}}{M_2^2 - |\mu|^2 + 2m_W^2 \cos 2\beta}, \quad (4.6.6)$$

and the phases are

$$\tan \varphi_U = -\frac{|\mu| \sin \varphi_\mu \sin \beta}{M_2 \cos \beta + |\mu| \cos \varphi_\mu \sin \beta}, \quad (4.6.7)$$

$$\tan \varphi_V = -\frac{|\mu| \sin \varphi_\mu \cos \beta}{M_2 \sin \beta + |\mu| \cos \varphi_\mu \cos \beta}, \quad (4.6.8)$$

$$\tan \phi_1 = \frac{|\mu| m_W^2 \sin 2\beta \sin \varphi_\mu}{M_2(m_{\chi^\pm}^2 - |\mu|^2) + |\mu| m_W^2 \sin 2\beta \cos \varphi_\mu}, \quad (4.6.9)$$

$$\tan \phi_2 = -\frac{|\mu| (m_{\chi^\pm}^2 - M_2^2) \sin \varphi_\mu}{M_2 m_W^2 \sin 2\beta + |\mu| (m_{\chi^\pm}^2 - M_2^2) \cos \varphi_\mu}, \quad (4.6.10)$$

where $\frac{-\pi}{2} \leq \theta_i \leq \frac{\pi}{2}$, $-\pi \leq \varphi_i, \phi_i \leq \pi$ with $i = U, V$.

In a similar way, one can write the neutralinos mass matrix as

$$M_N = \begin{pmatrix} |M_1|e^{i\varphi_{M_1}} & 0 & -m_Z \sin \theta_W \cos \beta & m_Z \sin \theta_W \sin \beta \\ 0 & |M_2|e^{i\varphi_{M_2}} & m_Z \cos \theta_W \cos \beta & -m_Z \cos \theta_W \sin \beta \\ -m_Z \sin \theta_W \cos \beta & m_Z \cos \theta_W \cos \beta & 0 & -|\mu|e^{i\varphi_\mu} \\ m_Z \sin \theta_W \sin \beta & -m_Z \cos \theta_W \sin \beta & -|\mu|e^{i\varphi_\mu} & 0 \end{pmatrix}, \quad (4.6.11)$$

4.6 CP Violation Effect on $\mathcal{R}(D)$ and $\mathcal{R}(D^*)$

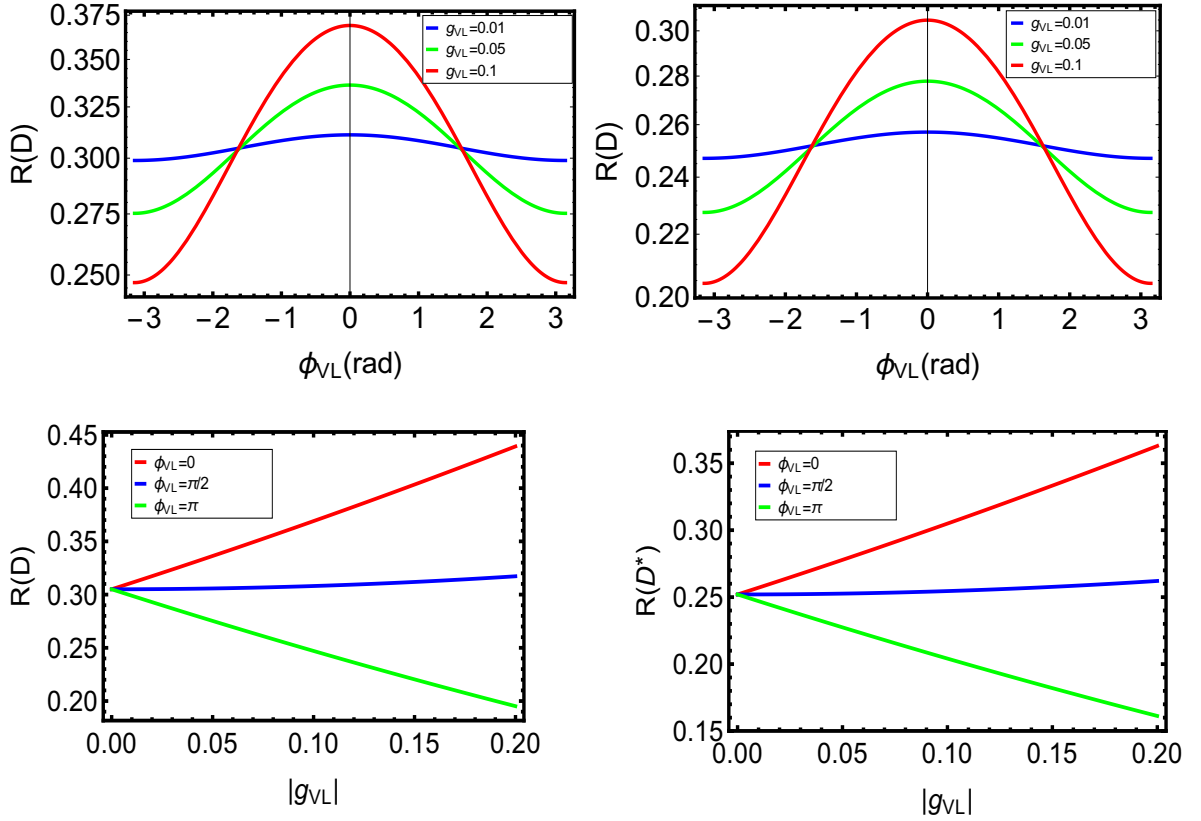


Figure 4.9: $\mathcal{R}(D)$ and $\mathcal{R}(D^*)$ as functions of the complex phase $\phi_{VL} \in [-\pi, \pi]$ (top) and $|g_{VL}|$ (bottom) for $|g_{VL}| = 0.01, 0.05, 0.1$ and $|\phi_{VL}| = 0, \pi/2, \pi$, respectively.

it can be diagonalized by a single 4×4 complex matrix Z_N such that

$$Z_N^* M_N Z_N^{-1} = \text{dig}(m_{\chi_1^0}, m_{\chi_2^0}, m_{\chi_3^0}, m_{\chi_4^0}). \quad (4.6.12)$$

Due to the complex diagonalizing matrices for chargino and neutralino masses, the Wilson coefficient could be complex, we write

$$g_{VL} = |g_{VL}| e^{i\phi_{VL}}. \quad (4.6.13)$$

Thus one finds that the observables $\mathcal{R}(D)$ and $\mathcal{R}(D^*)$ can be given by³

$$\mathcal{R}(D) = \mathcal{R}(D)^{\text{SM}} |1 + |g_{VL}| e^{i\phi_{VL}}|^2, \quad (4.6.14)$$

³We assume that $g_{SL(R)}$, g_{VR} and g_T are negligible.

$$\mathcal{R}(D^*) = \mathcal{R}(D^*)^{\text{SM}} |(1 + |g_{VL}|e^{i\phi_{VL}})|^2. \quad (4.6.15)$$

For model independent analysis, we assume that the dominant contribution coming from g_{VL} . This assumption allows us to determine the sensitivity of $\mathcal{R}(D)$ and $\mathcal{R}(D^*)$ to the complex phase ϕ_{VL} .

In Fig. 4.9 we show the complex phase effect on $\mathcal{R}(D)$ and $\mathcal{R}(D^*)$ for fixed values for $|g_{VL}|$. It is clear that $\mathcal{R}(D)$ and $\mathcal{R}(D^*)$ acquire the largest values for sizable $|g_{VL}|$ and vanishing phase. As can be seen from this plot, CP violation can not play an important role for enhancing $\mathcal{R}(D)$ and $\mathcal{R}(D^*)$. This implies that, the best senario is with vanishing phase.

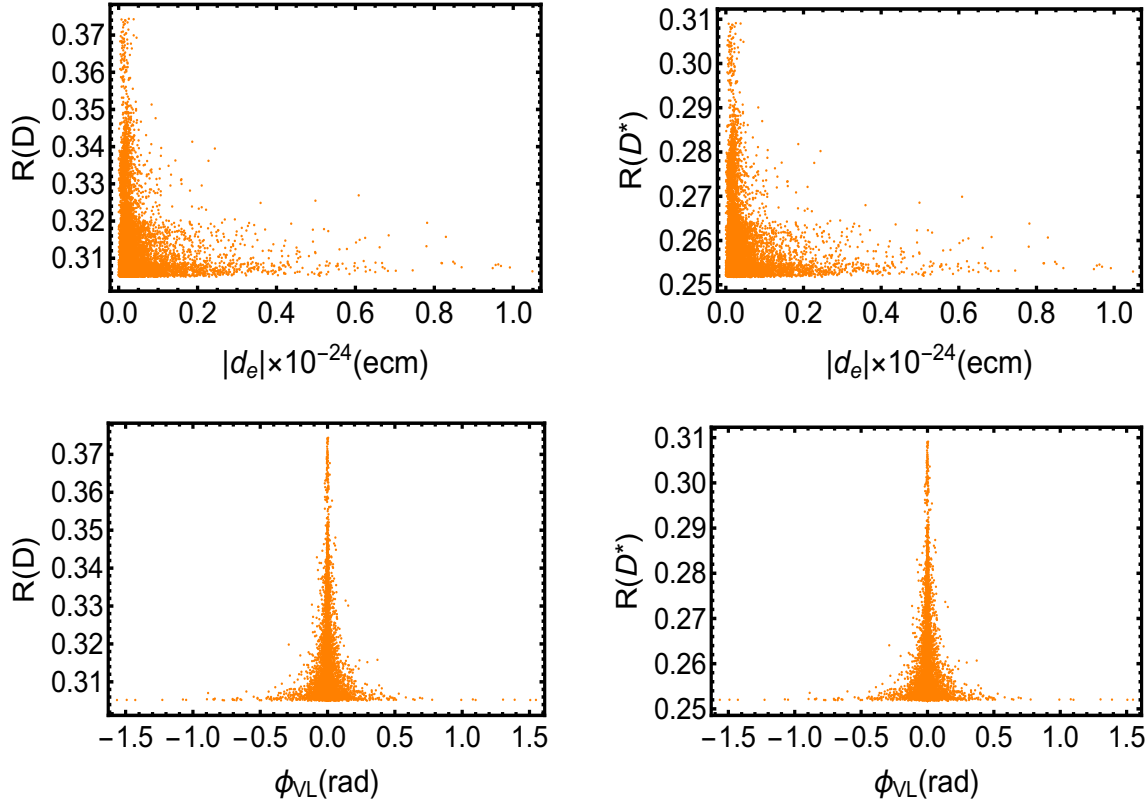


Figure 4.10: $\mathcal{R}(D)$ and $\mathcal{R}(D^*)$ as functions of eEDM (top) and complex phase ϕ_{VL} (bottom) in the MSSM.

As is discussed, the CP-violating MSSM contribution can be entering through soft

4.6 CP Violation Effect on $\mathcal{R}(D)$ and $\mathcal{R}(D^*)$

masses. Therefore, in our scan, the chosen complex soft masses are given by $\Re M_1, \Re M_2, \Im M_1, \Im M_2 \in [100, 300]$ GeV and $\Re \mu, \Im \mu \in [100, 300]$ GeV. Also, we choose the slepton soft mass terms $\Re m_{\tilde{L}}, \Im m_{\tilde{L}}$ and $\Re m_{\tilde{E}}, \Im m_{\tilde{E}} \in [100, 2000]$ GeV. While the other parameters remain as in CP-conserving case.

In Fig. 4.10 we present the scatter plot for CP violation effect on $\mathcal{R}(D)$ and $\mathcal{R}(D^*)$ in the MSSM. It can be seen from the plot that $\mathcal{R}(D)$ can be up to 0.37 while $\mathcal{R}(D^*)$ prolongs to 0.31 at $\phi_{VL} \sim 0$. However, the eEDM limit in Eq. (2.9.2) rules out this result. This argument shows up in the top panels. One has to note that the CP-violating phases do not affect both $\mathcal{R}(D)$ and $\mathcal{R}(D^*)$. Finally, we deduce that CP-conserving case is the best scenario for $\mathcal{R}(D)$ and $\mathcal{R}(D^*)$.

5

Conclusion

Through this thesis we have studied the phenomenological implications of supersymmetric theories on the light of the recent experimental result coming from B meson decays. We focused on the possible signals for supersymmetric events at the LHC, Belle and BaBar as well. For instance, we studied the processes $B \rightarrow \tau\bar{\mu} + \mu\bar{\tau}$, $\bar{B} \rightarrow D\tau\bar{\nu}_\tau$ and $\bar{B} \rightarrow D^*\tau\bar{\nu}_\tau$ in the MSSM, where we took into account the CP-violating and CP-conseving MSSM interactions for the last two processes.

Firstly, we analyzed the rare decay $B \rightarrow \tau\bar{\mu} + \mu\bar{\tau}$ in two possible extensions of the SM. In particular, we considered the decay in a model with an extended Higgs sector, the two Higgs doublet model (2HDM), where the decay is mediated by a light scalar which is identified with the recently observed state at the LHC. In the second example, the one loop contribution to this process in the MSSM was computed. We found that in the former scenario the branching ratio $B_s \rightarrow \tau\bar{\mu} + \mu\bar{\tau}$ can be as high as 10^{-6} ($B_d \rightarrow \tau\bar{\mu} + \mu\bar{\tau}$ can be as high as $\sim 10^{-7}$), while in the later scenario the $BR(B_s \rightarrow \tau\bar{\mu} + \mu\bar{\tau})$ could be $\sim 10^{-8}$. Therefore, one concludes that probing

this lepton flavor violating process at the LHCb would be a significant hint for a non-traditional new physics beyond the SM with large FCNC couplings.

Also, we have proven that the MSSM, in case of CP-conserving, has the potential to explain recent data produced by BaBar and Belle which revealed a rather significant excess above and beyond the best SM predictions available in the observed $\text{BR}(\bar{B} \rightarrow D\tau\bar{\nu}_\tau)$ and $\text{BR}(\bar{B} \rightarrow D^*\tau\bar{\nu}_\tau)$ relative to the light lepton cases. Most remarkably, within the MSSM, the excesses can be explained *simultaneously*, needless to say, over the same regions of parameter space. Further, the latter do not correspond to any particularly fine-tuned dynamics (possibly apart from a lightest neutralino/chargino mass degeneracy, plus a preference for a large $\tilde{\tau}_1$ and $\tilde{\nu}_1$ masses, recall Fig. 4.8) and a more than acceptable agreement with the Belle (especially) and BaBar (to a lesser extent) data can be reached via MSSM spectra easily compatible with current experimental constraints from a variety of sources (flavour physics, Higgs boson measurements, SUSY searches). Such a conclusion is obtained after the *first complete* tree-level plus (penguin dominated) one-loop calculation of all MSSM topologies entering the partonic $b \rightarrow c\tau\nu_\tau$ decay process matched with standard computational elements enabling the transition from the partonic to hadronic level. If forthcoming data will confirm the BaBar and Belle results, e.g., from the now running LHCb experiment at the LHC, our results are truly exciting as a variety of other (typically non-SUSY) models have been tried and tested as a possible explanation of the $\bar{B} \rightarrow D\tau\bar{\nu}_\tau$ and $\bar{B} \rightarrow D^*\tau\bar{\nu}_\tau$ anomalies and failed. On the one hand, our results can then be taken as a circumstantial evidence of SUSY. On the other hand, they pave the way to its direct discovery as they point to spectra in the sparticle sector of the MSSM that can be accessed at Run 2.



Notations and Definitions

A.1 Spinors

A.1.1 Weyl Spinors

The Weyl spinor can be defined as a two complex component object of $SL(2, \mathbb{C})$, $\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$, transforming under an element $M = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \in SL(2, \mathbb{C})$ as

$$\psi_\alpha \rightarrow \psi'_\alpha = M_\alpha^\beta \psi_\beta, \quad (\text{A.1.1})$$

where $\alpha, \beta = 1, 2$. For $SL(2, \mathbb{C})$ with elements $M = H.V$, $V \in SU(2)$ and $H = H^\dagger$ positive, $\det H = 1$. M and M^* give inequivalent representations. Therefore, a two-component $\bar{\psi}$ transforming as

$$\bar{\psi}_{\dot{\alpha}} \rightarrow \bar{\psi}'_{\dot{\alpha}} = (M^*)_{\dot{\alpha}}^{\dot{\beta}} \bar{\psi}_{\dot{\beta}}, \quad (\text{A.1.2})$$

is called a dotted spinor, while the above ψ is called an undotted one. The convention of representing all fermions in terms of left-handed Weyl fermions turns out to be very

useful for not only for supersymmetry but also for other theories of physics beyond the SM. A Weyl spinor with positive chirality (the eigenvalue of γ_5 is called the chirality of the spinor) is referred to as a right-handed spinor, and a Weyl spinor with negative chirality is referred to as a left-handed spinor. The left-handed Weyl spinors correspond with undotted spinors ψ_α which transform under the $(\frac{1}{2}, 0)$ representation of the Lorentz group, and the right-handed ones consistent with dotted spinors $\psi_{\bar{\alpha}}$ that transform under the $(0, \frac{1}{2})$ representation of the Lorentz group.

The spinor indices are raised and lowered using the antisymmetric two index tensors

$$\varepsilon_{\alpha\beta} = \varepsilon_{\dot{\alpha}\dot{\beta}} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \quad \varepsilon^{\alpha\beta} = \varepsilon^{\dot{\alpha}\dot{\beta}} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad (\text{A.1.3})$$

where

$$\psi^\alpha = \varepsilon^{\alpha\beta} \psi_\beta, \quad \psi_\alpha = \varepsilon_{\alpha\beta} \psi^\beta, \quad (\text{A.1.4})$$

$$\bar{\psi}^{\dot{\alpha}} = \varepsilon^{\dot{\alpha}\dot{\beta}} \bar{\psi}_{\dot{\beta}}, \quad \bar{\psi}_{\dot{\alpha}} = \varepsilon_{\dot{\alpha}\dot{\beta}} \bar{\psi}^{\dot{\beta}}. \quad (\text{A.1.5})$$

Note that $\varepsilon_{\alpha\beta} \varepsilon^{\beta\gamma} = \varepsilon^{\gamma\beta} \varepsilon_{\beta\alpha} = \delta_\alpha^\gamma$ and $\varepsilon_{\dot{\alpha}\dot{\beta}} \varepsilon^{\dot{\beta}\dot{\gamma}} = \varepsilon^{\dot{\gamma}\dot{\beta}} \varepsilon_{\dot{\beta}\dot{\alpha}} = \delta_{\dot{\alpha}}^{\dot{\gamma}}$. The scalar products $\psi\chi$ and $\bar{\psi}\bar{\chi}$ can be defined as

$$\psi\chi \equiv \psi^\alpha \chi_\alpha = \varepsilon^{\alpha\beta} \psi_\beta \chi_\alpha = -\varepsilon^{\alpha\beta} \chi_\alpha \psi_\beta = \varepsilon^{\beta\alpha} \chi_\alpha \psi_\beta = \chi^\beta \psi_\beta \equiv \chi\psi, \quad (\text{A.1.6})$$

$$\bar{\psi}\bar{\chi} \equiv \bar{\psi}_{\dot{\alpha}} \bar{\chi}^{\dot{\alpha}} = \varepsilon_{\dot{\alpha}\dot{\beta}} \bar{\psi}^{\dot{\beta}} \bar{\chi}^{\dot{\alpha}} = -\varepsilon_{\dot{\alpha}\dot{\beta}} \bar{\chi}^{\dot{\alpha}} \bar{\psi}^{\dot{\beta}} = \varepsilon_{\dot{\beta}\dot{\alpha}} \bar{\chi}^{\dot{\alpha}} \bar{\psi}^{\dot{\beta}} = \bar{\chi}_{\dot{\beta}} \bar{\psi}^{\dot{\beta}} \equiv \bar{\chi}\bar{\psi}, \quad (\text{A.1.7})$$

$$(\psi\chi)^\dagger = (\psi^\alpha \chi_\alpha)^\dagger = \bar{\psi}_{\dot{\alpha}} \bar{\chi}^{\dot{\alpha}} = \bar{\psi}\bar{\chi} = \bar{\chi}\bar{\psi}, \quad (\text{A.1.8})$$

where in particular

$$\psi\psi = \psi^\alpha \psi_\alpha = \varepsilon^{\alpha\beta} \psi_\beta \psi_\alpha = \psi_2 \psi_1 - \psi_1 \psi_2. \quad (\text{A.1.9})$$

Choosing the ψ_α to be anticommuting Grassmann numbers, $\psi_1 \psi_2 = \psi_2 \psi_1$, so $\psi\psi = 2\psi_2 \psi_1$. Thus $\psi_\alpha \psi_\beta = \frac{1}{2} \varepsilon^{\alpha\beta} (\psi\psi)$. In a similar way, one can find

$$\psi \sigma^\mu \bar{\chi} = \psi^\alpha \sigma_{\alpha\dot{\beta}}^\mu \bar{\chi}^{\dot{\beta}}, \quad \bar{\psi} \bar{\sigma}^\mu \chi = \bar{\psi}_{\dot{\alpha}} \bar{\sigma}^{\mu\dot{\alpha}\beta} \chi_\beta, \quad (\text{A.1.10})$$

$$\chi\sigma^\mu\bar{\psi} = -\bar{\psi}\bar{\sigma}^\mu\chi, \quad \chi\sigma^\mu\bar{\sigma}^\nu\phi = \psi\sigma^\nu\bar{\sigma}^\mu\chi, \quad (\text{A.1.11})$$

$$(\chi\sigma^\mu\bar{\psi})^\dagger = \psi\sigma^\mu\bar{\chi}, \quad (\chi\sigma^\mu\bar{\sigma}^\nu\phi)^\dagger = \bar{\psi}\bar{\sigma}^\nu\sigma^\mu\bar{\chi}, \quad (\text{A.1.12})$$

with

$$(\sigma^\mu)_{\alpha\dot{\alpha}} = (1, \sigma_i)_{\alpha\dot{\alpha}}, \quad (\text{A.1.13})$$

$$(\bar{\sigma}^\mu)^{\alpha\dot{\alpha}} = \varepsilon^{\dot{\alpha}\beta}\varepsilon^{\alpha\gamma}(\sigma^\mu)_{\beta\gamma} = (1, -\sigma_i)^{\alpha\dot{\alpha}}. \quad (\text{A.1.14})$$

A.1.2 Dirac Spinors

A four-component Dirac spinors are defined to be the direct sum of two Weyl spinors of opposite chirality such that two-component undoted and two-component dotted,

$$\psi_D = \begin{pmatrix} \psi_\alpha \\ \bar{\chi}^{\dot{\alpha}} \end{pmatrix}. \quad (\text{A.1.15})$$

It transforms as the reducible $(\frac{1}{2}, 0) \oplus (0, \frac{1}{2})$ representation of the Lorentz group. Therefore, $\begin{pmatrix} \psi_\alpha \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ \bar{\chi}^{\dot{\alpha}} \end{pmatrix}$ are chiral Dirac spinors. The Dirac matrices in the Weyl representation are defined as

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}, \quad \gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \quad (\text{A.1.16})$$

such that the action of γ_5 is given as

$$\gamma_5\psi_D = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \psi_\alpha \\ \bar{\chi}^{\dot{\alpha}} \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \psi_\alpha \\ 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ \bar{\chi}^{\dot{\alpha}} \end{pmatrix} = \begin{pmatrix} -\psi_\alpha \\ \bar{\chi}^{\dot{\alpha}} \end{pmatrix}, \quad (\text{A.1.17})$$

and thus, the chiral projections take a simple form,

$$\psi_L = \frac{1}{2}(1 - \gamma_5)\psi, \quad \psi_R = \frac{1}{2}(1 + \gamma_5)\psi, \quad (\text{A.1.18})$$

where ψ_L and ψ_R are called chirality eigenstates.

A.1.3 Majorana Spinors

A Majorana spinor is a Dirac spinor with $\psi \equiv \chi$, and satisfies the property of self-conjugation:

$$\Psi_M^c = \Psi_M. \quad (\text{A.1.19})$$

In Dirac theory the charge conjugated Dirac spinor is given by:

$$\Psi^c = C\bar{\Psi}^T, \quad (\text{A.1.20})$$

where C denotes the charge conjugation matrix,

$$C = i\gamma^0\gamma^2 = \begin{pmatrix} -i\sigma^2 & 0 \\ 0 & i\sigma^2 \end{pmatrix}. \quad (\text{A.1.21})$$

In terms of two-component spinors the charge conjugate of Ψ is:

$$\Psi^c = \begin{pmatrix} \chi_\alpha \\ \bar{\psi}^{\dot{\alpha}} \end{pmatrix}. \quad (\text{A.1.22})$$

Finally, one can write a Majorana spinor as

$$\Psi_M = \begin{pmatrix} \chi_\alpha \\ \bar{\chi}^{\dot{\alpha}} \end{pmatrix} = \begin{pmatrix} \chi \\ -i\sigma_2\chi^* \end{pmatrix} = \Psi_M^c. \quad (\text{A.1.23})$$

A.2 Superspace

Superspace for simple supersymmetry is defined by four ordinary spacetime coordinates x^μ and four anticommuting coordinates (or Grassmann variables) $(\theta_\alpha, \bar{\theta}^{\dot{\alpha}})$ where $\bar{\theta}^{\dot{\alpha}}$ are the complex conjugates of the θ_α associated to the supersymmetry generators $Q_\alpha, \bar{Q}^{\dot{\alpha}}$, and obtain an eight coordinate superspace labeled by $(x^\mu, \theta_\alpha, \bar{\theta}^{\dot{\alpha}})$. An important property of functions of Grassmann variables follows from the fact that any power series expansion in terms of the anticommuting coordinates always terminates because the square of any Grassmann variable is zero, so that

$$\{\theta_\alpha, \theta_\beta\} = \{\bar{\theta}^{\dot{\alpha}}, \bar{\theta}^{\dot{\beta}}\} = 0. \quad (\text{A.2.1})$$

Therefore, $\theta_\alpha \theta_\beta = \bar{\theta}^{\dot{\alpha}} \bar{\theta}^{\dot{\beta}} = 0$ for $\alpha = \beta$ and $\dot{\alpha} = \dot{\beta}$, where $\alpha, \beta = 1, 2$ and $\dot{\alpha}, \dot{\beta} = \dot{1}, \dot{2}$. Thus, the expansion of a generic function in θ (carrying no spinor indices) as a power series can be written as

$$f(\theta) = \sum_{k=1}^{\infty} f_k \theta^k = f_0 + f_1 \theta + f_2 \theta^2 + \dots = f_0 + f_1 \theta, \quad (\text{A.2.2})$$

where $\theta^2 = 0$ cancels all the terms except for two. So the most general function $f(\theta)$ is linear. Its derivative is given by $\frac{df}{d\theta} = f_1$.

To define an integration operation with respect to θ , we define

$$\int d\theta_\alpha = 0, \quad \int d\theta_\alpha \theta_\beta = \delta_{\alpha\beta}. \quad (\text{A.2.3})$$

Therefore, the integral over a function $f(\theta)$ is then equal to its derivative,

$$\int d\theta f(\theta) = \int d\theta (f_0 + f_1 \theta) = f_1 = \frac{df}{d\theta}. \quad (\text{A.2.4})$$

Now, let $\theta_\alpha, \bar{\theta}^{\dot{\alpha}}$ be spinors of Grassmann, their useful identities are defined by

$$\theta\theta = \theta^\alpha \theta_\alpha = -2\theta^1 \theta^2 = 2\theta_2 \theta_1 = -2\theta_1 \theta_2, \quad (\text{A.2.5})$$

$$\bar{\theta}\bar{\theta} = \bar{\theta}_{\dot{\alpha}} \bar{\theta}^{\dot{\alpha}} = 2\bar{\theta}^2 \bar{\theta}^1 = -2\bar{\theta}_2 \bar{\theta}_1 = 2\bar{\theta}_1 \bar{\theta}_2, \quad (\text{A.2.6})$$

$$\theta_\alpha \theta_\beta = \frac{1}{2} \varepsilon_{\alpha\beta} \theta\theta \quad \theta^\alpha \theta^\beta = -\frac{1}{2} \varepsilon^{\alpha\beta} \theta\theta, \quad (\text{A.2.7})$$

$$\bar{\theta}_{\dot{\alpha}} \bar{\theta}_{\dot{\beta}} = -\frac{1}{2} \varepsilon_{\dot{\alpha}\dot{\beta}} \bar{\theta}\bar{\theta} \quad \bar{\theta}^{\dot{\alpha}} \bar{\theta}^{\dot{\beta}} = \frac{1}{2} \varepsilon^{\dot{\alpha}\dot{\beta}} \bar{\theta}\bar{\theta}, \quad (\text{A.2.8})$$

$$\theta\sigma^\mu \bar{\theta} \theta\sigma^\nu \bar{\theta} = \frac{1}{2} \theta\theta \bar{\theta}\bar{\theta} g^{\mu\nu} \quad \theta\psi\theta\chi = -\frac{1}{2} \theta\theta\psi\chi. \quad (\text{A.2.9})$$

Derivatives in θ and $\bar{\theta}$ work in analogy to Minkowski coordinates:

$$\partial_\alpha \theta^\beta = \frac{\partial \theta^\beta}{\partial \theta^\alpha} = \delta_\alpha^\beta, \quad \bar{\partial}_{\dot{\alpha}} \bar{\theta}^{\dot{\beta}} = \frac{\partial \bar{\theta}^{\dot{\beta}}}{\partial \bar{\theta}^{\dot{\alpha}}} = \delta_{\dot{\alpha}}^{\dot{\beta}}, \quad (\text{A.2.10})$$

where $\{\partial_\alpha, \partial_\beta\} = \{\bar{\partial}_{\dot{\alpha}}, \bar{\partial}_{\dot{\beta}}\} = 0$.

In the other hand, the integration over superspace is defined as follows:

$$\int d\theta^1 d\theta^2 \theta^2 \theta^1 = \frac{1}{2} \int d\theta^1 d\theta^2 \theta\theta = \frac{1}{2} \int d^2\theta \theta\theta = 1, \quad (\text{A.2.11})$$

$$\int d\bar{\theta}^2 d\bar{\theta}^1 \bar{\theta}^1 \bar{\theta}^2 = \frac{1}{2} \int d\bar{\theta}^1 d\bar{\theta}^2 \bar{\theta} \bar{\theta} = \frac{1}{2} \int d^2 \bar{\theta} \bar{\theta} \bar{\theta} = 1, \quad (\text{A.2.12})$$

and

$$\frac{1}{4} \int d^2 \theta d^2 \bar{\theta} (\theta \theta) (\bar{\theta} \bar{\theta}) = 1. \quad (\text{A.2.13})$$

Recall that, in this case the integration of a chiral superfield, $F(x, \theta)$, proportional to $\theta \cdot \theta$ is usually called F-term and denoted by $F(x, \theta)|_{\theta\theta} \equiv F(x, \theta)|_F$. Also the integration of a chiral superfield, $S(x, \theta)$, proportional to combination of Grassmann variables $\theta\theta \cdot \bar{\theta}\bar{\theta}$ is known as the D-term and denoted by $S(x, \theta)|_{\theta\theta\bar{\theta}\bar{\theta}} \equiv S(x, \theta)|_D$.

A.3 Superfields

Superfields are functions of the superspace coordinates $(x^\mu, \theta_\alpha, \bar{\theta}^{\dot{\alpha}})$. Since $\theta_\alpha, \bar{\theta}^{\dot{\alpha}}$ anticommute, any product involving more than two θ 's or two $\bar{\theta}$'s vanishes. Hence, the most general superfield $\Phi(x, \theta, \bar{\theta})$ has the following simple Taylor-like expansion

$$\begin{aligned} \Phi(x, \theta, \bar{\theta}) = & \phi(x) + \theta\psi(x) + \bar{\theta}\bar{\chi}(x) + \theta\theta M(x) + \bar{\theta}\bar{\theta} N(x) + \theta\sigma^\mu\bar{\theta}v_\mu(x) \\ & + \theta\theta\bar{\theta}\bar{\lambda}(x) + \bar{\theta}\bar{\theta}\theta\rho(x) + \bar{\theta}\bar{\theta}\theta\bar{\theta}D(x). \end{aligned} \quad (\text{A.3.1})$$

The component fields $\phi(x), M(x), N(x), D(x), v_\mu(x)$ (four scalars and one vector) are complex bosonic fields, and $\psi(x), \chi(x), \lambda(x), \rho(x)$ are fermionic fields (spinors). Therefore the superfield $\Phi(x, \theta, \bar{\theta})$ has 32 components of x , $2 + 2 + 2 + 8 = 16$ bosonic components and $4 + 4 + 4 + 4 = 16$ fermionic components.

Supersymmetry transformation may be represented as a transformation over points in superspace. The well known procedure uses the fact that a group G may be realized as a transformation group over a coset space. For example, the case of Poincaré group $\mathcal{P}$ the coset decomposition with respect to the Lorentz subgroup L defines the coset space $\mathcal{P}/L$. In the case of the super-Poincaré group it is convenient to use an exponential parametrization of group elements,

$$\Omega(x, \xi, \bar{\xi}) = e^{i(-a^\mu P_\mu + \xi Q + \bar{\xi} \bar{Q})}, \quad (\text{A.3.2})$$

and take the fixed element of the coset space with respect to the Lorntz subgroup to be

$$G(x, \theta, \bar{\theta}) = e^{i(-x^\mu P_\mu + \theta Q + \bar{\theta} \bar{Q})}. \quad (\text{A.3.3})$$

Using the algebra of $N = 1$ supersymmetry generators and the Baker-Hausdorff-Campbell theorems:

$$e^A e^B = e^{A+B+\frac{1}{2}[A,B]+\dots}, \quad (\text{A.3.4})$$

one finds

$$\begin{aligned} G(x', \theta', \bar{\theta}') &= \Omega(x, \xi, \bar{\xi}) G(x, \theta, \bar{\theta}) \\ &= e^{i(-a^\mu P_\mu + \xi Q + \bar{\xi} \bar{Q})} e^{i(-x^\mu P_\mu + \theta Q + \bar{\theta} \bar{Q})} \\ &= e^{i[-(x^\mu + a^\mu)P_\mu + (\theta + \xi)Q + (\bar{\theta} + \bar{\xi})\bar{Q} + \frac{i}{2}[\xi Q, \bar{\theta} \bar{Q}] + \frac{i}{2}[\bar{\xi} \bar{Q}, \theta Q]]} \\ &= G(x^\mu + a^\mu - i\xi\sigma^\mu\bar{\theta} + i\theta\sigma^\mu\bar{\xi}, \theta + \xi, \bar{\theta} + \bar{\xi}), \end{aligned} \quad (\text{A.3.5})$$

where $[\xi Q, \bar{\theta} \bar{Q}] = 2\xi\sigma^\mu\bar{\theta}P_\mu$ and $[\bar{\xi} \bar{Q}, \theta Q] = -2\theta\sigma^\mu\bar{\xi}P_\mu$. Therefore, the supersymmetry transformation of the superspace coordinates:

$$x^\mu \rightarrow x'^\mu = x^\mu + a^\mu - i\xi\sigma^\mu\bar{\theta} + i\theta\sigma^\mu\bar{\xi}, \quad (\text{A.3.6})$$

$$\theta \rightarrow \theta' = \theta + \xi, \quad (\text{A.3.7})$$

$$\bar{\theta} \rightarrow \bar{\theta}' = \bar{\theta} + \bar{\xi}. \quad (\text{A.3.8})$$

Therefore, the supersymmetry transformation thus are realized as supertransformations over the arguments x^μ , θ_α and $\bar{\theta}^{\dot{\alpha}}$. Under the above transformations, the form the superfield $\Phi(x, \theta, \bar{\theta})$ at an infinitesimally class point $(x + \delta x, \theta + \delta\theta, \bar{\theta} + \delta\bar{\theta})$ is given by

$$\begin{aligned} \Phi(x + \delta x, \theta + \delta\theta, \bar{\theta} + \delta\bar{\theta}) &= \Phi(x, \theta, \bar{\theta}) + \delta x^\mu \partial_\mu \Phi(x, \theta, \bar{\theta}) + \delta\theta_\alpha \partial^\alpha \Phi(x, \theta, \bar{\theta}) \\ &\quad + \delta\bar{\theta}^{\dot{\alpha}} \partial_{\dot{\alpha}} \Phi(x, \theta, \bar{\theta}) \\ &= \Phi(x, \theta, \bar{\theta}) + (a^\mu + i\theta\sigma^\mu\bar{\xi} - i\xi\sigma^\mu\bar{\theta}) \frac{\partial \Phi}{\partial x^\mu} + \xi^\alpha \frac{\partial \Phi}{\partial \theta^\alpha} \end{aligned}$$

$$+ \bar{\xi}_{\dot{\alpha}} \frac{\partial \Phi}{\partial \bar{\theta}_{\dot{\alpha}}}. \quad (\text{A.3.9})$$

The super-Poincaé transformation for the superfield $\Phi(x, \theta, \bar{\theta})$ can be written as:

$$\Phi(x, \theta, \bar{\theta}) \rightarrow e^{i(-a^\mu P_\mu + \xi Q + \bar{\xi} \bar{Q})} \Phi(x, \theta, \bar{\theta}) \quad (\text{A.3.10})$$

Expanding to first order the exponential and setting this equal to Eq.(A.3.9), we get

$$P_\mu = i\partial_\mu, \quad (\text{A.3.11})$$

$$Q_\alpha = i(-\partial_\alpha + i\sigma_{\alpha\dot{\alpha}}^\mu \bar{\theta}^{\dot{\alpha}} \partial_\mu), \quad (\text{A.3.12})$$

$$\bar{Q}^{\dot{\alpha}} = i(-\bar{\partial}^{\dot{\alpha}} - i\bar{\sigma}^{\mu\dot{\alpha}\alpha} \theta_\alpha \partial_\mu), \quad (\text{A.3.13})$$

where $\partial_\mu = \frac{\partial}{\partial x^\mu}$, $\partial_\alpha = \frac{\partial}{\partial \theta^\alpha}$ and $\bar{\partial}^{\dot{\alpha}} = \frac{\partial}{\partial \bar{\theta}^{\dot{\alpha}}}$. Also we can define supercovariant derivatives D_α , $\bar{D}^{\dot{\alpha}}$, which anticommute with all Q 's as follows

$$D_\alpha = \partial_\alpha + i\sigma_{\alpha\dot{\alpha}}^\mu \bar{\theta}^{\dot{\alpha}} \partial_\mu, \quad (\text{A.3.14})$$

$$\bar{D}^{\dot{\alpha}} = \bar{\partial}^{\dot{\alpha}} + i\bar{\sigma}^{\mu\dot{\alpha}\alpha} \theta_\alpha \partial_\mu, \quad (\text{A.3.15})$$

where

$$\{D_\alpha, Q_\beta\} = \{D_\alpha, \bar{Q}_{\dot{\beta}}\} = \{\bar{D}_{\dot{\alpha}}, Q_\beta\} = \{\bar{D}_{\dot{\alpha}}, \bar{Q}_{\dot{\beta}}\} = 0, \quad (\text{A.3.16})$$

and thus

$$[D_\alpha, \xi Q + \bar{\xi} \bar{Q}] = 0. \quad (\text{A.3.17})$$

Also the supercovariant derivatives D and $\bar{D}$ satisfy the following anticommutation relations

$$\{D_\alpha, \bar{D}_{\dot{\beta}}\} = -2i(\sigma^\mu)_{\alpha\dot{\beta}} \partial_\mu, \quad (\text{A.3.18})$$

$$\{D_\alpha, D_\beta\} = 0, \quad (\text{A.3.19})$$

$$\{\bar{D}_{\dot{\alpha}}, \bar{D}_{\dot{\beta}}\} = 0. \quad (\text{A.3.20})$$

We can now give the action on the general superfield $\Phi(x, \theta, \bar{\theta})$ and determine the SUSY transformation. The SUSY transformation of a general scalar superfield:

$$\Phi'(x, \theta, \bar{\theta}) = e^{i(\xi Q + \bar{\xi} \bar{Q})} \Phi(x, \theta, \bar{\theta}) \quad (\text{A.3.21})$$

$$= \Phi(x, \theta, \bar{\theta}) + i(\xi Q + \bar{\xi} \bar{Q}) \Phi(x, \theta, \bar{\theta}), \quad (\text{A.3.22})$$

therefore

$$\begin{aligned} \delta\Phi(x, \theta, \bar{\theta}) = & \xi\psi + \bar{\xi}\bar{\chi} + \theta\sigma^\mu\bar{\xi}\partial_\mu\phi + 2\xi\theta M + \theta\sigma^\mu\bar{\xi}v_\mu - i\xi\sigma^\mu\bar{\theta}\partial_\mu\phi + 2\bar{\xi}\bar{\theta}N + \xi\sigma^\mu\bar{\theta}v_\mu \\ & + i(\theta\sigma^\mu\xi)\theta\partial_\mu\psi + 2\theta\bar{\theta}\bar{\xi}\lambda - i(\xi\sigma^\mu\bar{\theta})\bar{\theta}\sigma_\mu\bar{\chi} + \bar{\theta}\bar{\theta}\xi\rho - i\xi\sigma^\mu\bar{\theta}\partial_\mu\psi + i\theta\sigma^\mu\bar{\xi}\bar{\theta}\partial_\mu\bar{\chi} \\ & + \xi\theta\bar{\theta}\bar{\lambda} + 2\bar{\xi}\bar{\theta}\theta\rho - i\xi\sigma^\mu\bar{\theta}\theta\partial_\mu M + i\theta\sigma^\mu\bar{\xi}\theta\sigma^\nu\bar{\theta}\partial_\mu v_\nu + 2\theta\bar{\theta}\bar{\xi}D + i\theta\sigma^\mu\bar{\xi}\bar{\theta}\partial_\mu N \\ & - i\xi\sigma^\mu\bar{\theta}\theta\sigma^\nu\bar{\theta}\partial_\mu v_\nu + 2\xi\theta\bar{\theta}D - i\xi\sigma^\mu\bar{\theta}\theta\bar{\theta}\partial_\mu\bar{\lambda} + i\theta\sigma^\mu\bar{\xi}\bar{\theta}\bar{\theta}\partial_\mu\rho. \end{aligned} \quad (\text{A.3.23})$$

Reorganize expression into standard expansion of the field in components, the SUSY transformations of all component fields of the general superfield Eq. (A.3.1) are:

$$\delta_\xi\phi = \xi\psi + \bar{\xi}\bar{\chi}, \quad (\text{A.3.24})$$

$$\delta_\xi\psi_\alpha = 2\xi_\alpha M + \sigma_{\alpha\dot{\beta}}^\mu\bar{\xi}^{\dot{\beta}}(i\partial_\mu\phi + v_\mu), \quad (\text{A.3.25})$$

$$\delta_\xi\bar{\chi}^{\dot{\alpha}} = 2\bar{\xi}^{\dot{\alpha}}N + \xi^\beta\sigma_{\beta\dot{\rho}}^\mu\xi^{\dot{\rho}\dot{\alpha}}(i\partial_\mu\phi - v_\mu), \quad (\text{A.3.26})$$

$$\delta_\xi M = \bar{\xi}\bar{\lambda} - \frac{i}{2}\partial_\mu\phi\sigma^\mu\bar{\xi}, \quad (\text{A.3.27})$$

$$\delta_\xi N = \xi\rho + \frac{i}{2}\xi\sigma^\mu\partial_\mu\bar{\chi}, \quad (\text{A.3.28})$$

$$\delta_\xi v_\mu = \xi\sigma^\mu\bar{\lambda} + \rho\sigma^\mu\bar{\xi} + \frac{i}{2}\xi\partial_\mu\psi - \frac{i}{2}\partial_\mu\bar{\chi}\xi, \quad (\text{A.3.29})$$

$$\delta_\xi\bar{\lambda}^{\dot{\alpha}} = 2\bar{\xi}^{\dot{\alpha}}D + \frac{i}{2}\bar{\xi}^{\dot{\alpha}}\partial^\mu v_\mu + i(\xi\sigma^\mu\xi)^{\dot{\alpha}}\partial_\mu M, \quad (\text{A.3.30})$$

$$\delta_\xi\rho_\alpha = 2\xi_\alpha D - \frac{i}{2}\xi_\alpha\partial^\mu v_\mu + i(\sigma^\mu\bar{\xi})_\alpha\partial_\mu N, \quad (\text{A.3.31})$$

$$\delta_\xi D = \frac{i}{2}\partial_\mu(\rho\sigma^\mu\bar{\xi} + \sigma^\mu\bar{\lambda}). \quad (\text{A.3.32})$$

An unconstrained superfield provides a reducible representation of supersymmetry. To find irreducible representations we need to impose some constraints on $\Phi(x, \theta, \bar{\theta})$ that are preserved under supersymmetry.

A.4 Constrained Superfields

Since $\Phi(x, \theta, \bar{\theta})$ is not an irreducible representation of supersymmetry, it can be decomposed into a direct sum of three small representation. So we can eliminate some of its components keeping it still as a superfield. Therefore the imposed constraints on $\Phi(x, \theta, \bar{\theta})$, leading to smaller superfields that are irreducible representations of the SUSY algebra. We present two types such constraints leading to

- Chiral superfield Φ such that $\bar{D}_{\dot{\alpha}}\Phi = 0$.
- Antichiral superfield $\bar{\Phi}$ such that $D_{\alpha}\bar{\Phi} = 0$.
- Vector superfield $V = V^{\dagger}$.

A.4.1 Chiral Superfield

A chiral superfield Φ is a superfield that satisfies $\bar{D}_{\dot{\alpha}}\Phi = 0$, and the complex conjugate of a chiral superfield is an anti-chiral superfield $\bar{\Phi}$ satisfying $D_{\alpha}\bar{\Phi} = 0$. To solve this constrain, it is convenient to define

$$y^{\mu} = x^{\mu} - i\theta\sigma^{\mu}\bar{\theta}. \quad (\text{A.4.1})$$

Since $\bar{D}_{\dot{\alpha}}$ is a differential operator, then, if $\Phi = \Phi(y, \theta, \bar{\theta})$,

$$\bar{D}_{\dot{\alpha}}\Phi = (\bar{D}_{\dot{\alpha}}\theta^{\alpha})\frac{\partial\Phi}{\partial\theta^{\alpha}}|_{y,\bar{\theta}} + (\bar{D}_{\dot{\alpha}}y^{\mu})\frac{\partial\Phi}{\partial y^{\mu}}|_{\theta,\bar{\theta}} + (\bar{D}_{\dot{\alpha}}\bar{\theta}^{\dot{\beta}})\frac{\partial\Phi}{\partial\bar{\theta}^{\dot{\beta}}}|_{y,\theta}, \quad (\text{A.4.2})$$

thus, we have

$$\bar{D}_{\dot{\alpha}}\theta^{\alpha} = 0 \quad (\text{A.4.3})$$

$$\begin{aligned} \bar{D}_{\dot{\alpha}}y^{\mu} &= (\bar{\partial}_{\dot{\alpha}} - i\theta^{\alpha}\sigma_{\alpha\dot{\alpha}}^{\nu}\partial_{\nu})(x^{\mu} - i\theta\sigma^{\mu}\bar{\theta}) \\ &= -i\theta^{\alpha}\sigma_{\alpha\dot{\alpha}}^{\mu} + i\theta^{\beta}\sigma_{\beta\dot{\alpha}}^{\mu} \end{aligned} \quad (\text{A.4.4})$$

$$= i(\theta\sigma^{\mu})_{\dot{\alpha}} - i(\theta\sigma^{\mu})_{\dot{\alpha}} = 0. \quad (\text{A.4.5})$$

Therefore the chiral superfield condition becomes $\frac{\partial \Phi}{\partial \bar{\theta}^{\dot{\beta}}} = 0$. Thus Φ depends only on y and θ . One finds that the most general chiral superfield can be written as

$$\Phi(y, \theta) = \phi(y) + \sqrt{2}\theta\psi(y) + \theta\theta F(y), \quad (\text{A.4.6})$$

and similarly, the anti-chiral superfield $\bar{\Phi}(y^\dagger, \bar{\theta})$ which satisfies the condition $D_\alpha \bar{\Phi} = 0$ is given by

$$\bar{\Phi}(y^\dagger, \bar{\theta}) = \phi^*(y^\dagger) + \sqrt{2}\bar{\theta}\bar{\psi}(y^\dagger) + \bar{\theta}\bar{\theta} F^*(y^\dagger), \quad (\text{A.4.7})$$

where $\phi(y)$ is a complex scalar field, $\psi_\alpha(y)$ is a spinor field and $F(y)$ is an auxiliary field. Now there are 4 bosonic (complex ϕ, F) and 4 fermionic ψ_α .

In terms of the original coordinates $x, \theta, \bar{\theta}$, and by using the Taylor expansion in a power series around anticommuting coordinates, the above superfield becomes

$$\begin{aligned} \Phi(x, \theta, \bar{\theta}) &= e^{i(\theta\sigma^\mu\bar{\theta}\partial_\mu)}\Phi(x, \theta) \\ &= \phi(x) + \sqrt{2}\theta\psi(x) + \theta\theta F(x) + i\theta\sigma^\mu\bar{\theta}\partial_\mu\phi(x) - \frac{i}{\sqrt{2}}\theta\theta\sigma^\mu\bar{\theta}\partial_\mu\psi(x) \\ &\quad + \frac{1}{4}\theta\theta\bar{\theta}\bar{\theta}\partial_\mu\partial^\mu\phi(x), \end{aligned} \quad (\text{A.4.8})$$

$$\begin{aligned} \bar{\Phi}(x, \theta, \bar{\theta}) &= \phi^*(x) + \sqrt{2}\bar{\theta}\bar{\psi}(x) + \bar{\theta}\bar{\theta} F^*(x) - i\theta\sigma^\mu\bar{\theta}\partial_\mu\phi(x) + \frac{i}{\sqrt{2}}\bar{\theta}\bar{\theta}\sigma^\mu\theta\partial_\mu\bar{\psi}(x) \\ &\quad + \frac{1}{4}\theta\theta\bar{\theta}\bar{\theta}\partial_\mu\partial^\mu\phi^*(x). \end{aligned} \quad (\text{A.4.9})$$

Finally, let us find the SUSY transformation of chiral (or antichiral) superfield components. It is convenient to write the differential operators $Q_\alpha, \bar{Q}^{\dot{\alpha}}$ in the $(y^\mu, \theta, \bar{\theta})$ coordinate system as

$$Q_\alpha = -i\frac{\partial}{\partial\theta^\alpha}, \quad (\text{A.4.10})$$

$$\bar{Q}^{\dot{\alpha}} = i\left(\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} + 2i\bar{\sigma}^{\mu\alpha\dot{\alpha}}\theta_\alpha\frac{\partial}{\partial y^\mu}\right). \quad (\text{A.4.11})$$

Thus one finds that the infinitesimal SUSY transformation of chiral superfield so that

$$\delta_{\xi, \bar{\xi}}\Phi(y, \theta) = i(\xi Q + \bar{\xi}\bar{Q})\Phi(y, \theta)$$

$$\begin{aligned}
&= (\xi^\alpha \frac{\partial}{\partial \theta^\alpha} - 2i\theta^\alpha \sigma^\mu_{\alpha\dot{\beta}} \bar{\xi}^{\dot{\beta}} \frac{\partial}{\partial y^\mu}) \Phi(y, \theta) \\
&= \sqrt{2}\xi\psi + 2\xi\theta F - 2i\theta\sigma^\mu\bar{\xi}(\frac{\partial}{\partial y^\mu}\phi + \sqrt{2}\theta\frac{\partial}{\partial y^\mu}\psi) \\
&= \sqrt{2}\xi\psi + \sqrt{2}\theta(\sqrt{2}\xi F - i\sqrt{2}\sigma^\mu\bar{\xi}\frac{\partial}{\partial y^\mu}\phi) \\
&\quad - \theta\theta(i\sqrt{2}\bar{\xi}\sigma^\mu\frac{\partial}{\partial y^\mu}\psi). \tag{A.4.12}
\end{aligned}$$

Therefore, the expression for the SUSY transformation of the different field component of the chiral superfield Φ leads

$$\delta_\xi \phi = \sqrt{2}\xi\psi, \tag{A.4.13}$$

$$\delta_\xi \psi_\alpha = \sqrt{2}\xi F - i\sqrt{2}\sigma^\mu_{\alpha\dot{\beta}}\bar{\xi}^{\dot{\beta}}\partial_\mu\phi, \tag{A.4.14}$$

$$\delta_\xi F = i\sqrt{2}\partial_\mu\psi\sigma^\mu\bar{\xi}. \tag{A.4.15}$$

A.4.2 Vector Superfield

In order to have gauge interactions which describe the *spin* -1 gauge boson, we introduce vector superfield V . It is obtained by imposing the constraint

$$V(x, \theta, \bar{\theta}) = V^\dagger(x, \theta, \bar{\theta}),$$

where the expression of the vector superfield is

$$\begin{aligned}
V(x, \theta, \bar{\theta}) &= C(x) + i\theta\chi(x) - i\bar{\theta}\bar{\chi}(x) + \theta\sigma^\mu\bar{\theta}v_\mu + \frac{i}{2}\theta\theta(M(x) + iN(x)) \\
&\quad - \frac{i}{2}\bar{\theta}\bar{\theta}(M(x) - iN(x)) + i\theta\theta\bar{\theta}(\bar{\lambda}(x) + \frac{i}{2}\bar{\sigma}^\mu\partial_\mu\chi(x)) \\
&\quad - i\bar{\theta}\bar{\theta}\theta(\lambda(x) + \frac{i}{2}\sigma^\mu\partial_\mu\bar{\chi}(x)) + \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}(D(x) - \frac{1}{2}\partial^\mu\partial_\mu c(x)), \tag{A.4.16}
\end{aligned}$$

where C, M, N, D, V_μ are real scalars (8 bosonic component), χ_α and λ_α are Weyl spinors (4+4 fermionic ones). These are too many components to describe a single supermultiplet. To reduce their number, we introduce a generalized gauge transformation of vector fields as follows

$$V \rightarrow V + i(\Lambda - \Lambda^\dagger), \tag{A.4.17}$$

where $\Lambda(x, \theta, \bar{\theta})$ is a chiral superfield, then $i(\Lambda - \Lambda^\dagger)$ is a vector superfield. Its components are

$$C = i(\phi - \phi^\dagger), \quad (\text{A.4.18})$$

$$\chi = \sqrt{2}\psi, \quad (\text{A.4.19})$$

$$F = \frac{1}{2}(M + iN), \quad (\text{A.4.20})$$

$$v_\mu = -\partial_\mu(\phi + \phi^\dagger), \quad (\text{A.4.21})$$

$$\lambda = D = 0. \quad (\text{A.4.22})$$

The above transformation induces a standard transformation for the real vector v_μ :

$$v_\mu \rightarrow v_\mu - i\partial_\mu\alpha(x), \quad (\text{A.4.23})$$

where $\alpha(x)$ is scalar component of the chiral superfield Λ , which corresponds to an abelian gauge transformation.

The SUSY gauge, which is given in Eq. (A.4.17), allows to set $C = \chi = M = N = 0$. This defines the Wess Zumino gauge. In this physical gauge, the vector superfield reduces to the form

$$\begin{aligned} V_{WZ}(x, \theta, \bar{\theta}) &= (\theta\sigma^\mu\bar{\theta})v_\mu(x) + (\theta\theta)(\bar{\theta}\bar{\lambda}(x)) + (\bar{\theta}\bar{\theta})(\theta\lambda(x)) \\ &+ \frac{1}{2}(\bar{\theta}\bar{\theta})(\theta\theta)D(x), \end{aligned} \quad (\text{A.4.24})$$

where $D(x)$ is an auxiliary field, v_μ corresponding to gauge particles ($\gamma, W^\pm, Z, \text{gluon}$) and $\lambda, \bar{\lambda}$ are gauginos.

B

Amplitudes and Form Factors

The leptonic amplitudes can be calculated in q^2 rest-frame, using a four-component spinors [137], we get

- Scalar type

$$L^+ = -2\sqrt{q^2}v, \quad L^- = 0, \quad (\text{B.0.1})$$

where $v = \sqrt{1 - m_\tau^2/q^2}$.

- Vector type

$$L_\pm^+ = \pm\sqrt{2}m_\tau v \sin \theta_\tau, \quad L_0^+ = 2m_\tau v \cos \theta_\tau, \quad L_s^+ = -2m_\tau v, \quad (\text{B.0.2})$$

$$L_\pm^- = \sqrt{2}\sqrt{q^2}v(1 \pm \cos \theta_\tau), \quad L_0^- = -2\sqrt{q^2}v \sin \theta_\tau, \quad L_s^- = 0. \quad (\text{B.0.3})$$

- Tensor type

$$L_{0\pm}^+ = -L_{\pm 0}^+ = -2\sqrt{q^2}v \sin \theta_\tau, \quad (\text{B.0.4})$$

$$L_{+-}^+ = -L_{-+}^+ = L_{s0}^+ = -L_{0s}^+ = 2\sqrt{q^2}v \cos \theta_\tau, \quad (\text{B.0.5})$$

$$L_{\pm s}^+ = -L_{s\pm}^+ = \mp\sqrt{2}\sqrt{q^2}v \sin \theta_\tau, \quad (\text{B.0.6})$$

$$L_{0\pm}^- = -L_{\pm 0}^- = \mp\sqrt{2}m_\tau v(1 \pm \cos \theta_\tau), \quad (\text{B.0.7})$$

$$L_{+-}^- = -L_{-+}^- = L_{s0}^- = -L_{0s}^- = -2m_\tau v \sin \theta_\tau, \quad (\text{B.0.8})$$

$$L_{\pm s}^- = -L_{s\pm}^- = -\sqrt{2}m_\tau v(1 \cos \theta_\tau), \quad (\text{B.0.9})$$

$$L_{\lambda\lambda}^\pm = 0. \quad (\text{B.0.10})$$

The matrix elements of the $V-A$ structure and tensor contribution in $\bar{B} \rightarrow D^*$ transition depend on independent form factors, $V(q^2)$, $A_0(q^2)$, $A_1(q^2)$, $A_2(q^2)$, $T_1(q^2)$, $T_2(q^2)$ and $T_3(q^2)$ [138, 148], we have

$$\langle D^*(p_{D^*}, \varepsilon) | \bar{c}\gamma_\mu b | \bar{B}(p_B) \rangle = \frac{2iV(q^2)}{m_B + m_{D^*}} \varepsilon_{\mu\nu\alpha\beta} \varepsilon^{*\nu} p^\alpha p^\beta, \quad (\text{B.0.11})$$

$$\begin{aligned} \langle D^*(p_{D^*}, \varepsilon) | \bar{c}\gamma_\mu \gamma_5 b | \bar{B}(p_B) \rangle &= 2m_{D^*} \frac{\varepsilon^* q}{q^2} q_\mu A_0(q^2) + (m_B + m_{D^*}) \left(\varepsilon_\mu^* - \frac{\varepsilon^* q}{q^2} q_\mu \right) A_1(q^2) \\ &\quad - \frac{\varepsilon^* q}{m_B + m_{D^*}} \left((p_B + p_{D^*})_\mu - \frac{m_B^2 - m_{D^*}^2}{q^2} q_\mu \right) A_2(q^2), \end{aligned} \quad (\text{B.0.12})$$

with

$$A_0(0) = \frac{m_B + m_{D^*}}{2m_{D^*}} A_1(0) - \frac{m_B - m_{D^*}}{2m_{D^*}} A_2(0). \quad (\text{B.0.13})$$

Using the Dirac equation of motion for quark fields,

$$\begin{aligned} \partial_\mu (\bar{c}\gamma^\mu \gamma_5 b) &= (\partial_\mu \bar{c}) \gamma^\mu \gamma_5 b - \bar{c} \gamma_5 \gamma^\mu (\partial_\mu b) \\ &= i(m_b + m_c) \bar{c} \gamma_5 b, \end{aligned} \quad (\text{B.0.14})$$

then the pseudoscalar matrix elements can be written as

$$\langle D^*(p_{D^*}, \varepsilon) | \bar{c}\gamma_5 b | \bar{B}(p_B) \rangle = -\frac{2m_{D^*}}{m_b + m_c} A_0(q^2) \varepsilon^* q. \quad (\text{B.0.15})$$

The tensor matrix elements is [29]

$$\langle D^*(p_{D^*}, \varepsilon) | \bar{c}\sigma^{\mu\nu} b | \bar{B}(p_B) \rangle = \varepsilon_{\mu\nu\alpha\beta} \left[-\varepsilon^{*\alpha} (p_B + p_{D^*})^\beta T_1(q^2) \right]$$

$$\begin{aligned}
& + \frac{m_B^2 - m_{D^*}^2}{q^2} \varepsilon^{*\alpha} q^\beta (T_1(q^2) - T_2(q^2)) \\
& + 2 \frac{\varepsilon^* q}{q^2} p_B^\alpha p_{D^*}^\beta \left(T_1(q^2) - T_2(q^2) - \frac{q^2}{m_B^2 - m_{D^*}^2} T_3(q^2) \right) \Big].
\end{aligned} \tag{B.0.16}$$

Where, for the operator $\bar{c} \sigma^{\mu\nu} \gamma_5 b$ we use $\sigma^{\mu\nu} \gamma_5 = -i/2 \varepsilon^{\mu\nu\alpha\beta} \sigma_{\alpha\beta}$ identity.

The scalar, vector and tensor form factors of $\bar{B} \rightarrow D^*$ matrix elements, $F_0(q^2)$, $F_+(q^2)$ and $F_T(q^2)$ are defined as

$$\langle D(p_D) | \bar{c} b | \bar{B}(p_B) \rangle = \frac{m_B^2 - m_{D^*}^2}{m_b - m_c} F_0(q^2), \tag{B.0.17}$$

$$\langle D(p_D) | \bar{c} \gamma^\mu b | \bar{B}(p_B) \rangle = \left[p_B^\mu + p_D^\mu - \frac{m_B^2 - m_{D^*}^2}{q^2} q^\mu \right] F_+(q^2) + \frac{m_B^2 - m_{D^*}^2}{q^2} q^\mu F_0(q^2), \tag{B.0.18}$$

$$\langle D(p_D) | \bar{c} \sigma^{\mu\nu} b | \bar{B}(p_B) \rangle = -i(p_B^\mu p_D^\nu - p_D^\mu p_B^\nu) \frac{2F_T(q^2)}{m_B + m_{D^*}}, \tag{B.0.19}$$

The above form factors of both transitions $\bar{B} \rightarrow D^*$ and $\bar{B} \rightarrow D$ can be derived in the heavy-quark effective theory (HQET) [149], in which the meson momenta in terms of four-velocities are $v_B^\mu = p_B^\mu/m_B$ and $v_{D^*(D)}^\mu = p_{D^*(D)}^\mu/m_{D^*(D)}$. In this effective theory, the hadronic matrix elements are given by

$$\langle D(v') | \bar{c} \gamma^\mu b | \bar{B}(v) \rangle = \sqrt{m_B m_D} [h_+(w)(v + v')^\mu + h_-(w)(v - v')^\mu], \tag{B.0.20}$$

$$\langle D(v') | \bar{c} \sigma^{\mu\nu} b | \bar{B}(v) \rangle = -i\sqrt{m_B m_D} h_T(w) [v^\mu v'^\nu - v^\nu v'^\mu], \tag{B.0.21}$$

$$\langle D^*(v', \varepsilon') | \bar{c} \gamma_\mu b | \bar{B}(v) \rangle = i\sqrt{m_B m_{D^*}} h_V(w) \varepsilon_{\mu\nu\alpha\beta} \varepsilon'^{\nu\alpha} v'^\alpha v'^\beta, \tag{B.0.22}$$

$$\langle D^*(v', \varepsilon') | \bar{c} \gamma_\mu \gamma_5 b | \bar{B}(v) \rangle = \sqrt{m_B m_{D^*}} [h_{A_1}(w)(w + 1) \varepsilon'_\mu - (\varepsilon'^* \cdot v)(h_{A_2}(w)v_\mu + h_{A_3}(w)v'_\mu)], \tag{B.0.23}$$

$$\begin{aligned}
\langle D^*(v', \varepsilon') | \bar{c} \sigma_{\mu\nu} b | \bar{B}(v) \rangle = & -\sqrt{m_B m_{D^*}} \varepsilon_{\mu\nu\alpha\beta} [h_{T_1}(w) \varepsilon^{*\alpha} (v + v')^\beta + h_{T_2}(w) \varepsilon^{*\alpha} (v - v')^\beta \\
& + h_{T_3}(w) (\varepsilon^* \cdot v)(v + v')^\alpha (v - v')^\beta],
\end{aligned} \tag{B.0.24}$$

where $w = v_B \cdot v_{D^*}' = (m_B^2 + m_{D^*}^2 - q^2)/2m_B m_{D^*}$.

The form factors $F_0(q^2)$, $F_+(q^2)$ and $F_T(q^2)$ can be extracted by comparing the Eqs. (B.0.18), (B.0.19) and (B.0.20), (B.0.21) while for the form factors $V(q^2)$, $A_0(q^2)$, $A_1(q^2)$, $A_2(q^2)$, $T_1(q^2)$, $T_2(q^2)$ and $T_3(q^2)$ we take the Eqs (B.0.11), (B.0.12), (B.0.16) and (B.0.22), (B.0.23), (B.0.24). Thus we obtain the following relations

$$F_0(q^2) = \frac{1}{2\sqrt{m_B m_D}} \left[\frac{(m_B + m_D)^2 - q^2}{m_B + m_D} h_+(w(q^2)) - \frac{(m_B - m_D)^2 - q^2}{(m_B - m_D)} h_-(w(q^2)) \right], \quad (\text{B.0.25})$$

$$F_1(q^2) = \frac{1}{2\sqrt{m_B m_D}} \left[(m_B + m_D) h_+(w(q^2)) - (m_B - m_D) h_-(w(q^2)) \right], \quad (\text{B.0.26})$$

$$F_T(q^2) = \frac{m_B + m_D}{2\sqrt{m_B m_D}} h_T(w(q^2)), \quad (\text{B.0.27})$$

$$V(q^2) = \frac{m_B + m_{D^*}}{2\sqrt{m_B m_{D^*}}} h_V(w(q^2)), \quad (\text{B.0.28})$$

$$A_0(q^2) = \frac{1}{2\sqrt{m_B m_{D^*}}} \left[\frac{(m_B + m_{D^*})^2 - q^2}{2m_{D^*}} h_{A_1}(w(q^2)) - \frac{m_B^2 - m_{D^*}^2 + q^2}{2m_{D^*}} h_{A_2}(w(q^2)) - \frac{m_B^2 - m_{D^*}^2 - q^2}{2m_{D^*}} h_{A_3}(w(q^2)) \right], \quad (\text{B.0.29})$$

$$A_1(q^2) = \frac{(m_B + m_{D^*})^2 - q^2}{2\sqrt{m_B m_{D^*}} (m_B + m_{D^*})} h_{A_1}(w(q^2)), \quad (\text{B.0.30})$$

$$A_2(q^2) = \frac{(m_B + m_{D^*})^2}{2\sqrt{m_B m_{D^*}}} \left[h_{A_3}(w(q^2)) + \frac{m_{D^*}}{m_B} h_{A_2}(w(q^2)) \right], \quad (\text{B.0.31})$$

$$T_1(q^2) = \frac{1}{2\sqrt{m_B m_{D^*}}} \left[(m_B + m_{D^*}) h_{T_1}(w(q^2)) - (m_B - m_{D^*}) h_{T_2}(w(q^2)) \right], \quad (\text{B.0.32})$$

$$T_2(q^2) = \frac{1}{2\sqrt{m_B m_{D^*}}} \left[\frac{(m_B + m_{D^*})^2 - q^2}{m_B + m_{D^*}} h_{T_1}(w(q^2)) - \frac{(m_B - m_{D^*})^2 - q^2}{m_B - m_{D^*}} h_{T_2}(w(q^2)) \right], \quad (\text{B.0.33})$$

$$T_3(q^2) = \frac{1}{2\sqrt{m_B m_{D^*}}} \left[(m_B - m_{D^*}) h_{T_1}(w(q^2)) - (m_B + m_{D^*}) h_{T_2}(w(q^2)) - \frac{2(m_B^2 - m_{D^*}^2)}{m_B} h_{T_3}(w(q^2)) \right], \quad (\text{B.0.34})$$

Finally, the hadronic amplitudes are presented as [28]

-
- $\bar{B} \rightarrow D\tau\bar{\nu}_\tau$

$$H_S^s(q^2) = H_{SL}^s(q^2) = H_{SR}^s(q^2) = m_B\sqrt{r}(1+w)S_1(w), \quad (\text{B.0.35})$$

$$H_{VL,\pm}^s(q^2) = H_{VR,\pm}^s(q^2) = 0, \quad (\text{B.0.36})$$

$$H_{V,0}^s(q^2) = H_{VL,0}^s(q^2) = H_{VR,0}^s(q^2) = m_B\sqrt{r}(1+r)\frac{\sqrt{w^2-1}}{\sqrt{\hat{q}^2(w)}}V_1(w), \quad (\text{B.0.37})$$

$$H_{V,s}^s(q^2) = H_{VL,s}^s(q^2) = H_{VR,s}^s(q^2) = m_B\sqrt{r}(1-r)\frac{w+1}{\sqrt{\hat{q}^2(w)}}S_1(w), \quad (\text{B.0.38})$$

$$H_{+-}^s(q^2) = H_{0s}^s(q^2) = m_B\sqrt{r}\frac{\sqrt{w^2-1}}{\hat{q}^2(w)}[-(1+r)^2V_1(w) + 2r(1+w)S_1(w)], \quad (\text{B.0.39})$$

- $\bar{B} \rightarrow D^*\tau\bar{\nu}_\tau$

$$H_{SR}^\pm(q^2) = H_{SL}^\pm(q^2) = 0, \quad (\text{B.0.40})$$

$$H_S(q^2) = H_{SR}^0(q^2) = -H_{SL}^0(q^2) = \frac{m_B\sqrt{w^2-1}}{2\sqrt{r}(1+r)}A_1(w)[-2r(1+w) + (1-r^2)R_2(w) - \hat{q}^2(w)R_3(w)], \quad (\text{B.0.41})$$

$$H_{VL,\pm}(q^2) = H_{VL,\pm}^\pm(q^2) = -H_{VR,\mp}^\mp(q^2) = m_B\sqrt{r}A_1(w)[1+w \mp \sqrt{w^2-1}R_1(w)], \quad (\text{B.0.42})$$

$$H_{VL,0}(q^2) = H_{VL,0}^0(q^2) = -H_{VR,0}^0(q^2) = m_B\sqrt{r}\frac{w+1}{\sqrt{\hat{q}^2(w)}}A_1(w)[-w+r+(w-1)R_2(w)], \quad (\text{B.0.43})$$

$$H_{VL,s}(q^2) = H_{VL,s}^0(q^2) = -H_{VR,s}^0(q^2) = \frac{m_B}{2\sqrt{r}}\frac{\sqrt{w^2-1}}{\sqrt{\hat{q}^2(w)}}A_1(w)[-2r(1+w) + (1-r^2)R_2(w) - \hat{q}^2(w)R_3(w)], \quad (\text{B.0.44})$$

$$H_{\pm 0}(q^2) = H_{\pm 0}^\pm(q^2) = \pm H_{\pm s}^\pm(q^2) = \frac{m_B\sqrt{r}}{\sqrt{\hat{q}^2(w)}}A_1(w)[\pm(1-r)(1+w) + (1+r)\sqrt{w^2-1}R_1(w)], \quad (\text{B.0.45})$$

$$H_{+-}(q^2) = H_{+-}^0(q^2) = H_{0s}^0(q^2) = \frac{m_B\sqrt{r}}{1+r}A_1(w)[-(w+r)(1+w) + (w^2-1)R_3(w)], \quad (\text{B.0.46})$$

$$H_{\lambda\lambda'}^{\lambda_M}(q^2) = -H_{\lambda'\lambda}^{\lambda_M}(q^2) \quad (\text{B.0.47})$$

where $r = m_M/m_B$ and $\hat{q}^2(w) = 1 + r^2 - 2rw$. The form factors expressed by [22, 149]

$$V_1(w) = V_1(1)[1 - 8\rho_D^2 z(w) + (51\rho_D^2 - 10)z^2(w) - (252\rho_D^2 - 84)z^3(w)], \quad (\text{B.0.48})$$

$$S_1(w) = V_1(w)[1 + (0.041(w - 1) - 0.015(w - 1)^2 - 0.019)], \quad (\text{B.0.49})$$

$$A_1(w) = A_1(1)[1 - 8\rho_{A_1}^2 z(w) + (53\rho_{A_1}^2 - 15)z^2(w) - (231\rho_{A_1}^2 - 91)z^3(w)], \quad (\text{B.0.50})$$

$$R_1(w) = R_1(1) - 0.12(w - 1) + 0.05(w - 1)^2, \quad (\text{B.0.51})$$

$$R_2(w) = R_2(1) + 0.11(w - 1) - 0.06(w - 1)^2, \quad (\text{B.0.52})$$

$$R_3(w) = 1.22 - 0.052(w - 1) + 0.026(w - 1)^2, \quad (\text{B.0.53})$$

where $z(w) = (\sqrt{w+1} - \sqrt{2})/(\sqrt{w+1} + \sqrt{2})$. The numerical values of the free parameters ρ_D^2 , $\rho_{A_1}^2$, $R_1(1)$ and $R_2(1)$ are given [117]

$$\rho_D^2 = 1.186 \pm 0.054, \quad \rho_{D^*}^2 = 1.207 \pm 0.026, \quad (\text{B.0.54})$$

$$R_1(1) = 1.403 \pm 0.033, \quad R_2(1) = 0.854 \pm 0.020, \quad (\text{B.0.55})$$

and $V_1(1)$ and $A_1(1)$ are

$$V_1(1)|V_{cb}| = (43.0 \pm 1.9 \pm 1.4)10^{-3}, \quad A_1(1)|V_{cb}| = (34.6 \pm 0.2 \pm 1.0). \quad (\text{B.0.56})$$

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