People's Democratic Republic of Algeria
Ministry of Higher Education and Scientific Research

Physics 2: Electricity and Magnetism

Course Handout and Exercises

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Faculty:

Faculty of Exact Sciences and Informatics

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This handout is intended for first-year LMD Mathematics and Informatics students.

About This Course

Course Objectives:

This course aims to provide the fundamental principles of electricity and magnetism, focusing on elementary knowledge of electricity and magnetism (calculation of electric and magnetic fields, current calculation, etc.), enabling students to analyze and interpret related phenomena. The course is structured into four chapters:

- **Chapter I (Electrostatics):** This chapter covers phenomena related to static electric charges, such as the electric field and electrostatic potential.
- **Chapter II (Conductors in Equilibrium):** This section establishes the properties of a conductor in equilibrium, followed by detailed definitions of capacitors, their associations, and capacitance.
- **Chapter III (Electrokinetics):** This chapter focuses on the motion of mobile charges, studied in terms of currents and voltages.
- **Chapter IV (Magnetostatics):** This chapter examines the interactions between moving charged particles. The order of these chapters is justified by the fact that magnetostatics is constantly based on concepts acquired in electrostatics and electrodynamics.

Numerous solved exercises, strategically placed at the end of each chapter, provide students with valuable opportunities to apply the concepts studied. These well-designed, varied exercises not only help solidify theoretical understanding but also serve as effective preparation for exams.

Prerequisites:

- Familiarity with basic electricity experiments and their explanations.
- Understanding of some physical concepts.
- Knowledge of the source of electricity.
- Familiarity with some mathematical operators for calculating electric fields and forces.

Target Audience:

This course is intended for first-year LMD students in the second semester.
Field: Mathematics and Informatics – **Department:** Common Core.

Contents

Chapter 1

Electrostatics

1.1 Definition

Electrostatics is the study of electric charges at rest, their interactions, and the electric fields and forces they produce.

1.2 Electrical charge

Observation: When a balloon is rubbed with hair and then brought close to a wall, it becomes statically charged and can stick to the wall for a significant period.
Numerous experiments have demonstrated that objects can acquire a property known as "electric charge," which leads to interactions referred to as "electrostatic forces."
In fact, all objects can be electrified, either through rubbing or by direct contact with a charged object or by being connected to one terminal of a power source.
Electrostatic forces arise when an object possesses a specific quantity of charge, denoted as q, which plays a role similar to that of mass in gravitational interactions. While mass enables objects to attract other massive bodies through gravitational forces, electric charge enables objects to attract or repel other charged objects. Just as mass affects objects via gravitational forces, electric charge affects them through electrostatic forces.

1.3 Basic charge and quantization electrical charge.

The atom consists of a nucleus surrounded by a cloud of electrons. Electrons are assigned a negative charge and occupy regions around the nucleus, often described as orbitals. The nucleus itself is composed of protons, which carry a positive charge, and neutrons, which are electrically neutral.
Electrons and protons have equal but opposite electrical charges. The magnitude of this charge is the smallest known unit of electric charge, referred to as the *elementary charge*, denoted by e, and is approximately 1.6×10^{-19} coulombs (C). Coulombs measure the amount of electric charge that an object possesses.

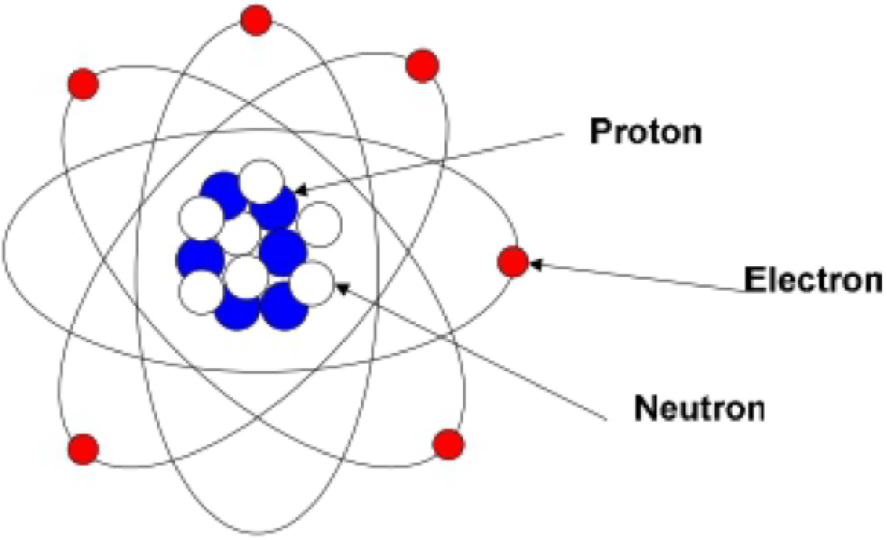


Figure 1.1: The Structure of an Atom.

Example 1: how many electrons are in one coulomb?

Solution: We are given that the charge of a single electron is:

$$e = 1.6 \times 10^{-19}\,\mathrm{C}$$

We need to determine how many electrons (denoted N) are required to make up a total charge of $1\,\mathrm{C}$.

The total charge Q is the sum of the charge of all the electrons. Therefore, we can write the equation:

$$Q = N \times e$$

where:

- Q is the total charge (in coulombs),
- N is the number of electrons,
- e is the charge of one electron.

Since we want to find the number of electrons required for $Q = 1\,\mathrm{C}$, we substitute into the equation:

$$1\,\mathrm{C} = N \times 1.6 \times 10^{-19}\,\mathrm{C}$$

Rearranging the equation to solve for N, we get:

$$N = \frac{1\,\mathrm{C}}{1.6 \times 10^{-19}\,\mathrm{C}}$$

$$N = \frac{1}{1.6 \times 10^{-19}} = 6.25 \times 10^{18}\,\text{electrons}$$

Thus, the number of electrons required to make up one coulomb of charge is:

$$N \approx 6.25 \times 10^{18}\,\text{electrons}$$

An electric charge cannot take an arbitrary numerical value. In fact, every electric charge is an integer multiple of the elementary charge, which reflects the fundamental principle of charge quantization. Mathematically, the electric charge q can be expressed as:

$$q = \pm n \cdot e \quad (n \in \mathbb{N})$$

where e is the magnitude of the elementary charge, approximately equal to $1.6 \times 10^{-19}\,\mathrm{C}$. This principle implies that the electric charge of any object is always a discrete multiple of the elementary charge e, making charge a quantized rather than a continuous quantity.

Observations

- When electrons are removed from a body, it becomes positively charged; when electrons are added, it becomes negatively charged.
- Objects carrying charges of the same sign repel each other, while objects carrying opposite charges attract.

1.4 Coulomb's law

Coulomb's Law describes the force of attraction or repulsion between two point charges. If two charges q_1 and q_2 are separated by a distance r, the magnitude of the force F between them is given by:

$$F = k\frac{|q_1 q_2|}{r^2}$$

where:

- F is the magnitude of the electrostatic force (in newtons, N);
- $k = 8.9876 \times 10^9\,\mathrm{N{\cdot}m^2/C^2}$ is Coulomb's constant.

This is the force that each charge exerts on the other, as per Newton's Third Law. The constant k can also be expressed as:

$$k = \frac{1}{4\pi\epsilon_0}$$

where $\epsilon_0 = 8.85419 \times 10^{-12}\,\mathrm{C^2/(N{\cdot}m^2)}$ is the permittivity of free space.

Force Direction:

- If q_1 and q_2 have the same sign (both positive or both negative), the force is **repulsive**, and each charge pushes the other away.
- If q_1 and q_2 have opposite signs (one positive and one negative), the force is **attractive**, and each charge pulls the other toward it.

This relationship is illustrated in the following Figure:

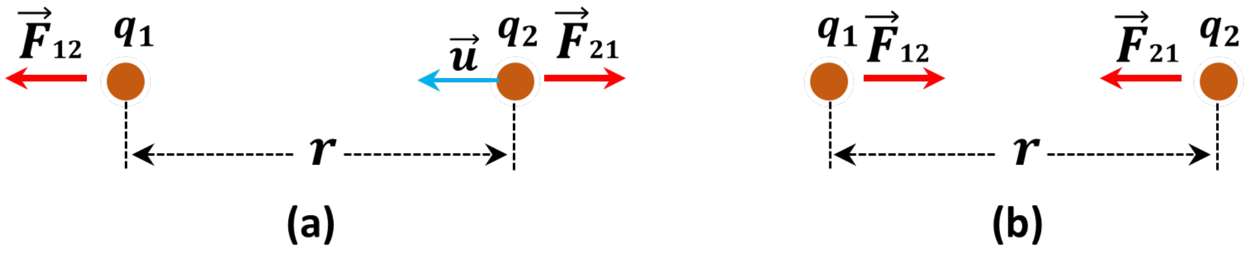


Figure 1.2: (a) Repulsive force between like charges (q_1 and q_2). (b) Attractive force between opposite charges (q_1 and q_2).

In accordance with Newton's Third Law, we have:

$$\vec{\mathbf{F}}_{12} = -\vec{\mathbf{F}}_{21}$$

where $\mathbf{F}_{12}$ is the force exerted by q_2 on q_1, and $\mathbf{F}_{21}$ is the force exerted by q_1 on q_2. Coulomb's law can be expressed in vector form as follows:

$$\vec{\mathbf{F}}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\vec{\mathbf{u}}$$

where $\vec{\mathbf{u}}$ is a unit vector directed from q_2 to q_1 (see Figure 2.a).

Example 2: Force Between Two Positive Charges Consider two charges, $q_1 = 2\,\mu C = 2 \times 10^{-6}\,C$ and $q_2 = 3\,\mu C = 3 \times 10^{-6}\,C$, separated by a distance $r = 0.5\,\text{m}$. The force between the charges is:

$$F = k\frac{|q_1 q_2|}{r^2}$$

Substituting the values:

$$F = (8.9876 \times 10^9)\frac{(2 \times 10^{-6})(3 \times 10^{-6})}{(0.5)^2}$$

$$F = (8.9876 \times 10^9)\frac{6 \times 10^{-12}}{0.25}$$

$$F = (8.9876 \times 10^9) \times 24 \times 10^{-12}$$

$$F = 0.2157\,\text{N}$$

The force is repulsive since both charges are positive.

Example 3: Force Between Opposite Charges Consider two charges, $q_1 = 5\,\mu C = 5 \times 10^{-6}\,C$ and $q_2 = -4\,\mu C = -4 \times 10^{-6}\,C$, separated by a distance $r = 1\,\text{m}$. The force between the charges is:

$$F = k\frac{|q_1 q_2|}{r^2}$$

Substituting the values:

$$F = (8.9876 \times 10^9)\frac{|(5 \times 10^{-6})(-4 \times 10^{-6})|}{1^2}$$

$$F = (8.9876 \times 10^9) \times 20 \times 10^{-12}$$

$$F = 0.1798\,\text{N}$$

The force is attractive since the charges are of opposite signs.

1.5 Electric Forces and the Superposition Principle

The electric force is subject to the **principle of superposition**, which is valid only in the case of *electrostatic* charges (charges at rest). The total electric force $\vec{F}$ acting on a charge q_0 due to multiple other charges $q_1, q_2, q_3, \ldots, q_N$ is equal to the vector sum of the individual forces exerted by each charge. Mathematically:

$$\vec{F} = \sum_{i=1}^{N} \vec{F}_{i0} = \vec{F}_{10} + \vec{F}_{20} + \cdots + \vec{F}_{N0}.$$

Here, $\vec{F}_{i0}$ represents the force exerted by the charge q_i on q_0.

1.6 6. Electric field

1.6.1 Electric Field Created by a Point Charge

When a point charge q is located at a specific position, it creates an electric field in the surrounding space. The electric field experienced by a small test charge q_0 placed at a point in the field is proportional to both the magnitude of the point charge q and the inverse square of the distance r from the charge. Mathematically, the electric field $\vec{E}$ created by a point charge q at a distance r is given by:

$$\vec{E} = \frac{k|q|}{r^2}\,\vec{u}$$

where:

- k is Coulomb's constant, $k = 8.9876 \times 10^9\,\mathrm{N{\cdot}m^2/C^2}$;
- $|q|$ is the magnitude of the point charge;
- r is the distance from the point charge;
- $\vec{u}$ is the unit vector pointing from the charge to the point where the electric field is being measured.

The force $\vec{F}$ experienced by a small test charge q_0 in the electric field $\vec{E}$ is related to the electric field by the following equation:

$$\vec{F} = q_0\vec{E}$$

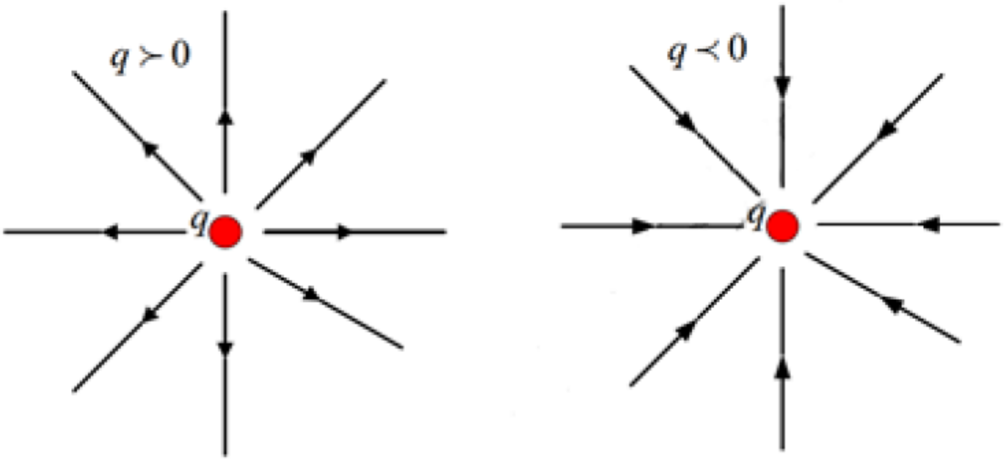


Figure 1.3: Electric field resulting from a point charge.

Example 4: What is the magnitude and direction of the electric field created by a negative point charge $Q = 1 \times 10^{-4}\,\text{C}$ at a point 50 cm away from it? Also, what is the electric force acting on a small positive charge $q = 4\,\mu\text{C}$ at that point?
Solution: The electric field created by a point charge Q is given by the equation:

$$E = \frac{kQ}{r^2}$$

$$E = \frac{(8.9876 \times 10^9) \times (1 \times 10^{-4})}{(0.5)^2} \approx 3.6 \times 10^6\,\text{N/C}$$

Since the charge Q is negative, the electric field will point towards Q, as electric field lines point towards negative charges.
Now, to calculate the electric force acting on a positive charge $q = 4\,\mu\text{C}$, we use the relationship:

$$F = q \cdot E = 4 \times 10^{-6} \times 3.6 \times 10^6 = 14.4\,\text{N}$$

The direction of the force is the same as the direction of the electric field, which is towards the negative charge Q, since the test charge q is positive. This is in accordance with the fact that opposite charges attract.

1.6.2 Electric Field Created by Multiple Point Charges

For multiple point charges, the total electric field at a given point is the vector sum of the electric fields due to each individual charge. This follows from the principle of superposition, which states that the resultant electric field is the sum of the electric fields created by each charge, as if each charge acted independently. Mathematically, the total electric field $\vec{E}_{\text{total}}$ at a point is:

$$\vec{E}_{\text{total}} = \sum_{i=1}^{N} \vec{E}_i$$

where $\vec{E}_i$ is the electric field due to the i-th charge, and the summation is taken over all the charges. The direction and magnitude of the resultant field depend on the positions and magnitudes of all the charges involved.

1.7 Electric Potential

1.7.1 Electrostatic Potential Due to a Point Charge

The electric potential V due to a point charge q at a distance r is given by:

$$V = \frac{Kq}{r}$$

where:

- V is the electric potential in **volts (V)**;
- K is Coulomb's constant, defined as:

$$K = \frac{1}{4\pi\epsilon_0} \quad \text{(in units of N} \cdot \text{m}^2/\text{C}^2\text{);}$$

- q is the charge in **coulombs (C)**;
- r is the distance from the charge in **meters (m)**.

1.7.2 Electrostatic Potential Due to Multiple Point Charges

Since V is a scalar quantity, the electric potential $V(M)$ at a point M due to multiple point charges is given by the following expression:

$$V(M) = K \sum_i \frac{q_i}{r_i}$$

where r_i is the distance between the charge q_i and the point M, and it is important to consider the sign of q_i, as q_i can be either positive or negative.

1.8 Electrostatic Flux

We define the electric flux through a surface as the quantity:

$$\phi = \int \vec{\mathbf{E}} \cdot d\vec{\mathbf{S}}$$

where $\vec{\mathbf{E}}$ is the electric field vector and $d\vec{\mathbf{S}}$ is the differential surface vector, which is always perpendicular to the surface and directed outward from the volume enclosed by the surface.

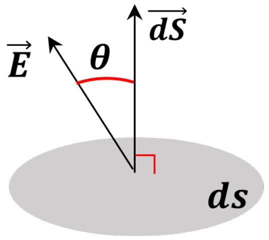


Figure 1.4: Flux through a differential surface.

If the angle between the electric field vector $\vec{\mathbf{E}}$ and the differential surface vector $d\vec{\mathbf{S}}$ is θ, then the electric flux is:

$$\phi = \int \mathbf{E} \cdot d\mathbf{S} \cos\theta$$

The unit of electric flux is the Weber (Wb), which is equivalent to the amount of flux passing through a surface.

1.9 Gauss's Law

Gauss's Law states that the electric flux, Φ, through a closed surface is equal to the algebraic sum of the charges enclosed within the volume bounded by the surface, divided by the permittivity of free space, ε_0. Mathematically, this can be expressed as:

$$\phi = \int \vec{\mathbf{E}} \cdot d\vec{\mathbf{S}} = \frac{Q_{\text{enc}}}{\varepsilon_0}$$

where:

- Φ is the electric flux through the closed surface S,
- $\vec{E}$ is the electric field vector,
- $d\vec{S}$ is the differential surface area vector, which is directed outward and perpendicular to the surface,
- Q_{enc} is the total charge enclosed within the surface S,
- ε_0 is the permittivity of free space ($\varepsilon_0 = 8.854 \times 10^{-12}\,\mathrm{C}^2/\mathrm{N}\cdot\mathrm{m}^2$).

This law is useful because it simplifies the process of calculating the electric field generated by a symmetric distribution of charges.

1.9.1 Application of Gauss's Theorem

1.9.1.1 Electric Field Due to a Point Charge

Consider a positive charge q located at the center of a spherical surface with radius r. The electric field $\mathbf{E}$ is radial and directed outward, so $\cos(0°) = 1$.

The electric flux through the spherical surface is given by:

$$\phi = \int \vec{\mathbf{E}} \cdot d\vec{\mathbf{S}} \Rightarrow \phi = \int \mathbf{E} \cdot d\mathbf{S} \cdot \cos(0°) = \frac{q}{\epsilon_0} = \mathbf{E} \cdot S$$

The surface area of the sphere is $S = 4\pi r^2$, so the electric field at a distance r from the charge is:

$$\mathbf{E} = \frac{q}{4\pi\epsilon_0 r^2}$$

1.9.1.2 Electric Field Due to a Uniformly Charged Infinite Wire

To determine the electric field due to an infinitely long, uniformly charged wire, we choose an appropriate Gaussian surface. The most suitable choice is a cylindrical surface of radius R and length l, coaxial with the charged wire.

This cylindrical Gaussian surface consists of three parts:

- Two flat circular surfaces: S_1 and S_2 (the bases of the cylinder).

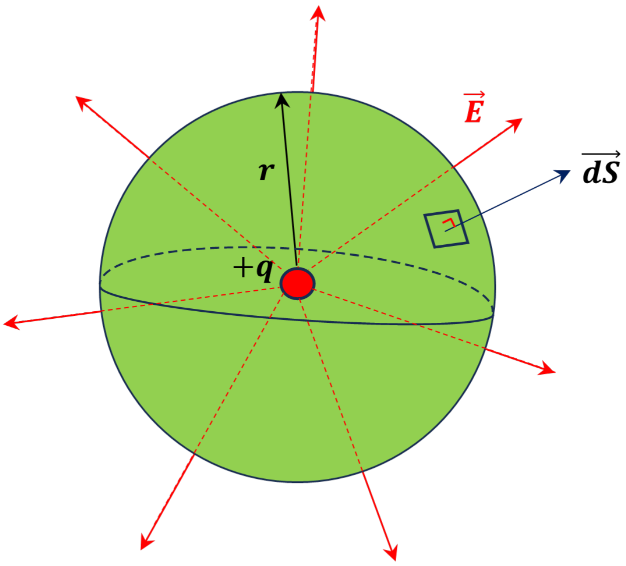


Figure 1.5: Electric field of a point charge with a spherical Gaussian surface.

  - A curved lateral surface: S_3.

Gauss's Law states that the total electric flux through a closed surface is proportional to the charge enclosed within the surface:

$$\Phi = \int \vec{\mathbf{E}} \cdot d\vec{\mathbf{S}} = \frac{Q_{\text{enc}}}{\varepsilon_0}$$

The total flux through the Gaussian surface is the sum of the flux contributions from the three surfaces:

$$\Phi = \int_{S_1} \vec{\mathbf{E}} \cdot d\vec{\mathbf{S}} + \int_{S_2} \vec{\mathbf{E}} \cdot d\vec{\mathbf{S}} + \int_{S_3} \vec{\mathbf{E}} \cdot d\vec{\mathbf{S}}$$

On the circular surfaces S_1 and S_2, the electric field vector $\vec{\mathbf{E}}$ is radial, while the surface element vector $d\vec{\mathbf{S}}$ is normal to these surfaces. Since the angle between $\vec{\mathbf{E}}$ and $d\vec{\mathbf{S}}$ is 90°, we have:

$$\vec{\mathbf{E}} \cdot d\vec{\mathbf{S}} = E\, dS \cos(90^\circ) = 0$$

Thus, there is no electric flux through S_1 and S_2.

On the lateral surface S_3, both the electric field vector $\vec{\mathbf{E}}$ and the surface element vector $d\vec{\mathbf{S}}$ are directed radially outward. Hence, the angle between them is 0°, meaning:

$$\vec{\mathbf{E}} \cdot d\vec{\mathbf{S}} = E\, dS \cos(0^\circ) = E\, dS$$

The total flux through the lateral surface is then:

$$\Phi = \int_{S_3} \mathbf{E} \cdot d\mathbf{S} = E \int_{S_3} d\mathbf{S} = ES_3 \quad \text{(Because } \mathbf{E} \text{ is constant over } S_3\text{)}$$

where the lateral surface area of the cylinder, which has a radius R, is given by:

$$S_3 = 2\pi R l$$

Since the enclosed charge is given by:

$$Q_{\text{enc}} = \lambda l$$

where λ is the linear charge density (charge per unit length), Gauss's Law gives:

$$E(2\pi R l) = \frac{\lambda l}{\varepsilon_0}$$

Therefor:

$$E = \frac{\lambda}{2\pi\varepsilon_0 R}$$

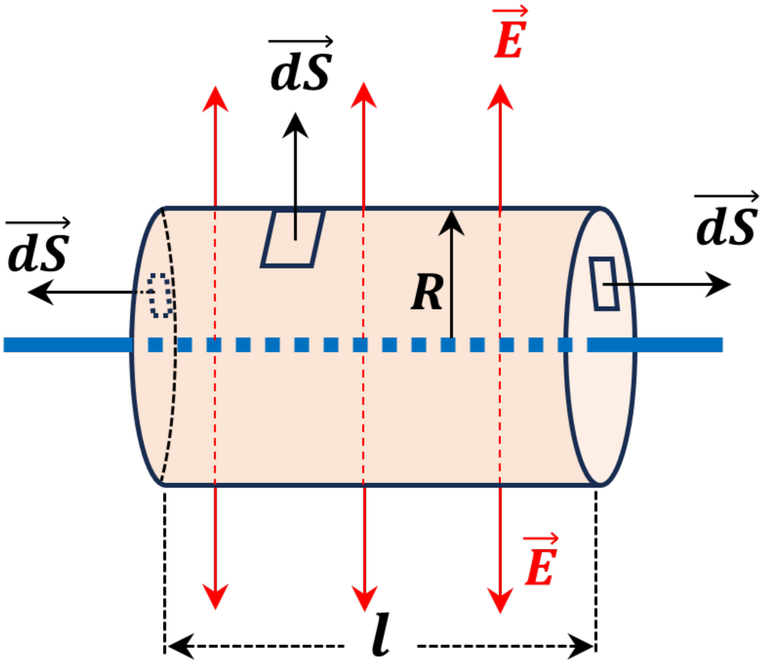


Figure 1.6: Electric field of a uniformly charged infinite wire with a cylindrical Gaussian surface.

1.10 Electric Dipole

An electrostatic dipole is a system consisting of two point charges of equal magnitude but opposite sign, $+q$ and $-q$, separated by a distance d.

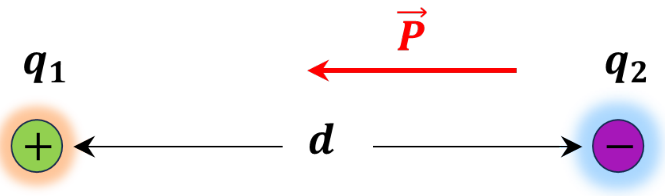


Figure 1.7: Electric dipole.

1.10.1 Electric Dipole Moment

The electric dipole moment is a vector $\vec{\mathbf{P}}$ that is equal to the product of the charge q and the displacement vector $\vec{\mathbf{d}}$, directed from the negative charge towards the positive charge:

$$\vec{\mathbf{P}} = q\vec{\mathbf{d}}$$

Where:

- $\vec{\mathbf{P}}$ is the electric dipole moment vector (with units of Coulomb meters, C m);
- q is the magnitude of one of the charges (since they are equal and opposite);
- $\vec{\mathbf{d}}$ is the displacement vector pointing from the negative charge to the positive charge.

1.10.2 Potential of an Electric Dipole

The point M where we wish to calculate the potential is located using polar coordinates: $r = OM$, $\theta = \angle(Ox, OM)$. We assume $r \gg d = AB$, where O is the midpoint of AB. The potential V at M due to the dipole (see Figure 8) is given by:

$$V = \frac{1}{4\pi\epsilon_0}\left(\frac{-q}{AM} + \frac{q}{BM}\right) = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{BM} - \frac{1}{AM}\right)$$

where:

$$\begin{aligned} BM^2 &= OM^2 + OB^2 - 2 \cdot OB \cdot OM \cdot \cos\theta \quad \text{(Law of Cosines)} \\ &= r^2 + \frac{d^2}{4} - dr\cos\theta \\ \Rightarrow BM &= r\sqrt{1 - \frac{d}{r}\cos\theta + \frac{d^2}{4r^2}} \end{aligned}$$

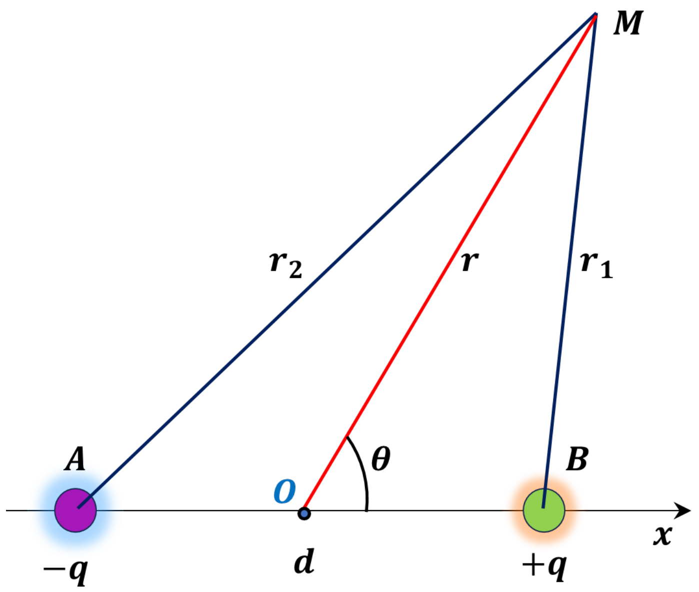


Figure 1.8: Electric potential of a dipole.

Similarly, for AM (by replacing θ with $\pi - \theta$):

$$\begin{aligned} AM^2 &= OM^2 + OB^2 + 2 \cdot OB \cdot OM \cdot \cos\theta \quad \text{(Law of Cosines)} \\ &= r^2 + \frac{d^2}{4} + dr\cos\theta \\ \Rightarrow AM &= r\sqrt{1 + \frac{d}{r}\cos\theta + \frac{d^2}{4r^2}} \end{aligned}$$

Thus, the potential becomes:

$$V = \frac{q}{4\pi\varepsilon_0 r}\left[\left(1 - \frac{d}{r}\cos\theta + \frac{d^2}{4r^2}\right)^{-\frac{1}{2}} - \left(1 + \frac{d}{r}\cos\theta + \frac{d^2}{4r^2}\right)^{-\frac{1}{2}}\right]$$

Since $r \gg d$, the term $\frac{d^2}{4r^2}$ is very small and can be neglected:

$$V \approx \frac{q}{4\pi\varepsilon_0 r}\left[\left(1 - \frac{d}{r}\cos\theta\right)^{-\frac{1}{2}} - \left(1 + \frac{d}{r}\cos\theta\right)^{-\frac{1}{2}}\right]$$

Using the first-order approximation:

$$(1+x)^n \approx 1 + nx, \quad \text{for } x \ll 1,$$

with $n = -\frac{1}{2}$, we expand:

$$\left(1 - \frac{d}{r}\cos\theta\right)^{-\frac{1}{2}} \approx 1 + \frac{1}{2}\frac{d}{r}\cos\theta,$$

$$\left(1 + \frac{d}{r}\cos\theta\right)^{-\frac{1}{2}} \approx 1 - \frac{1}{2}\frac{d}{r}\cos\theta.$$

Substituting these approximations:

$$V \approx \frac{q}{4\pi\varepsilon_0 r}\left[\left(1+\frac{1}{2}\frac{d}{r}\cos\theta\right)-\left(1-\frac{1}{2}\frac{d}{r}\cos\theta\right)\right]$$

$$V \approx \frac{q}{4\pi\varepsilon_0 r}\left(\frac{d}{r}\cos\theta\right)$$

Thus, we obtain:

$$V = \frac{qd\cos\theta}{4\pi\varepsilon_0 r^2} = \frac{\vec{p}\cdot\vec{r}}{4\pi\varepsilon_0 r^3}$$

where $\vec{p} = q\vec{d}$ is the dipole moment and $\vec{r} = \vec{OM}$.

Chapter 2

Conductors in Equilibrium

2.1 Conductors

A conductor is a material in which electric charges can move freely due to the presence of mobile electrons. These electrons, while still associated with the material, are weakly attached to individual atoms and can move under the influence of an external electric field. This property enables conductors to efficiently support the flow of electric current with minimal resistance. Common examples of conductors include metals such as copper, silver, aluminum, and gold, which are widely used in electrical wiring and circuits.

2.2 Conductors in Electrostatic Equilibrium

In conductors, electrostatic equilibrium is the state where there is no net motion of electric charges. This means that while individual charge carriers (typically electrons) may experience random thermal motion, there is no overall, or average, directed flow of charge. The random motions of individual charges cancel each other out, resulting in no bulk movement of charge. This is characterized by a zero electric field within the conductor and a constant electric potential throughout the conductor.

2.3 Properties of Conductors in Electrostatic Equilibrium

When a conductor reaches electrostatic equilibrium (i.e., there is no net charge motion), it exhibits the following properties:

2.3.1 The Electric Field Inside a Conductor is Zero

A fundamental characteristic of electrostatic equilibrium is that the electric field inside a conductor is exactly zero ($E_{\text{inside}} = 0$). This condition arises because free charges within the conductor redistribute themselves on the surface in such a way that they completely cancel any external electric field inside the conductor. As a result, ***electrostatic shielding*** occurs, meaning that the conductor's interior is fully protected from external electric fields. This shielding effect has significant practical applications, including Faraday cages and shielded cables, which are used to protect sensitive electronic equipment from electromagnetic interference.

2.3.2 Excess Charge Resides on the Surface

Because like charges repel, any excess charge placed on a conductor will distribute itself to maximize the distance between individual charges. This results in the excess charge migrating to the outer surface of the conductor. This can be rigorously demonstrated using Gauss's Law:

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{Q_{\text{enc}}}{\epsilon_0}$$

If we consider a Gaussian surface entirely within the conductor (just below the surface), where the electric field is zero ($E_{\text{inside}} = 0$) due to electrostatic equilibrium, the left side of Gauss's Law becomes zero:

$$0 = \frac{Q_{\text{enc}}}{\epsilon_0}$$

This implies that the enclosed charge (Q_{enc}) within this Gaussian surface must also be zero. Since this is true for any Gaussian surface we might draw within the conductor, it means there can be no net charge within the bulk of the conductor. Therefore, any excess charge must reside on the conductor's outer surface.

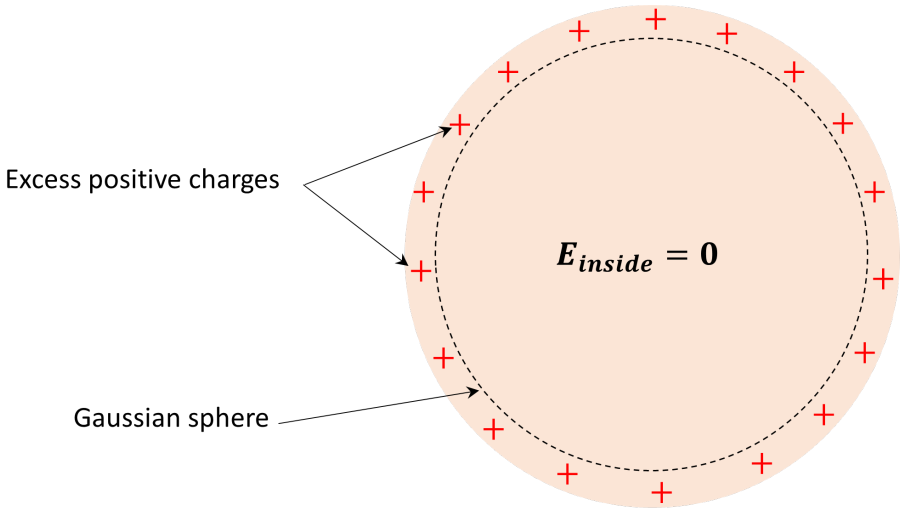


Figure 2.1: Excess charge resides on the surface of a solid conducting sphere.

2.3.3 The Electric Field Just Outside a Conductor is Perpendicular to its Surface

In electrostatic equilibrium, the electric field at the surface of a conductor must be perpendicular to the surface. If there were a tangential (parallel) component of the electric field, free charges on the surface would experience a force and move, contradicting the equilibrium condition.

Thus, just outside the conductor, the electric field is given by:

$$\mathbf{E}_{\textbf{just outside}} = \frac{\sigma}{\epsilon_0}$$

where:

- σ is the local surface charge density (charge per unit area at a specific point on the surface);
- ϵ_0 is the permittivity of free space.

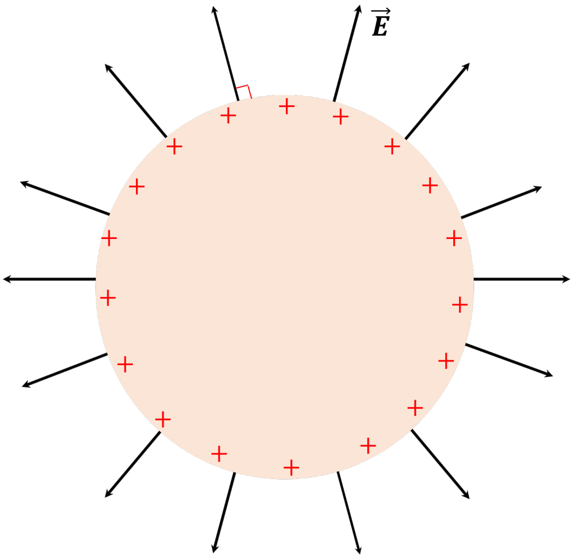


Figure 2.2: The electric field just outside a conductor in electrostatic equilibrium.

2.3.4 Charge Accumulates More Densely at Sharp Points

In electrostatic equilibrium, the surface charge distribution on an irregularly shaped conductor is non-uniform. Charge tends to accumulate more densely in regions of high curvature (small radius of curvature), such as sharp edges and pointed tips. This results in locally stronger electric fields, as described by the relation:

$$E = \frac{\sigma}{\epsilon_0}$$

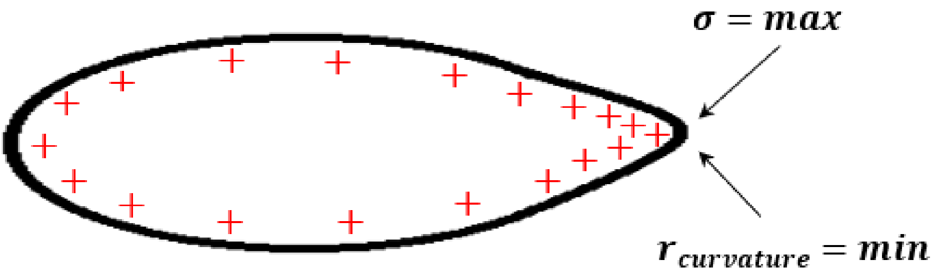


Figure 2.3: Charge distribution on an irregularly shaped conductor in electrostatic equilibrium.

2.4 The Phenomenon of Influence Between Conductors

2.4.1 Partial Influence

Consider an electrically neutral conductor A placed near a positively charged conductor B. The presence of conductor B generates an external electric field

$\vec{E}_B$, which extends into the surrounding space, including the interior of conductor A. As a result, the free electrons within A experience a force due to $\vec{E}_B$ and migrate toward the surface facing B, leading to an accumulation of negative charge ($-Q$). Conversely, the opposite surface of A experiences a deficiency of electrons, resulting in a localized positive charge (+Q).

This charge separation, known as electrostatic induction, produces an induced electric field $\vec{E}_i$, which opposes the external field $\vec{E}_B$. The redistribution of charge continues until electrostatic equilibrium is achieved, at which point the total electric field inside conductor A becomes zero ($\vec{E}_B + \vec{E}_i = 0$). This phenomenon, referred to as partial influence, occurs when only a fraction of the external field's electric field lines terminate on the induced charges of A, meaning that not all field lines from B are captured by A.

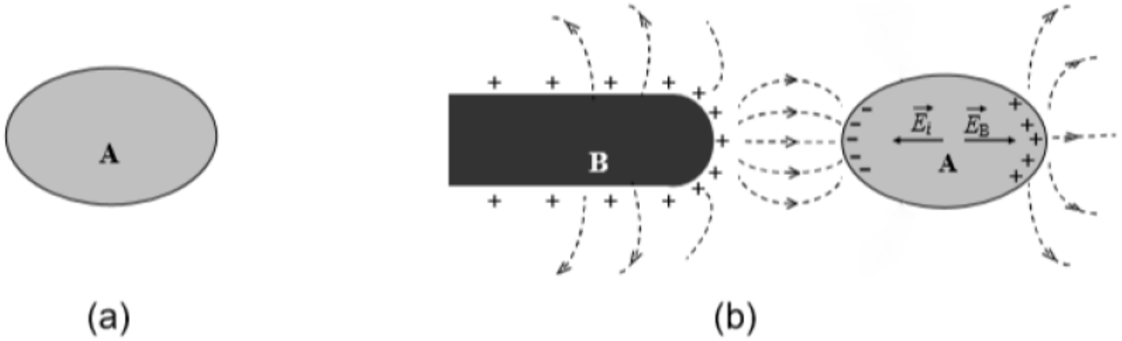


Figure 2.4: Partial influence.

2.4.2 Total Influence

Total influence arises when a charged conductor B is completely surrounded by a neutral conductor A. In this configuration, all the electric field lines emanating from B terminate on the inner surface of A, resulting in a redistribution of charges that ensures electrostatic equilibrium within A.

At equilibrium, the electric field inside conductor A becomes zero due to the complete redistribution of charges. The inner surface of A_i (A_{intern}) acquires a charge Q_{A_i} that is equal in magnitude but opposite in sign to the charge Q_B on B, such that:

$$Q_B = -Q_{A_i}$$

The outer surface of A_e (A_{extern}) carries a charge Q_{A_e}, whose value depends on whether A was initially neutral or charged.

If A is initially neutral, the principle of charge conservation, which states that the total electric charge in an isolated system remains constant, ensures that the total charge on A remains zero:

$$Q_{A_i} + Q_{A_e} = 0$$

This implies that the charge on the outer surface is:

$$Q_{A_e} = -Q_{A_i} = Q_B$$

If A carries an initial charge Q_0, the principle of charge conservation requires:

$$Q_{A_e} + Q_{A_i} = Q_0$$

Substituting $Q_{A_i} = -Q_B$, we find:

$$Q_{A_e} = Q_0 + Q_B$$

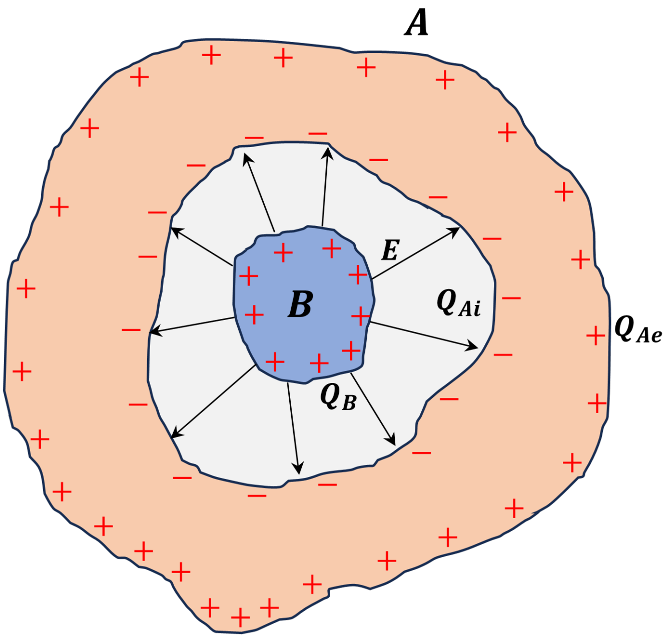


Figure 2.5: Total influence.

2.5 Resistance of Conductors

The resistance of a conductor is a fundamental property that determines its ability to oppose the flow of electric current. It is influenced by several factors, including the material properties, geometry, and temperature.

2.5.1 Resistivity and Conductor Resistance

The resistance R of a conductor is related to its intrinsic material property known as resistivity ρ, which defines how strongly the material opposes current flow. The resistance is given by:

$$R = \rho \frac{L}{A}$$

where:

- ρ is the resistivity of the material $(\Omega \cdot \mathrm{m})$,
- L is the length of the conductor (m),
- A is the cross-sectional area of the conductor (m^2).

Resistivity is an intrinsic property of the material and varies with temperature.

2.5.2 Generalized Ohm's Law

The macroscopic form of Ohm's law, commonly used in circuit analysis, states that the voltage across a conductor is proportional to the current passing through it, given by $V = IR$. This form is useful for circuit calculations but does not describe the underlying physics of conduction. However, a more fundamental description of Ohm's Law exists at the microscopic level. This microscopic form, often referred to as the generalized Ohm's Law, expresses the local relationship between the electric field **E** and the current density **J**:

$$\mathbf{J} = \sigma \mathbf{E}$$

where:

- **J** is the current density (A/m^2);
- σ is the electrical conductivity of the material ($\Omega^{-1}\text{m}^{-1}$).

Since conductivity σ is related to resistivity ρ by $\sigma = \frac{1}{\rho}$:

$$\mathbf{J} = \frac{1}{\rho}\mathbf{E}$$

2.5.3 Effect of Temperature on Resistance

Temperature affects how electrons move inside a conductor. When the temperature increases, the atoms in the conductor vibrate more. This makes it harder for electrons to move freely, causing more collisions. As a result, the material's resistivity increases, which leads to a higher resistance in the conductor.

The resistivity at a temperature T can be estimated using the formula:

$$\rho = \rho_0[1 + \alpha(T - T_0)]$$

where:

- ρ_0 is the resistivity at a temperature T_0,
- α is the temperature coefficient of resistivity (in K^{-1} or $°\text{C}^{-1}$, depending on the unit of temperature used), which depends on the type of material,
- T is the actual temperature (in K or °C).

Similarly, the resistance at temperature T is given by:

$$R = R_0[1 + \alpha(T - T_0)]$$

where R_0 is the resistance at the reference temperature T_0.

2.6 Self-Capacitance of an Isolated Conductor

For an isolated conductor in electrostatic equilibrium, the charge Q on the conductor is proportional to its electric potential V, such that:

$$C = \frac{Q}{V}$$

where C is the self-capacitance of the conductor. In the International System of Units (SI), capacitance is measured in farads (F).

Example 01: Self-Capacitance of an Isolated spherical conductor

Consider a spherical conductor of radius R carrying a total charge Q. The electrostatic potential at its surface is given by:

$$V = \frac{KQ}{R} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}$$

From the definition of capacitance, we obtain:

$$C = \frac{Q}{V} = 4\pi\varepsilon_0 R$$

2.7 Capacitors

A capacitor is a system consisting of two conductors (commonly referred to as plates or electrodes) separated by an insulating medium or vacuum. When a voltage is applied across the conductors, charges of equal magnitude but opposite polarity (+Q and Q) accumulate on their surfaces, establishing an electric field directed from the positively charged conductor to the negatively charged conductor. The capacitance C of the system quantifies its ability to store charge per unit voltage and is given by:

$$C = \frac{Q}{V}$$

where:

- Q is the magnitude of charge on one of the conductors;
- V is the potential difference between them.

2.8 Types of Capacitors

2.8.1 Parallel-Plate Capacitor

A parallel-plate capacitor consists of two conducting plates, each of area A, separated by a distance d and carrying equal and opposite charges $+Q$ and $-Q$.

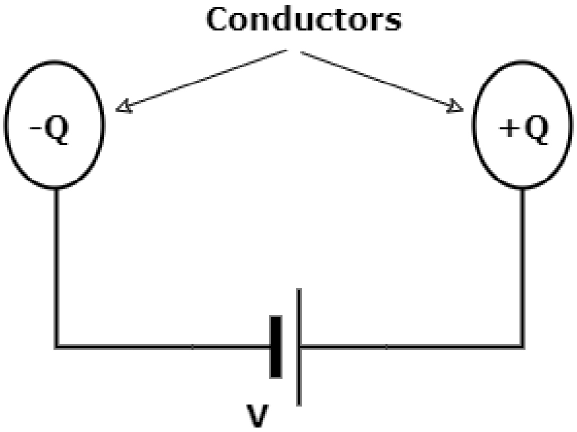


Figure 2.6: Capacitor circuit diagram.

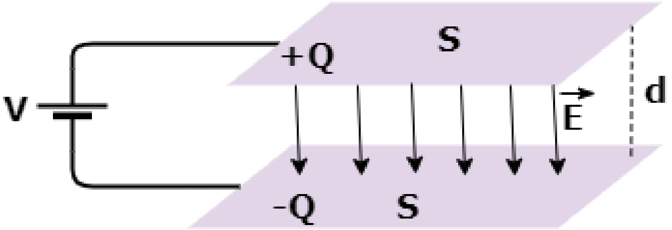


Figure 2.7: Parallel-plate capacitor.

By definition, the capacitance C is given by:

$$C = \frac{Q}{V}$$

From Gauss's law, the magnitude of the electric field between two infinite parallel plates with uniform charge distribution is:

$$E = \frac{\sigma}{\varepsilon_0}$$

The potential difference V between the plates is given by:

$$V = Ed = \frac{\sigma d}{\varepsilon_0}$$

Since the surface charge density $\sigma = \frac{Q}{A}$, we obtain:

$$V = \frac{Qd}{A\varepsilon_0}$$

Rearranging for capacitance:

$$C = \frac{Q}{V} = \frac{Q}{\frac{Qd}{A\varepsilon_0}} = \frac{A\varepsilon_0}{d}$$

Thus, the capacitance of a parallel-plate capacitor is:

$$C = \frac{\varepsilon_0 A}{d}$$

2.8.2 Cylindrical Capacitor

A cylindrical capacitor consists of two coaxial conducting cylinders of radii R_1 (inner) and R_2 (outer), separated by a vacuum.

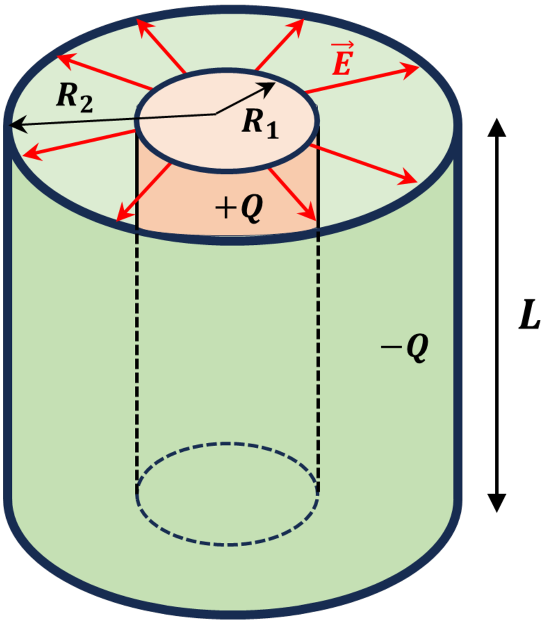


Figure 2.8: Cylindrical capacitor.

By definition, capacitance is given by

$$C = \frac{Q}{V}$$

Using Gauss's law, for a cylindrical Gaussian surface of radius r and length L, the total enclosed charge is Q, giving:

$$\oint E \cdot dA = \frac{Q}{\varepsilon_0} \Rightarrow E(2\pi rL) = \frac{Q}{\varepsilon_0} \Rightarrow E = \frac{Q}{2\pi\varepsilon_0 rL}.$$

The potential difference between the cylinders is obtained by integrating the electric field from $r = R_2$ to $r = R_1$:

$$V = -\int_{R_2}^{R_1} E\, dr = -\int_{R_2}^{R_1} \frac{Q}{2\pi\varepsilon_0 rL}\, dr = -\frac{Q}{2\pi\varepsilon_0 L}\int_{R_2}^{R_1} \frac{1}{r}\, dr.$$

Evaluating the integral,

$$V = -\frac{Q}{2\pi\varepsilon_0 L}\ln\left(\frac{R_1}{R_2}\right) = \frac{Q}{2\pi\varepsilon_0 L}\ln\left(\frac{R_2}{R_1}\right).$$

From $C = \frac{Q}{V}$, substituting for V:

$$C = \frac{Q}{\frac{Q}{2\pi\varepsilon_0 L}\ln\left(\frac{R_2}{R_1}\right)} = \frac{2\pi\varepsilon_0 L}{\ln\left(\frac{R_2}{R_1}\right)}.$$

Thus, the capacitance of a Cylindrical capacitor is:

$$C = \frac{2\pi\varepsilon_0 L}{\ln\left(\frac{R_2}{R_1}\right)}$$

2.8.3 Spherical Capacitor

A spherical capacitor consists of two concentric spherical conducting shells with radii R_1 (inner) and R_2 (outer).

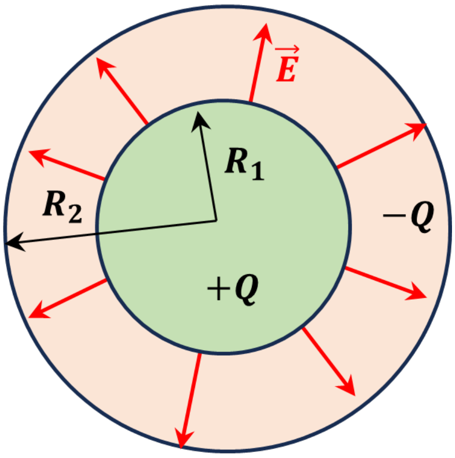


Figure 2.9: Spherical capacitor.

By definition, the capacitance is given by

$$C = \frac{Q}{V}$$

Using Gauss's law, the electric field at a radial distance r from the center, for $R_1 \leq r \leq R_2$, is given by

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}.$$

The potential difference is obtained by integrating the electric field from R_2 to R_1:

$$\begin{aligned} V &= -\int_{R_2}^{R_1} E\,dr = -\int_{R_2}^{R_1} \frac{Q}{4\pi\varepsilon_0 r^2}\,dr = -\frac{Q}{4\pi\varepsilon_0}\int_{R_2}^{R_1}\frac{1}{r^2}\,dr \\ &= -\frac{Q}{4\pi\varepsilon_0}\left[-\frac{1}{r}\right]_{R_2}^{R_1} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{R_1}-\frac{1}{R_2}\right). \end{aligned}$$

From the definition of capacitance, substituting for V:

$$C = \frac{Q}{V} = \frac{Q}{\frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)} = \frac{4\pi\varepsilon_0 R_1 R_2}{R_2 - R_1}.$$

Thus, the capacitance of a spherical capacitor is:

$$C = \frac{4\pi\varepsilon_0 R_1 R_2}{R_2 - R_1}.$$

From our study of parallel plate, cylindrical, and spherical capacitors, we notice that capacitance depends on the geometry of the conductors and the permittivity of the insulating medium between them.

2.9 Connections of Capacitors in Electric Circuits

In an electric circuit, capacitors can be connected in different configurations, primarily in series and parallel.

2.9.1 Series Connection

When capacitors are connected in series, they are arranged sequentially, forming a single path for charge flow. In this arrangement, the charge (Q) is the same across all capacitors. However, the voltage across each capacitor differs, denoted as $V_1, V_2, V_3, \ldots$.

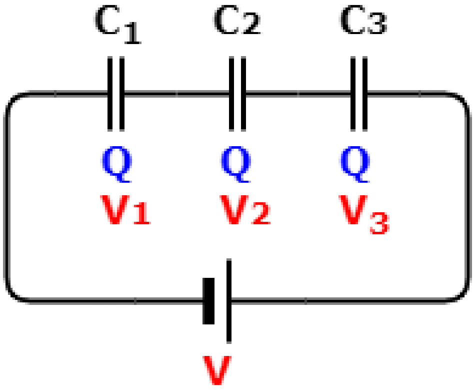


Figure 2.10: Capacitors in series configuration.

The total voltage (V) across the series combination is the sum of the individual voltages:

$$V = V_1 + V_2 + V_3 + \ldots$$

Since the charge is the same for all capacitors, the voltage across each capacitor can be expressed as $V_1 = \frac{Q}{C_1}, V_2 = \frac{Q}{C_2}, V_3 = \frac{Q}{C_3}$, and so forth. Substituting these into the total voltage equation yields:

$$\frac{Q}{C_{eq}} = Q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots\right)$$

Therefore, the equivalent capacitance (C_{eq}) for capacitors in series is given by:

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots$$

It is important to note that the equivalent capacitance in a series connection is always smaller than the smallest individual capacitance in the series.

2.9.2 Parallel Connection

In a parallel connection, the same voltage is applied across each capacitor, while the charge stored on each capacitor varies directly with its capacitance, denoted as $Q_1, Q_2, Q_3, \dots$.

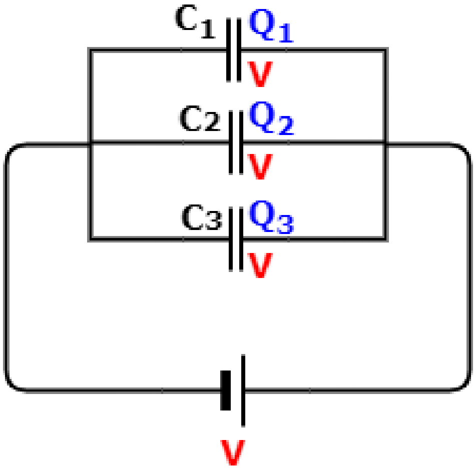


Figure 2.11: Capacitors in parallel configuration.

The total charge (Q) stored in the parallel combination is the sum of the individual charges:

$$Q = Q_1 + Q_2 + Q_3 + \dots$$

Since the voltage is the same for all capacitors, we can express the charge on each capacitor as $Q_1 = C_1V, Q_2 = C_2V, Q_3 = C_3V$, and so on. Substituting these into the total charge equation gives:

$$C_{eq}V = V\,(C_1 + C_2 + C_3 + \dots)$$

Therefore, the equivalent capacitance (C_{eq}) for capacitors in parallel is given by:

$$C_{\text{eq}} = C_1 + C_2 + C_3 + \dots$$

The equivalent capacitance in a parallel connection is always greater than the largest individual capacitance in the parallel combination.

Chapter 3

Electrokinetics

3.1 Electric Current

3.1.1 Definition

Electric current is the net movement of free electrons through a conductor. As illustrated in Figure 1(a), in the absence of an electric field, free electrons move randomly, resulting in no net directional flow. However, when a potential difference is applied—such as when a wire is connected to a battery (see Figure 1(b))— the electric field directs the motion of these electrons, producing a net directional flow that constitutes an electric current.

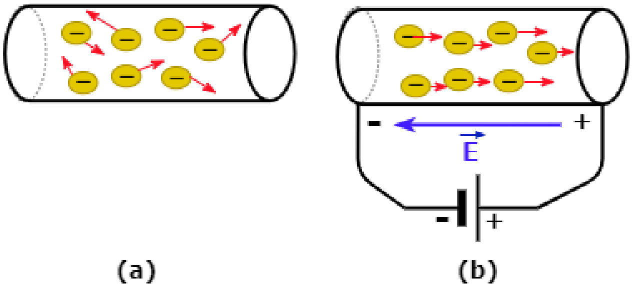


Figure 3.1: (a) Random motion in the absence of an electric field; (b) Directed flow under an applied potential difference.

The average electrical current, I_{ave}, quantifies the rate at which charge passes through a given cross-sectional area of a conductor over a specified time interval:

$$I_{\text{ave}} = \frac{\Delta Q}{\Delta t} \quad \left(\text{Unit:} \frac{\text{Coulombs (C)}}{\text{Seconds (S)}} = \text{Amperes (A)}\right)$$

The instantaneous electrical current, or simply the ***electric current***, is given by:

$$I = \frac{dQ}{dt}$$

Example: Consider a charge function given by:

$$Q(t) = t^2 + 5t - 3$$

To find the instantaneous current at $t = 2$ seconds, we differentiate $Q(t)$ with respect to t:

$$I = \frac{dQ}{dt} = \frac{d}{dt}(t^2 + 5t - 3) = 2t + 5 = 9\,A$$

Thus, the current at $t = 2$ seconds is 9 A.

3.1.2 Current Density

Current density is a vector quantity that represents the electric current per unit cross-sectional area of a conductor. It is expressed as:

$$J = \frac{I}{A}$$

where:

- J is the current density in amperes per square meter (A/m^2)
- I is the electric current in amperes (A)
- A is the cross-sectional area in square meters (m^2)

3.1.3 Direction of Electric Current

The direction of current can be described in two different ways:

3.1.3.1 Conventional Current Direction

Conventional Current Direction: By historical convention, electric current is considered to flow from the positive terminal of a power source to the negative terminal. This convention was established before the discovery of electrons and is still widely used in circuit analysis.

3.1.3.2 Electron Flow Direction

In reality, electric current in conductors (such as metals) is due to the movement of free electrons, which carry negative charge. These electrons move from the negative terminal to the positive terminal of a power source.

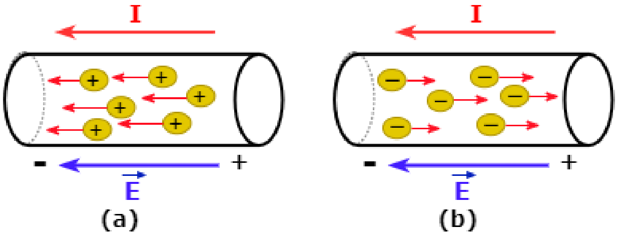


Figure 3.2: (a) Motion of positive charges in the direction of the electric field, corresponding to the conventional current direction; (b) Motion of negative charges opposite to the electric field direction. The conventional current flows in the opposite direction of negative charge movement, commonly referred to as electron flow.

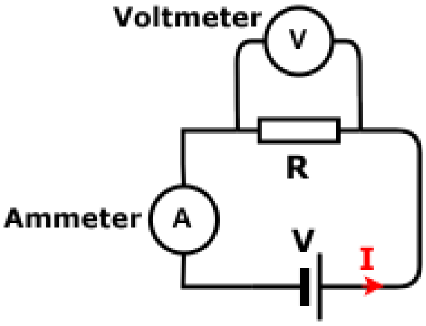


Figure 3.3: Simple electric circuit for demonstrating Ohm's law.

3.2 Ohm's Law

Consider a simple electrical circuit consisting of a voltage source (battery) and a resistor connected in series, as illustrated in Figure 3.

The measured voltage and current values are presented in the following table:

Voltage (V)	Current (I)
0	0
V_1	I_1
V_2	I_2
V_3	I_3
V_4	I_4

Table 3.1: Measured voltage and current values.

Analysis of the data reveals that the ratio V/I remains constant, within experimental uncertainty, for all recorded measurements.

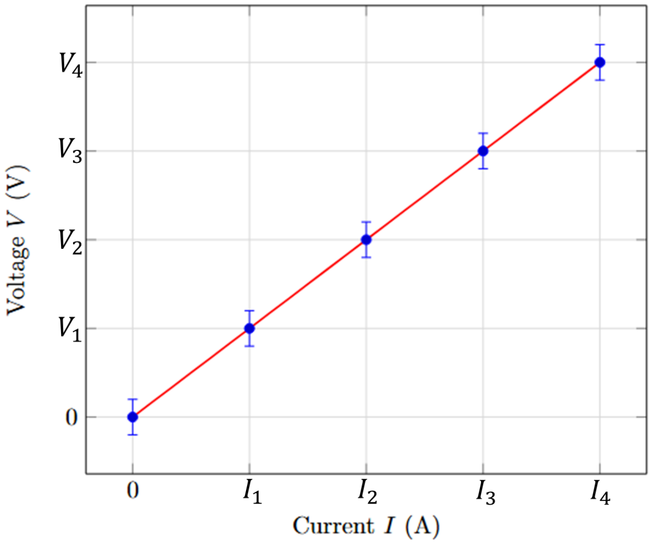


Figure 3.4: Relationship between voltage and current in a resistor.

Figure 4 illustrates the direct proportionality between voltage (V) and current (I). This relationship is linear and can be expressed as:

$$V = RI$$

where R represents the slope of the line and corresponds to the electrical resistance of the resistor. This linear relationship between voltage and current is known as ***Ohm's Law***.

3.3 Electrical Power

Electrical power is defined as the rate at which electrical energy is transferred or converted per unit time. It quantifies the amount of energy transferred or converted per unit time, regardless of its form (work, heat, light, mechanical motion, etc.). Mathematically, electrical power can be expressed as:

$$P = \frac{E}{t}$$

where:

- P is the power (in watts, W),
- E is the energy (in joules, J),
- t is the time (in seconds, S).

3.3.1 Power in Electrical Circuits

Electrical circuits involve two primary power types: generated power (from sources like generators) and dissipated power (in elements such as resistors)

3.3.1.1 Generated Power

As illustrated in Figure 5, the generator provides a voltage V and delivers a current I. The generated power is then given by:

$$P_{\text{generated}} = VI$$

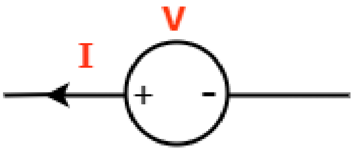


Figure 3.5: Generator delivering voltage V and current I.

3.3.1.2 Power Dissipation in Resistors

Resistive elements convert electrical energy into heat via the ***Joule heating effect***. Applying Ohm's Law ($V = IR$), power dissipation in a resistor can be expressed in the following equivalent forms:

$$P = VI = I^2R = \frac{V^2}{R}$$

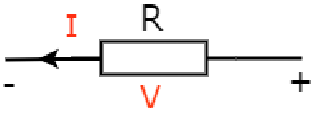


Figure 3.6: Resistor subjected to voltage V and current I.

- $P = VI$: The general expression applicable to all electrical elements.
- $P = I^2R$: Useful when the current I and resistance R are known.
- $P = \frac{V^2}{R}$: Useful when the voltage V and resistance R are known.

3.4 Basic Circuit Elements

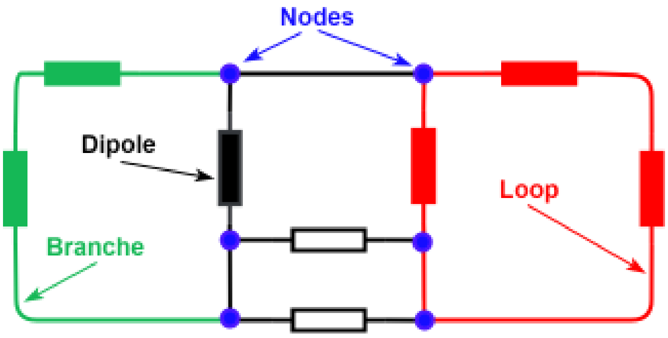


Figure 3.7: Basic circuit elements.

Figure 7 illustrates fundamental circuit concepts by showing the connections between electrical components. It highlights key elements used in circuit analysis:

- **Nodes** are points where two or more circuit elements meet, allowing current to flow between them. The blue-labeled points in the figure represents nodes.
- **Branches** are paths that connect two nodes and may contain one or more electrical components. The green-labeled sections show these branches, which guide the flow of current.
- **Loops**, marked in red, are closed paths within the circuit.
- **Dipoles** are two-terminal components such as resistors, capacitors, or voltage sources. They either supply, store, or dissipate energy in the circuit. The yellow-highlighted dipole in the figure represents such an element.

- **Wires** are conductive paths that connect different circuit elements. Their resistance is considered negligible compared to the resistance of other two-terminal components.

Note: When analyzing two-terminal components, it is essential to distinguish between two conventions for voltage and current reference directions:

- **Generator (Source) (Figure 8a):** A component acts as a generator (or source) when the reference direction of current is from the positive (higher potential) to the negative (lower potential) terminal. In this case, voltage and current have the same direction across the terminals, indicating that the component supplies power to the circuit.
- **Receiver (Load Element) (Figure 8b):** A component acts as a receiver (or load) when the reference direction of current is from the positive to the negative terminal. However, voltage and current are opposite in direction across the terminals, meaning the component absorbs power from the circuit.

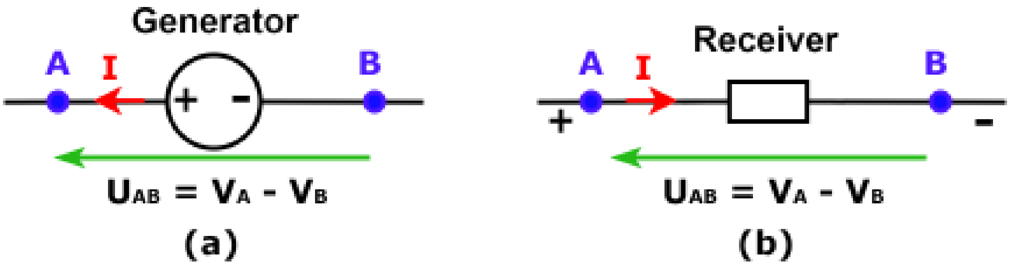


Figure 3.8: (a) A generator; (b) A receiver.

3.5 Electromotive Force (EMF)

Electromotive force (EMF) is the energy supplied per unit charge by a source to drive current in a circuit. It is measured in volts (V) and denoted by ε.

3.5.1 Sources of Electromotive Force (EMF)

Several devices generate EMF through different processes:

- **Batteries:** Convert chemical energy into electrical energy through electrochemical reactions.
- **Generators:** Produce EMF using mechanical motion and magnetic fields based on electromagnetic induction.
- **Solar Cells:** Generate electrical energy from sunlight through the photovoltaic effect.
- **Fuel Cells:** Convert chemical energy directly into electrical energy through continuous electrochemical reactions.

- **Thermocouples:** Generate voltage due to temperature differences at the junction of two dissimilar metals, known as the thermoelectric effect.

3.5.2 The Electromotive Force (EMF) of a Battery

An ideal battery (Figure 9a) maintains a constant EMF, meaning its terminal voltage V_t is always equal to ε, regardless of the current flowing. However, real batteries have internal resistance (r) (Figure 9b), which affects their behavior during charge and discharge.

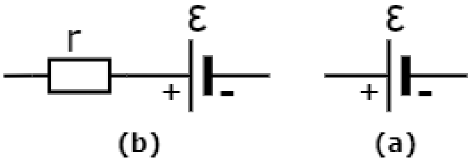


Figure 3.9: (a) Ideal battery; (b) Real battery.

When a battery supplies energy (Figure 10a) to an external circuit (discharges), the current flows from the positive terminal. The battery's internal resistance causes a voltage drop within the battery itself, resulting in a terminal voltage (V_t) that is lower than the EMF (ε). This relationship is expressed as:

$$V_t = \varepsilon - rI$$

During charging (when energy is stored in the battery (Figure 10b)), current flows into the positive terminal from an external source. The internal resistance opposes this incoming current, causing the terminal voltage to be higher than the EMF:

$$V_t = \varepsilon + rI$$

This additional voltage compensates for the internal resistance and allows energy to be stored in the battery.

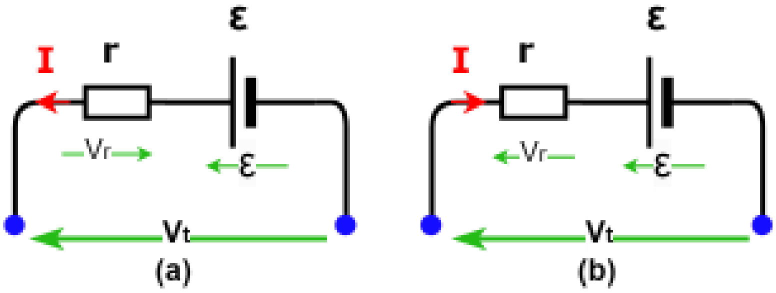


Figure 3.10: (a) Battery during discharging; (b) Battery during charging.

In both cases, the internal resistance plays a key role in determining the actual voltage observed at the battery terminals.

3.6 Connections of Resistors in Electric Circuits

Resistors in electric circuits can be connected in various configurations to achieve desired circuit characteristics, such as controlling current flow or setting specific voltage drops. The two fundamental connection types are series and parallel:

3.6.1 Series Connection

When resistors are connected in series, they are arranged sequentially, forming a single path for current flow. In this configuration, the current (I) remains the same across all resistors, while the voltage is divided among them, denoted as $V_1, V_2, V_3, \ldots$.

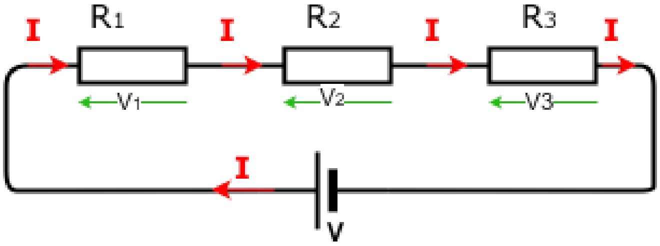


Figure 3.11: Resistors in series connection.

The total voltage (V) across the series combination is the sum of the individual voltage drops:

$$V = V_1 + V_2 + V_3 + \ldots$$

Using Ohm's Law $(V = IR)$:

$$IR_s = I(R_1 + R_2 + R_3 + \ldots) \implies R_s = R_1 + R_2 + R_3 + \ldots$$

The total resistance in a series circuit is always greater than the largest individual resistance in the series.

3.6.2 Parallel Connection

In a parallel connection, all resistors share the same voltage, but the current through each resistor varies depending on its resistance.

The total current (I) in the parallel network is the sum of the individual currents:

$$I = I_1 + I_2 + I_3 + \ldots$$

Using Ohm's Law:

$$\frac{V}{R_p} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} + \ldots \implies \frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$$

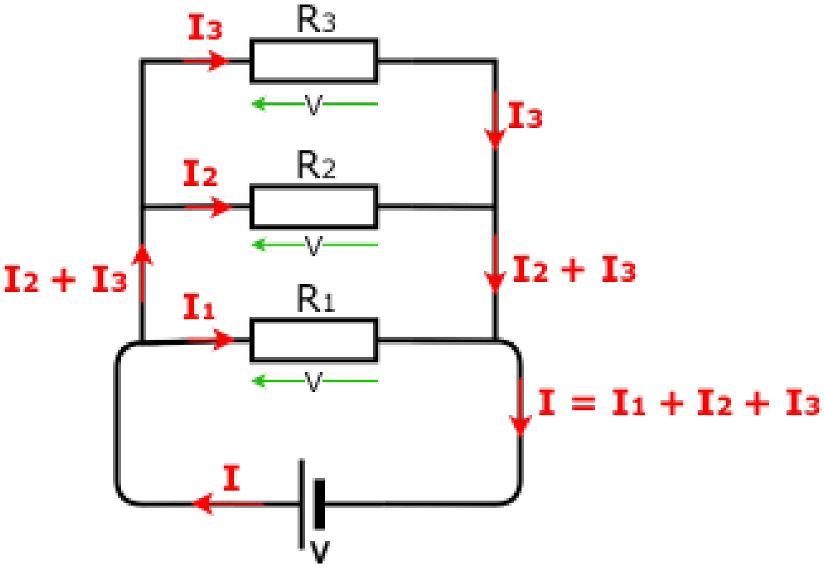


Figure 3.12: Resistors in parallel connection.

The equivalent resistance in a parallel connection is always smaller than the smallest individual resistance in the network.

3.7 Kirchhoff's Laws

In electrical circuits, determining unknown currents and voltages is crucial for designing and analyzing circuits effectively. However, in complex circuits with multiple branches, loops, and nodes, Ohm's Law alone is not sufficient for comprehensive analysis. Kirchhoff's Laws provide fundamental principles for analyzing such circuits. These laws are based on two fundamental principles:

3.7.1 Kirchhoff's Current Law (KCL)

Kirchhoff's Current Law states that the total current entering a junction is equal to the total current leaving the junction. Mathematically, it is expressed as:

$$\sum I_{\text{in}} = \sum I_{\text{out}}$$

3.7.2 Kirchhoff's Voltage Law (KVL)

Kirchhoff's Voltage Law states that the sum of all voltage drops and rises around any closed loop in a circuit is zero. Mathematically, it is given by:

$$\sum V = 0$$

Example 01:

Consider the circuit in Figure 13. Calculate I_1, I_2, and I_3 using Kirchhoff's Laws.

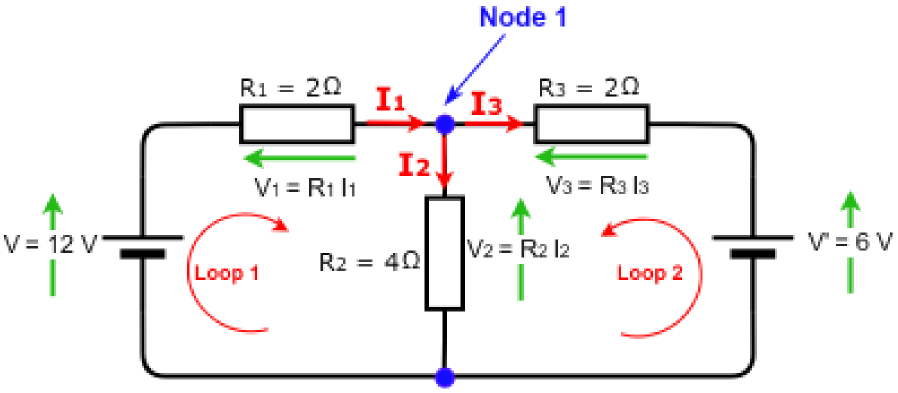


Figure 3.13: Current and voltage analysis in an electrical circuit.

Solution

Before solving the example, we first define:

- ***Current directions:*** We assume current directions arbitrarily for each branch. If our assumption is wrong, the calculated value will be negative, meaning the actual direction is opposite.
- ***Potential Differences:*** In a voltage source, the potential is higher at the positive terminal and lower at the negative terminal. In passive components like resistors, the potential drops in the direction of current flow, meaning the point where current enters has a higher potential than where it exits.
- ***Loop Orientation:*** The direction of loops in circuit analysis is chosen arbitrarily, whether clockwise or counterclockwise. The final results remain unchanged as long as Kirchhoff's Voltage Law (KVL) is applied consistently throughout the circuit.

Since we have three unknowns (I_1, I_2, I_3), we need to establish three independent equations using Kirchhoff's Laws.

Step 1: Kirchhoff's Current Law (KCL) at Node 1

At Node 1, three currents meet: I_1 enters; I_2 and I_3 leave, thus:

$$I_1 = I_2 + I_3 \tag{3.1}$$

Step 2: Kirchhoff's Voltage Law (KVL) from Loop 1

From Loop 1 (left loop), we apply Kirchhoff's Voltage Law:

$$V - V_1 - V_2 = 0$$

Using Ohm's Law ($V = IR$):

$$15 - R_1 I_1 - R_2 I_2 = 0 \tag{3.2}$$

Step 3: Kirchhoff's Voltage Law (KVL) from Loop 2

From Loop 2 (right loop):

$$V' + V_3 - V_2 = 0$$

Substituting $V = IR$:

$$5 + R_3 I_3 - R_2 I_2 = 0 \tag{3.3}$$

Substituting Equation (1) into Equation (2):

$$12 - 2I_2 - 2I_3 - 4I_2 = 0$$

$$12 - 6I_2 - 2I_3 = 0 \tag{3.4}$$

Adding Equations (4) and (3):

$$\begin{aligned}(12 - 6I_2 - 2I_3) + (6 + 2I_3 - 4I_2) &= 0\\ 12 - 6I_2 - 2I_3 + 6 + 2I_3 - 4I_2 &= 0\\ I_2 &= 1.8A\end{aligned}$$

Substituting $I_2 = 1.8A$ into Equation (3):

$$\begin{aligned}6 + 2I_3 - 4(1.8) &= 0\\ I_3 &= 0.6A\end{aligned}$$

Using Equation (1):

$$\begin{aligned}&I_1 = I_2 + I_3 = 1.8 + 0.6\\ &I_1 = 2.4A\end{aligned}$$

3.8 Thevenin's Theorem

Thevenin's Theorem is a fundamental and powerful principle in circuit analysis that simplifies the study of complex linear circuits (circuits containing elements where voltage and current have a direct proportional relationship, such as resistors). It is particularly useful when analyzing a specific portion of a circuit, often called the ***load***, without requiring the solution of the entire circuit.

The theorem states that any linear circuit, regardless of its complexity, can be replaced by a simpler equivalent circuit containing only two elements: a ***Thévenin voltage*** ($V_{\mathbf{th}}$) and a ***Thévenin resistance*** ($R_{\mathbf{th}}$).

Once the Thevenin equivalent circuit is determined, the current through the load resistor (R_L) can be easily calculated using Ohm's Law:

$$I = \frac{V_{\text{th}}}{R_{\text{th}} + R_L}$$

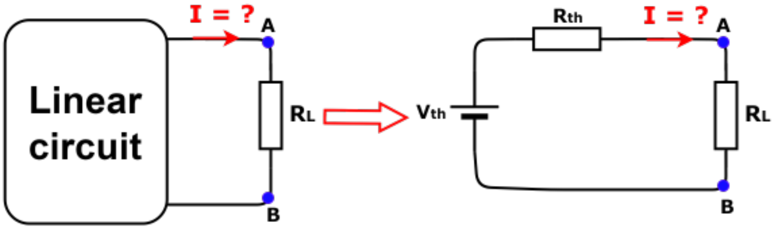


Figure 3.14: Thevenin equivalent circuit with load resistor.

Example 02:

Find the Thevenin equivalent circuit for the given circuit (Figure 15) and determine the current through the load resistor R_L.

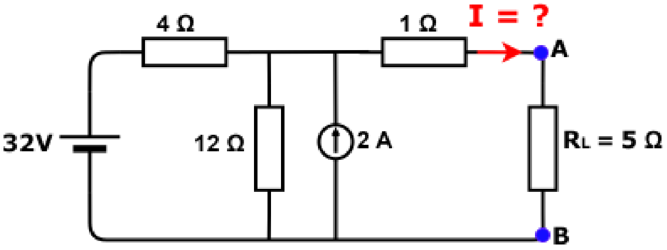


Figure 3.15: A circuit for Thevenin's theorem analysis.

Solution

Step 1: Remove the Load Resistor R_L

To determine the Thevenin equivalent circuit, we first remove R_L from the circuit, leaving two open terminals where R_L was previously connected.

Step 2: Find Thevenin Resistance (R_{th})

To find R_{th}, we deactivate all independent sources:

- Short-circuit the 32V voltage source
- Open-circuit the 2A current source

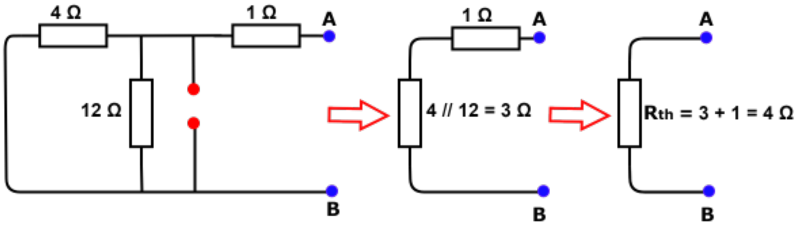


Figure 3.16: Determination of R_{th}.

The 4Ω resistor is connected in parallel with the 12Ω resistor:

$$R_{\text{eq}} = \frac{(12)(4)}{12+4} = \frac{48}{16} = 3\Omega$$

This equivalent resistance is in series with the 1Ω resistor:

$$R_{\text{th}} = 3 + 1 = 4\Omega$$

Step 3: Find Thevenin Voltage (V_{th})

To find V_{th}, we calculate the open-circuit voltage across the terminals where R_L was removed.

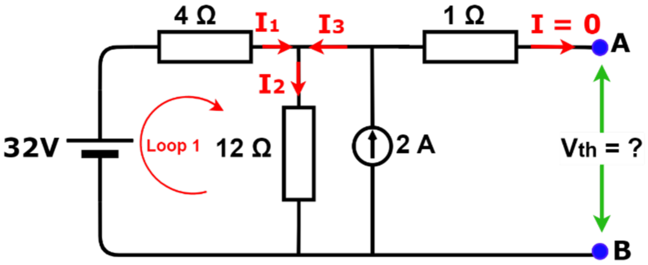


Figure 3.17: Determination of V_{th}.

Applying Kirchhoff's Current Law (KCL):

$$I_2 = I_1 + I_3$$

Since $I_3 = 2A$, we can express:

$$I_2 = I_1 + 2$$

Applying KVL to Loop 1:

$$32 - (4 \cdot I_1) - (12 \cdot I_2) = 0$$

Substituting $I_2 = I_1 + 2$:

$$32 - (4 \cdot I_1) - (12 \cdot (I_1 + 2)) = 0$$

Thus:

$$I_1 = 0.5A$$

Now, we substitute $I_1 = 0.5A$ into $I_2 = I_1 + 2$:

$$I_2 = 0.5 + 2 = 2.5A$$

The Thevenin voltage V_{th} is the voltage across the open terminal at the right side. This is simply the voltage across the 12 resistor. Using Ohm's Law:

$$V_{\text{th}} = I_2 \times 12 = 2.5 \times 12 = 30V$$

Now, the entire circuit can be replaced by its Thevenin equivalent circuit, as shown in Figure 18.

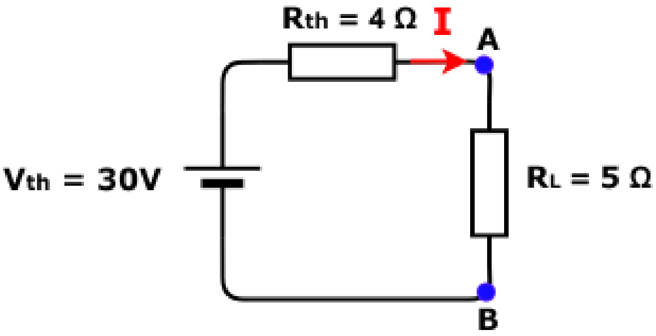


Figure 3.18: Thevenin equivalent circuit.

The current through R_L can be found using Ohm's law:

$$I = \frac{V_{\text{th}}}{R_{\text{th}} + R_L} = \frac{30}{4 + 5} = 3.33A.$$

3.9 Norton's Theorem

Norton's theorem states that any linear circuit, regardless of its complexity, can be replaced by a simpler equivalent circuit containing only two elements: a Norton current (I_N) and a Norton resistance (R_N) (Figure 19).

Once the Norton equivalent circuit is determined, the current through the load resistor (R_L) can be easily calculated using the current division rule:

$$I = I_N \times \frac{R_N}{R_N + R_L}$$

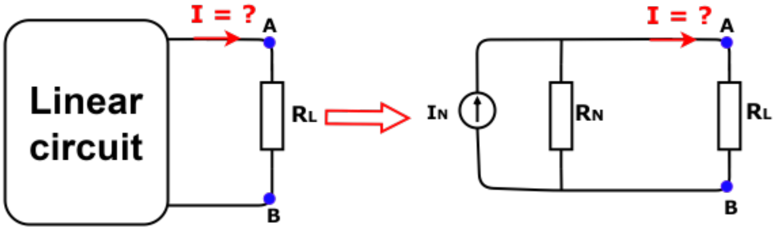


Figure 3.19: Norton equivalent circuit with load resistor.

Example 03:

Find the Norton equivalent circuit for the given circuit (Figure 20) and determine the current through the load resistor R_L.

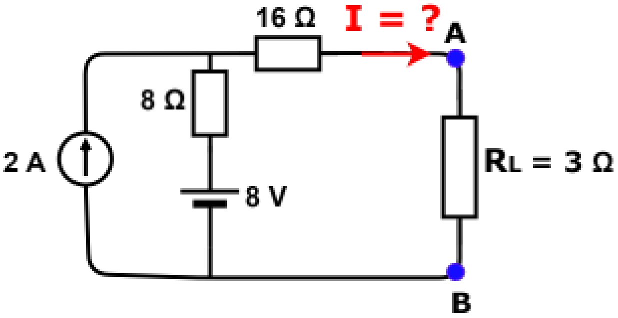


Figure 3.20: A circuit for Norton's theorem analysis.

Solution

Step 1: Remove the Load Resistor R_L

To determine the Norton equivalent circuit, we first remove R_L from the circuit, leaving two open terminals where R_L was previously connected.

Step 2: Find Norton Resistance (R_N)

To find R_N, we deactivate all independent sources:

1. Short-circuit the 32V voltage source.
2. Open-circuit the 2A current source.

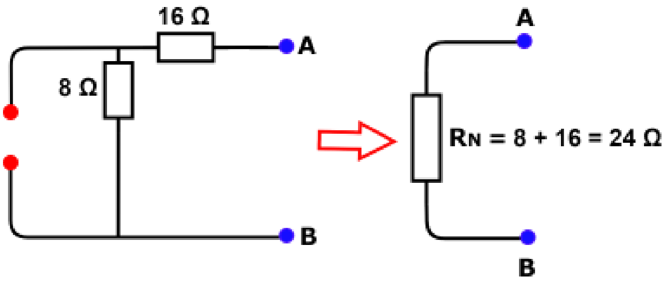


Figure 3.21: Determination of R_{N}.

The 8 resistor is in series with the 16 resistor:

$$R_N = 8 + 16 = 24\Omega$$

Thus, the Norton resistance is:

$$R_N = 24\Omega$$

Step 3: Find Norton Current (I_N)

To determine the Norton current (I_{N}), the load (R_{L}) is removed from the circuit, and a short circuit is placed across the terminals where the load was connected. I_{N} is the current that flows through this short circuit.

Applying Kirchhoff's Voltage Law (KVL) to Loop 1, the voltage equation is formulated as:

$$8 + 8I_2 - 16I_N = 0$$

Using Kirchhoff's Current Law (KCL) at Node 1, we establish the current relationship:

$$I_1 = I_2 + I_N \implies I_2 = 2 - I_N$$

Replacing I_2 in the voltage equation:

$$8 + 8(2 - I_N) - 16I_N = 0$$

Expanding the terms:

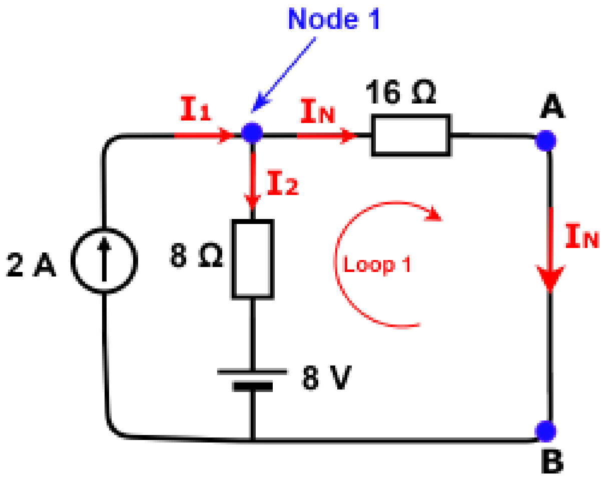


Figure 3.22: Determination of I_N.

$$8 + 16 - 8I_N - 16I_N = 0$$

$$24 - 24I_N = 0$$

Therefore:

$$I_N = \frac{24}{24} = 1A$$

The circuit can now be represented by its Norton equivalent, as shown in Figure 23.

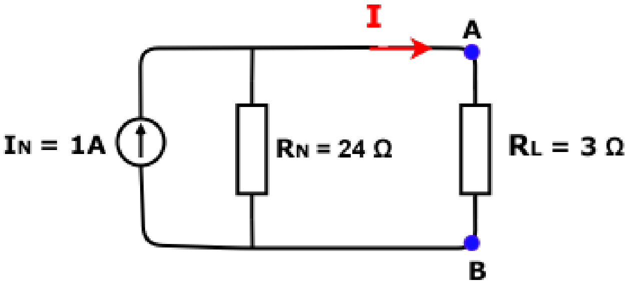


Figure 3.23: Norton equivalent circuit.

The current flowing through R_L is determined using current division:

$$I = I_N \times \frac{R_N}{R_N + R_L} = 1 \times \frac{24}{24+3} = 0.89A.$$

Chapter 4

Magnetostatics

4.1 Introduction

Magnetostatics is the study of static magnetic fields generated by steady (constant) currents. It is the magnetic counterpart of electrostatics, which examines electric fields produced by stationary charges. These static magnetic fields exert forces on moving charges and other current-carrying conductors.

4.2 Magnetostatic Forces

Magnetostatic forces arise from the interaction between magnetic fields and either steady currents or moving charges. The two primary forces in magnetostatics are:

4.2.1 Magnetic Force on a Moving Electric Charge (Lorentz Force)

At the end of the 19th century, the Dutch physicist Hendrik Lorentz formulated the equation describing the force $\vec{F}$ acting on a point charge q moving with velocity $\vec{v}$ in the presence of both an electric field $\vec{E}$ and a magnetic field $\vec{B}$:

$$\vec{F} = \vec{F}_E + \vec{F}_B = q\vec{E} + q(\vec{v} \times \vec{B}) = q(\vec{E} + \vec{v} \times \vec{B})$$

where:

- $\vec{F}_E$ is the electric force;
- $\vec{F}_B$ is the magnetic force;
- $\times$ represents the vector cross product.

Special Case: Presence of Only a Magnetic Field

If there is no electric field ($\vec{E} = 0$), the Lorentz force simplifies to:

$$\vec{F} = \vec{F}_B = q(\vec{v} \times \vec{B})$$

The magnitude of the magnetic force in this case is given by:

$$F = qvB\sin(\theta)$$

where θ is the angle between $\vec{v}$ and $\vec{B}$.

Properties of the Magnetic Force

- The magnetic force $\vec{F}_B$ is always perpendicular to both the velocity vector $\vec{v}$ and the magnetic field $\vec{B}$.
- The direction of the force is determined using the **right-hand rule with three fingers**:

  - **For a positive charge** ($q > 0$):
    - **Index finger** → Points in the direction of the velocity $\vec{v}$.
    - **Middle finger** → Points in the direction of the magnetic field $\vec{B}$.
    - **Thumb** → Points in the direction of the magnetic force $\vec{F}_B$.

$$(\text{Index: } \vec{v},\ \text{Middle: } \vec{B},\ \text{Thumb: } \vec{F}_B)$$

  - **For a negative charge** ($q < 0$):
    - First, apply the right-hand rule as if the charge were positive.
    - Then, reverse the direction of the force $\vec{F}_B$.

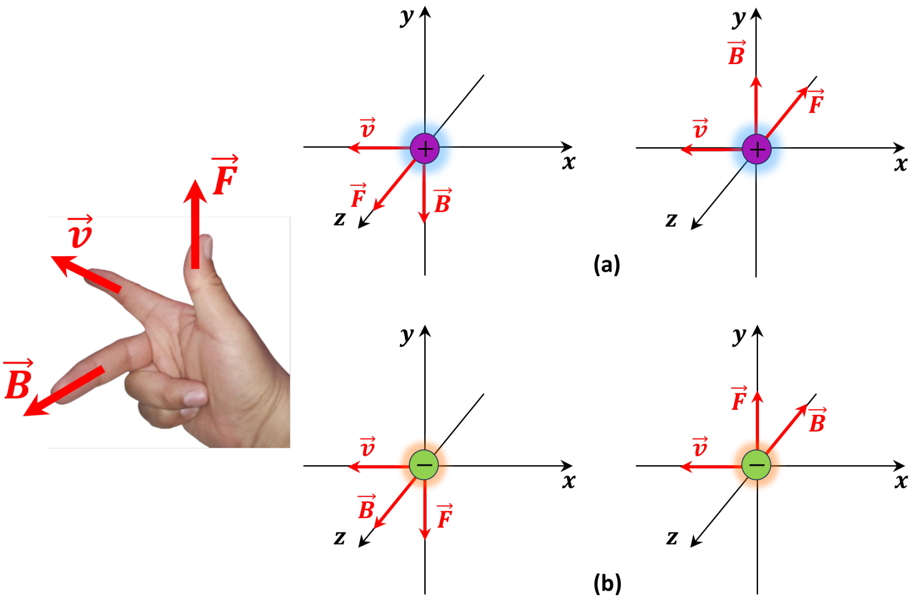


Figure 4.1: Right-Hand Rule for the magnetic force:
(a) Positive charge; (b) Negative charge.

- The magnetic force reaches its maximum value when $\vec{v}$ and $\vec{B}$ are perpendicular ($\theta = \frac{\pi}{2}$), since the cross product is maximized when the vectors are orthogonal. In this case, the force simplifies to:

$$F_B = qvB$$

- The magnetic force is zero ($F = 0$) in the following cases:
  - When the velocity is zero ($v = 0$), meaning the charge is at rest.
  - When the velocity and magnetic field are parallel or antiparallel ($\theta = 0°$ or $\theta = 180°$), since $\sin(0) = \sin(180) = 0$, making the cross product zero.

4.2.2 Magnetic Force on a Current-Carrying Conductor (Laplace Force)

The Laplace force describes the magnetic force acting on a current-carrying conductor placed in a magnetic field. It is the macroscopic equivalent of the Lorentz force, which acts on individual moving charges.

When an electric current flows through a conductor inside a magnetic field, each moving charge within the conductor experiences a Lorentz force. The total force on the conductor is the sum of the forces on all individual charges, leading to the Laplace force.

Consider a conductor carrying a steady current I in the presence of a uniform magnetic field $\vec{B}$. The force $\vec{F}$ acting on a small segment of the conductor of length $d\vec{\ell}$ is given by:

$$d\vec{F} = I(d\vec{\ell} \times \vec{B})$$

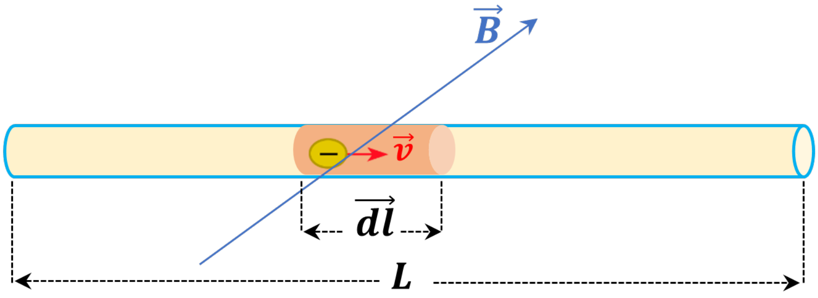


Figure 4.2: Current-carrying conductor in a magnetic field.

where:

- $d\vec{F}$ is the magnetic force acting on the small segment of the conductor;
- $d\vec{\ell}$ is a small length element of the conductor, which is a vector along the direction of the current;
- $\times$ represents the vector cross product;

Force on a Finite-Length Conductor

For a conductor of finite length carrying a current in a uniform magnetic field, the total force is obtained by integrating along the conductor:

$$\vec{F} = I \int (d\vec{\ell} \times \vec{B})$$

Since the integral sums up all small displacement elements $d\vec{\ell}$, we obtain:

$$\int d\vec{\ell} \times \vec{B} = \left(\int d\vec{\ell} \right) \times \vec{B} = \vec{L} \times \vec{B}$$

Substituting this result:

$$\vec{F} = I(\vec{L} \times \vec{B})$$

where $\vec{L}$ is the length vector of the conductor, pointing in the direction of the current.

The magnitude of the force is given by:

$$F = ILB\sin(\theta)$$

where θ is the angle between the length vector $\vec{L}$ (aligned with the current) and the magnetic field $\vec{B}$.

Properties of the Laplace Force

- The Laplace force $\vec{F}$ is always perpendicular to both the current direction ($\vec{L}$) and the magnetic field ($\vec{B}$).
- The direction of the force is determined using the right-hand rule:
  - **Index finger** → Points in the direction of the current (I or $\vec{L}$).
  - **Middle finger** → Points in the direction of the magnetic field ($\vec{B}$).
  - **Thumb** → Points in the direction of the force ($\vec{F}$).
- The Laplace force reaches its maximum value when $\vec{L}$ and $\vec{B}$ are perpendicular ($\theta = \frac{\pi}{2}$):

$$F = ILB$$

- The Laplace force is zero ($F = 0$) in the following cases:
  - When there is no current ($I = 0$).
  - When the length vector $\vec{L}$ (current direction) and the magnetic field $\vec{B}$ are parallel or antiparallel ($\theta = 0°$ or $\theta = 180°$), since $\sin(0) = \sin(180) = 0$, making the cross product zero.

4.3 Magnetic Fields

A **magnetic field** is generated by the movement of **electric charges** (such as an **electric current**) or by the **intrinsic magnetic properties** of materials. Additionally, a **time-varying electric field** can also induce a magnetic field, as described by **Maxwell's equations**. However, in this chapter, the focus will be exclusively on the **magnetic field generated by electric currents**.

Magnetic fields, denoted by $\vec{B}$, describe the influence of **moving charges or currents** on other **moving charges or magnetized materials** in space. These fields are commonly **visualized using magnetic field lines**, which represent the **field's direction and strength**. The strength of a magnetic field is measured in **Tesla (T)**.

Magnetic Field Lines

Magnetic field lines form closed loops, unlike electric field lines. They provide a visual representation of the magnetic field's direction and strength. The magnetic field around a current-carrying wire decreases with distance. This is visualized by the increasing spacing of magnetic field lines, indicating a decrease in field strength farther from the wire, as shown in Figure 3(b). The right-hand rule is essential for determining the direction of the magnetic field due to currents (Figure 3(a)).

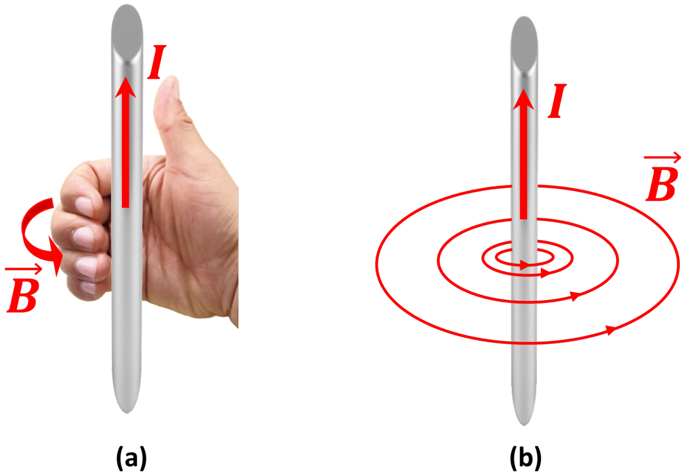


Figure 4.3: (a) Right-hand rule; (b) Magnetic field around a straight wire, forming circular loops.

4.4 Biot-Savart Law

The Biot-Savart law is a fundamental equation that describes the magnetic field generated by a small current element. It states that the infinitesimal contribution to the magnetic field $d\vec{B}$ at a point P, due to a current element $Id\vec{\ell}$, is given by:

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{\ell} \times \hat{r}}{r^2}$$

where:

- μ_0: Permeability of free space, with a value of $4\pi \times 10^{-7}$ $\mathrm{T \cdot m/A}$.
- $d\vec{\ell}$: Infinitesimal segment of the current-carrying conductor.
- r: Distance from the current element to the point where $\vec{B}$ is being calculated.
- $\hat{r}$: Unit vector pointing from the current element to the observation point.

This law applies to any current distribution, meaning that it can be used to compute the magnetic field of wires of **any shape**, such as straight, circular, or arbitrary paths.

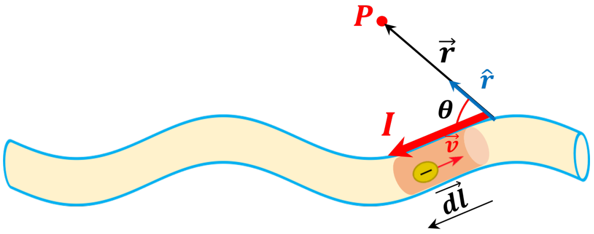


Figure 4.4: Illustration of the Biot-Savart Law: magnetic field due to a current element.

4.5 Magnetic Field Due to a Straight Current-Carrying Wire

Consider a straight wire carrying a current I. We want to determine the infinitesimal magnetic field contribution dB at a point P. To do this, we consider a perpendicular distance a from the wire as the distance between the observation point P and the wire. The distance from the current element to P is given by r.

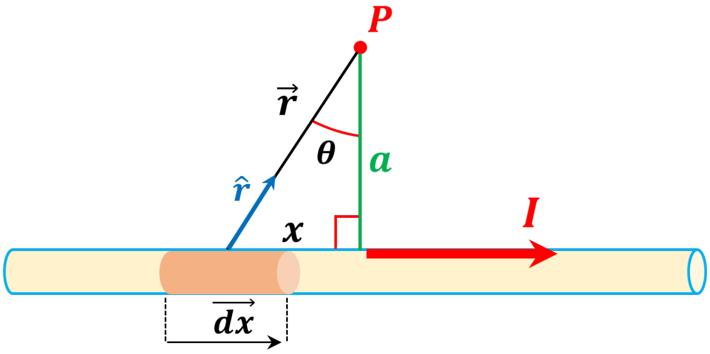


Figure 4.5: Magnetic field of a straight wire.

From Figure 5, the infinitesimal contribution to the magnetic field $d\vec{B}$ at a point P due to a current element is given by:

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I d\vec{x} \times \hat{r}}{r^2}$$

Since $d\vec{x} \times \hat{r}$ introduces a sine term, we rewrite it as:

$$dB = \frac{\mu_0 I}{4\pi}\frac{dx \sin(90^\circ - \theta)}{r^2} = \frac{\mu_0 I}{4\pi}\frac{dx \cos(\theta)}{r^2} \tag{4.1}$$

Using the geometry:

$$\cos\theta = \frac{a}{r} \Rightarrow r = \frac{a}{\cos\theta}$$

$$x = a\tan\theta \Rightarrow dx = a\sec^2\theta\, d\theta$$

Substituting these into the Equation 1:

$$dB = \frac{\mu_0 I}{4\pi}\frac{a\sec^2\theta\, d\theta\ \cos\theta\ \cos^2\theta}{a^2}$$

Since we recall that:

$$\sec^2\theta = \frac{1}{\cos^2\theta}$$

This simplifies to:

$$dB = \frac{\mu_0 I}{4\pi a}\cos\theta\, d\theta$$

Integrating from θ_1 to θ_2, we obtain the general expression for the magnetic field:

$$B = \frac{\mu_0 I}{4\pi a}\int_{\theta_1}^{\theta_2}\cos\theta\, d\theta = \frac{\mu_0 I}{4\pi a}[\sin\theta]_{\theta_1}^{\theta_2} = \frac{\mu_0 I}{4\pi a}(\sin\theta_2 - \sin\theta_1)$$

Depending on the length of the wire, we analyze the following cases:

Case 1: Magnetic Field of a Finite Wire

For a wire of finite length, we apply the limits of integration to get:

$$B = \frac{\mu_0 I}{4\pi a}(\sin\theta_2 - \sin\theta_1)$$

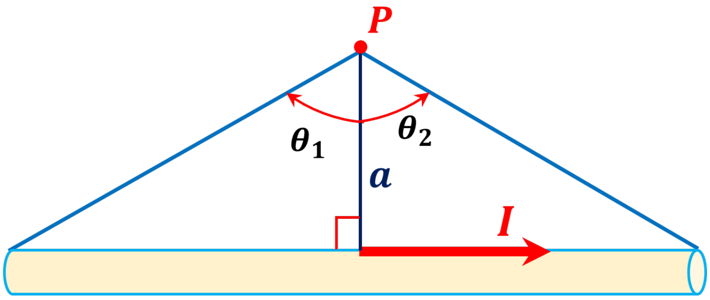


Figure 4.6: Magnetic field of a finite wire.

Case 2: Magnetic Field of a Semi-Infinite Wire

For a semi-infinite wire, we consider the limits of integration as $\theta_1 = 0°$ and $\theta_2 = 90°$. This represents a wire that extends infinitely in one direction.

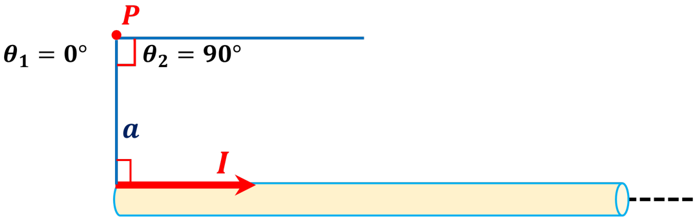


Figure 4.7: Magnetic field of a semi-infinite wire.

$$B = \frac{\mu_0 I}{4\pi a}(\sin 90° - \sin 0°) = \frac{\mu_0 I}{4\pi a}$$

Thus, the magnetic field due to a **semi-infinite** current-carrying wire is:

$$B = \frac{\mu_0 I}{4\pi a}$$

Case 3: Magnetic Field of an Infinite Wire

For an infinitely long wire, we consider the limits of integration as $\theta_1 = -90°$ and $\theta_2 = 90°$. This represents the wire extending infinitely in both directions.

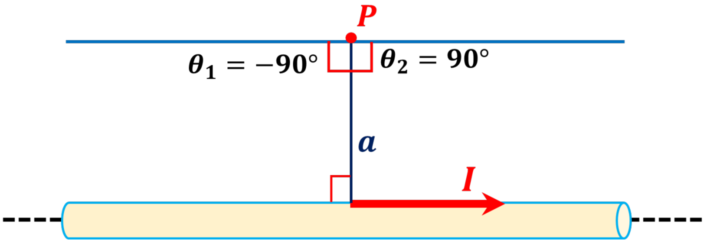


Figure 4.8: Magnetic field of an infinite wire.

$$B = \frac{\mu_0 I}{4\pi a}\left(\sin 90° - \sin(-90°)\right) = \frac{\mu_0 I}{4\pi a}(1-(-1)) = \frac{\mu_0 I}{2\pi a}$$

Therefore, the magnetic field due to an infinitely long straight wire is:

$$B = \frac{\mu_0 I}{2\pi a}$$

Note: The angle θ is measured from the perpendicular distance a (the shortest distance from the current-carrying wire to the observation point P). The sign convention for θ is defined as follows:

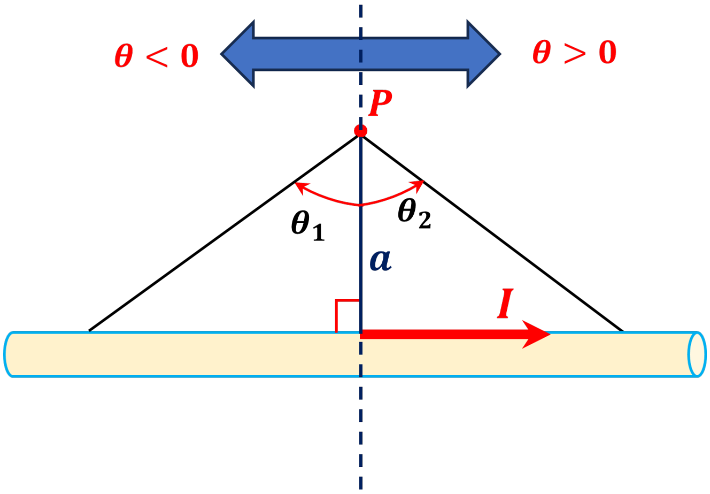


Figure 4.9: Sign convention for the angle θ.

- **Positive angle** $(\theta > 0)$: Measured in the direction of the current I, as illustrated by θ_2 in Figure 9.
- **Negative angle** $(\theta < 0)$: Measured opposite to the direction of the current I, as shown by θ_1 in Figure 9.

Series of Exercises

Series of Exercises 01

Exercise 01

Consider two point charges positioned along the x-axis as follows:

- A charge $q_1 = +3\mu C$ is located at $x_1 = 0\ cm$.
- A charge $q_2 = -2\mu C$ is positioned at $x_2 = 10\ cm$.

Determine:

1. The magnitude of the electrostatic force exerted by charge q_1 on charge q_2.
2. The magnitude of the electrostatic force exerted by charge q_2 on charge q_1.
3. The direction of the force acting on each charge.
4. The magnitude and direction of the electric field at the position of charge q_1 due to q_2.
5. The magnitude and direction of the electric field at the position of charge q_2 due to q_1.

Exercise 02

Consider two point charges positioned 0.2 meters apart. Charge q_1 is $+4.0 \times 10^{-9}\ C$ and charge q_2 is $-3.5 \times 10^{-9}\ C$. A point P is located at a distance of 0.1 meters from charge q_1 and 0.15 meters from charge q_2.

Determine the electric potential at point P due to both charges.

Exercise 03

Consider three charges, $q_1 = -1.5\,\mathrm{mC}$, $q_2 = 0.5\,\mathrm{mC}$, and $q_3 = 0.2\,\mathrm{mC}$, positioned as depicted in Figure 1, where the distance between q_1 and q_3 is $r_1 = 1.2\,\mathrm{m}$ and the distance between q_2 and q_3 is $r_2 = 0.5\,\mathrm{m}$.

- Calculate the resultant force acting on the charge q_3.
- Find the direction of the resultant force.

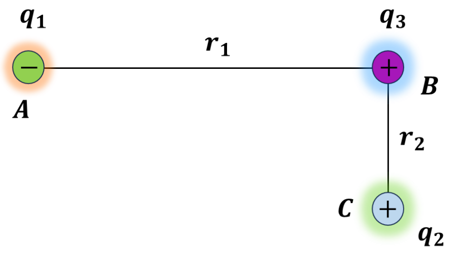


Figure 4.10: Arrangement of charges and separation distances.

Exercise 04

Consider two point charges, q_1 and q_2, placed along the x-axis. The charge $q_1 = 5\,\mathrm{C}$ is located at the origin, while $q_2 = -1\,\mathrm{C}$ is placed at a distance $d = 2\,\mathrm{m}$ along the positive x-axis. We seek to determine the resultant electric field at point P, located at a distance $h = 1\,\mathrm{m}$ along the positive y-axis from the origin.

Compute the electric field at point P due to both charges.

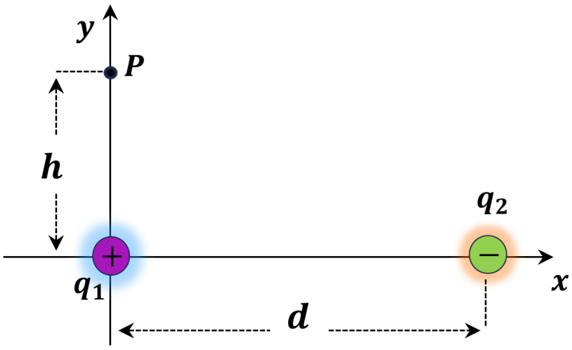


Figure 4.11: Arrangement of charges and separation distances.

Exercise 05

Determine the electric field and its direction at point P, as illustrated in Figure 3. Then, compute its magnitude for the given values:

$$q = 5 \times 10^{-6}\,\mathrm{C}, \quad r = 0.3\,\mathrm{m}$$

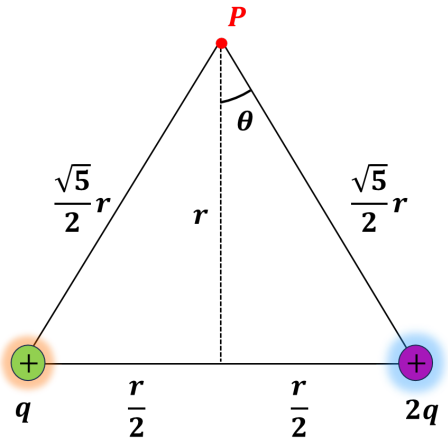
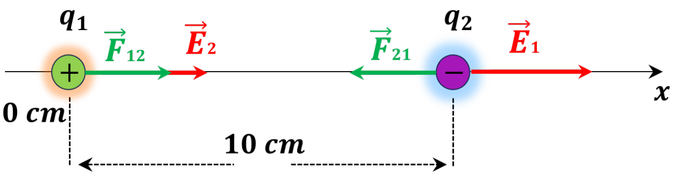


Figure 4.12: Arrangement of charges and separation distances.

Solution of Series 01

Solution of Exercise 01



Electrostatic Force (Coulomb's Law)

The electrostatic force between two point charges is given by Coulomb's Law:

$$F = k\frac{|q_1 q_2|}{r^2}$$

Substituting the given values:

$$F = 8.9876 \times 10^9 \times \frac{(3 \times 10^{-6})(2 \times 10^{-6})}{(0.1)^2} = 5.39\ N.$$

Since q_1 is positive and q_2 is negative, the force between them is attractive.

Magnitude of the Force Exerted by q_2 on q_1

By Newton's Third Law, the force exerted by q_2 on q_1 has the **same magnitude** as the force exerted by q_1 on q_2:

$$\vec{\mathbf{F}}_{12} = -\vec{\mathbf{F}}_{21}$$

Therefore:

$$F_{12} = F_{21} = 5.39\ N.$$

By Newton's Third Law, the forces are equal in magnitude but opposite in direction:

$$\mathbf{F_{12}} = -\mathbf{F_{21}}.$$

Electric Field at q_1 Due to q_2

The electric field produced by a point charge is given by:

$$E = k\frac{|q|}{r^2}.$$

For the field at q_1 due to q_2:

- Charge producing the field: $q_2 = -2 \times 10^{-6}\ C$.
- Distance: $r = 0.1\ m$.

$$E_2 = (8.9876 \times 10^9) \times \frac{2 \times 10^{-6}}{(0.1)^2} = 1.8 \times 10^6\ \text{N/C}.$$

Since q_2 is negative, the electric field at q_1 is directed toward q_2.

Electric Field at q_2 Due to q_1

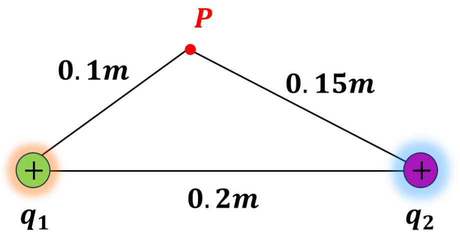
For the field at q_2 due to q_1:

- Charge producing the field: $q_1 = +3 \times 10^{-6}\ C$.
- Distance: $r = 0.1\ m$.

$$E_1 = (8.9876 \times 10^9) \times \frac{3 \times 10^{-6}}{(0.1)^2} = 2.7 \times 10^6\ \text{N/C}.$$

Since q_1 is positive, the electric field at q_2 is directed away from q_1.

Solution of Exercise 02



The electric potential V at a given point due to a single point charge is defined by the expression:

$$V = k\frac{q}{r}$$

Electric Potential Contribution from Charge q_1 at P

$$V_1 = \left(8.9876 \times 10^9\right) \times \frac{4.0 \times 10^{-9}}{0.1} = 359.5\ \text{V}$$

Electric Potential Contribution from Charge q_2 at P

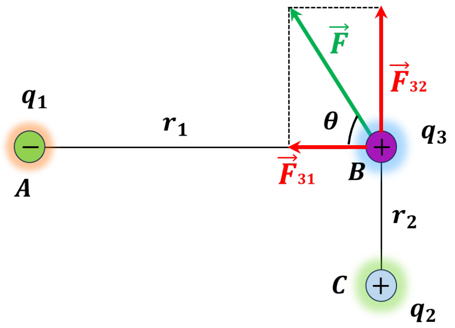
$$V_2 = \left(8.9876 \times 10^9\right) \times \frac{-3.5 \times 10^{-9}}{0.15} = -209.7\ \text{V}$$

Total Electric Potential at Point P

Since electric potential is a scalar quantity, the total potential at point P is obtained by summing the individual contributions:

$$V_P = V_1 + V_2 = 359.5 + (-209.7) = 149.8 \text{ V}.$$

Solution of Exercise 03



Step 1: Calculate the Force Between q_1 and q_3

Since q_1 and q_3 have opposite signs, the force will be attractive:

$$F_{31} = k\frac{|q_1 q_3|}{r_1^2} = \frac{8.9876 \times 10^9 \times 1.5 \times 10^{-3} \times 0.2 \times 10^{-3}}{(1.2)^2} = 1.87 \times 10^3 \,\text{N}$$

Step 2: Calculate the Force Between q_2 and q_3

Since q_2 and q_3 have opposite signs, the force will be repulsive:

$$F_{32} = k\frac{|q_2 q_3|}{r_2^2} = \frac{8.9876 \times 10^9 \times 0.5 \times 10^{-3} \times 0.2 \times 10^{-3}}{(0.5)^2} = 3.6 \ \times 10^3 \,\text{N}$$

Step 3: Calculate the Resultant Force F

$$F = \sqrt{F_{31}^2 + F_{32}^2} = \sqrt{(1.87 \times 10^3)^2 + (3.6 \times 10^3)^2} = 4.06 \times 10^3 \,\text{N}$$

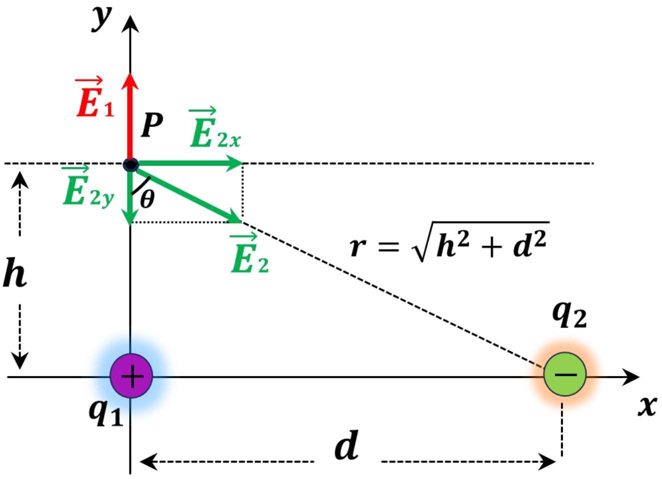
Step 4: Find the Direction of the Resultant Force

Calculate the angle θ between F and the line BA:

$$\tan\theta = \frac{F_{32}}{F_{31}} = 1.93 \Rightarrow \theta = \tan^{-1}(1.93) = 62.61^\circ$$

Therefore, the resultant force is directed at an angle of 62.61° with respect to the line BA, measured clockwise, as shown in the figure.

Solution of Exercise 04



The electric field at P due to each charge is calculated separately and then summed as vectors.

Electric Field Due to q_1

The electric field magnitude is:

$$E_1 = \frac{k|q_1|}{h^2} = \frac{8.9876 \times 10^9 \times 5}{1^2} = 44.9 \times 10^9 \, \mathrm{N/C}$$

Since q_1 is positive, the electric field points radially outward, which in this case is in the positive y-direction:

$$\vec{\mathbf{E}}_1 = 45 \times 10^9 \, \vec{\jmath}$$

Electric Field Due to q_2

The distance from q_2 to P is determined using the Pythagorean theorem:

$$r = \sqrt{h^2 + d^2} = \sqrt{1^2 + 2^2} = \sqrt{1 + 4} = \sqrt{5} \, \mathrm{m}$$

Since the electric field is a vector, we can resolve it into two components based on the angle θ:

$$E_{2x} = E_2 \sin(\theta) \quad \text{and} \quad E_{2y} = E_2 \cos(\theta)$$

We have:

$$\cos\theta = \frac{h}{r} = \frac{1}{\sqrt{5}}, \quad \sin\theta = \frac{d}{r} = \frac{2}{\sqrt{5}}$$

and

$$E_2 = \frac{k|q_2|}{r^2} = 8.9876 \times 10^9 \frac{1}{(\sqrt{5})^2} \approx 1.8 \times 10^9 \,\mathrm{N/C}$$

Thus, the components are:

$$E_{2x} = E_2 \sin\theta = 1.8 \times 10^9 \times \frac{2}{\sqrt{5}} = 1.61 \times 10^9 \,\mathrm{N/C},$$

$$E_{2y} = E_2 \cos\theta = 1.8 \times 10^9 \times \frac{1}{\sqrt{5}} = 0.81 \times 10^9 \,\mathrm{N/C}$$

Therefore, the vector form is:

$$\vec{E}_2 = 1.61 \times 10^9 \,\vec{i} - 0.81 \times 10^9 \,\vec{j}$$

Total Electric Field at P

Summing both contributions:

$$E_x = E_2x = 1.61 \times 10^9 \,\mathrm{N/C},$$
$$E_y = E_1 - E_2y = 45 \times 10^9 - 0.81 \times 10^9 = 44.19 \times 10^9 \,\mathrm{N/C}$$

Final vector form:

$$\vec{E} = 1.61 \times 10^9 \,\vec{i} + 44.19 \times 10^9 \,\vec{j}$$

Magnitude of the Electric Field

The magnitude of the electric field E at point P is:

$$E = \sqrt{E_x^2 + E_y^2} = \sqrt{(1.61 \times 10^9)^2 + (44.19 \times 10^9)^2} \approx 44.23 \times 10^9 \,\mathrm{N/C}$$

Solution of Exercise 05

Given values:

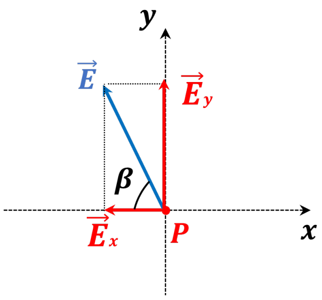
$$q = 5 \times 10^{-6} \text{ C}, \quad r = 0.3 \text{ m}$$

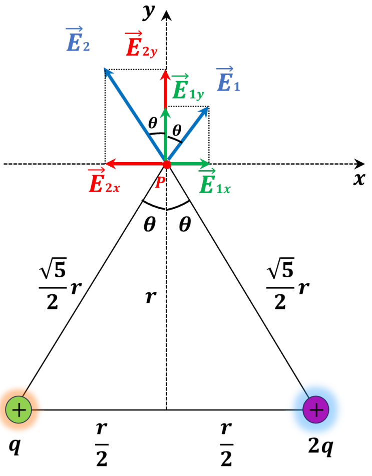
1. The electric field due to charge q at point P:

$$E_1 = \frac{kq}{\left(\frac{\sqrt{5}}{2} r\right)^2} = \frac{4kq}{5r^2}$$

Similarly, for charge $2q$:

$$E_2 = \frac{2kq}{\left(\frac{\sqrt{5}}{2} r\right)^2} = \frac{8kq}{5r^2} = 2E_1$$





2. From the diagram, the electric fields make an angle θ with the vertical axis. The components are:

For E_1:

$$E_{1x} = E_1 \sin\theta, \quad E_{1y} = E_1 \cos\theta$$

For E_2:

$$E_{2x} = E_2 \sin\theta, \quad E_{2y} = E_2 \cos\theta$$

Using the given trigonometric values:

$$\sin\theta = \frac{\frac{r}{2}}{\frac{\sqrt{5}}{2}r} = \frac{1}{\sqrt{5}} = 0.447, \quad \cos\theta = \frac{r}{\frac{\sqrt{5}}{2}r} = \frac{2}{\sqrt{5}} = 0.894$$

3. The total components are:

$$E_x = E_{1x} - E_{2x} = E_1 \sin\theta - E_2 \sin\theta = E_1 \sin\theta - 2E_1 \sin\theta = -E_1 \sin\theta$$

$$E_y = E_{1y} + E_{2y} = E_1 \cos\theta + E_2 \cos\theta = E_1 \cos\theta + 2E_1 \cos\theta = 3E_1 \cos\theta$$

Substituting values:

$$E_x = -E_1 \sin\theta = -3.99 \times 10^5 \times 0.447 = -1.78 \times 10^5 \text{ N/C}$$

$$E_y = 3E_1 \cos\theta = 3 \times 3.99 \times 10^5 \times 0.894 = 10.7 \times 10^5 \text{ N/C}$$

4. Magnitude and Direction of the Resultant Field

$$E = \sqrt{E_x^2 + E_y^2}$$

$$E = \sqrt{(1.78 \times 10^5)^2 + (10.7 \times 10^5)^2} = 10.85 \times 10^5 \text{ N/C}$$

The direction of the electric field is given by:

$$\tan\beta = \frac{E_y}{E_x} \approx 6$$

$$\beta = \tan^{-1}(6) = 80.54^\circ$$

Thus, the direction of **E** is 80.54° above the negative x-axis (or equivalently, 99.46° counterclockwise from the positive x-axis).

Series of Exercises 02

Exercise 01

Consider a solid spherical conductor of radius $R = 5\,\text{cm}$ that is in electrostatic equilibrium and carries a total charge of $Q = 10\,\mu\text{C}$.

1. Calculate the electric field at a distance $r = 3\,\text{cm}$ from the center of the sphere.
2. Calculate the electric field at $r = 5\,\text{cm}$ (on the surface) and at $r = 20\,\text{cm}$.
3. Draw the electric field lines around the conductor.

Exercise 02

A copper wire has a length of $10\,\text{m}$ and a cross-sectional area of $2 \times 10^{-6}\,\text{m}^2$. The resistivity of copper is $1.68 \times 10^{-8}\,\Omega \cdot \text{m}$.

1. Calculate the resistance of the wire.
2. If the wire is stretched to twice its original length while maintaining the same volume, what is its new resistance?

Exercise 03

A tungsten wire has a resistance of $10\,\Omega$ at 20°C. The temperature coefficient of resistivity for tungsten is $\alpha = 4.5 \times 10^{-3}\,{}^\circ\text{C}^{-1}$.

1. Calculate the resistance of the wire at 100°C.
2. If the wire is cooled to -10°C, what is its new resistance?

Exercise 04

1. **Capacitance and Charge Storage**
   - (a) Calculate the capacitance of a parallel-plate capacitor with vacuum between its plates, where each plate has an area of $A = 1.00\ \text{m}^2$ and the separation is $d = 1.00$ mm. Use $\varepsilon_0 = 8.854 \times 10^{-12}\,\text{C}^2/(\text{N} \cdot \text{m}^2)$.
   - (b) Determine the charge stored when a voltage of 3×10^3 V is applied.
2. **Capacitor Design**
   Determine the required plate area to achieve a capacitance of 1 nF, given a plate separation of 1.0 mm in a vacuum.

Exercise 05

Two conductors with net charges of $+10.0\,\mu$C and $-10.0\,\mu$C have a potential difference of 10.0 V between them.

1. Determine the capacitance of the system.
2. If the charges on each conductor are increased to $+100\,\mu$C and $-100\,\mu$C, calculate the new potential difference between them.

Exercise 06

Consider three capacitors with capacitances $C_1 = 4\,\mu$F, $C_2 = 6\,\mu$F, and $C_3 = 8\,\mu$F. These capacitors are connected to a 12 V power supply.

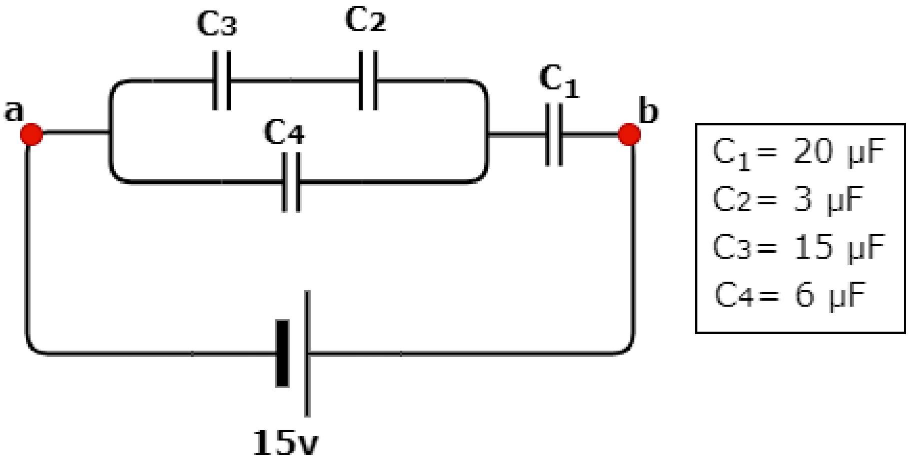
(a) Determine the equivalent capacitance, the charge on each capacitor, and the voltage across each capacitor when the capacitors are connected in series.

(b) Determine the equivalent capacitance, the charge on each capacitor, and the voltage across each capacitor when the capacitors are connected in parallel.

Exercise 07

Four capacitors are connected in a specific configuration, as shown in the Figure bellow.

1. Find the equivalent capacitance between points a and b.
2. Given that the potential difference $V_{ab} = 15.0$ V, calculate the charge on each capacitor in the network.



Solution of Series 02

Solution of Exercise 01

1. **Electric Field at $r = 3\,\text{cm}$**

For any point inside a conductor in electrostatic equilibrium ($r < R$), Gauss's Law states that the electric field must be zero:

$$E(3\,\text{cm}) = 0$$

2. Electric Field at $r = 5\,\text{cm}$ (on the surface) and $r = 20\,\text{cm}$ We apply Gauss's Law, which states:

$$\oint E \cdot dS = \frac{Q_{\text{enc}}}{\epsilon_0}$$

For a spherically symmetric charge distribution, we choose a Gaussian surface of radius r (where $r \geq R$). Since E is radial and constant over the sphere, we simplify:

$$E \oint dS = \frac{Q}{\epsilon_0}$$

$$E(4\pi r^2) = \frac{Q}{\epsilon_0} \implies E = \frac{Q}{4\pi\epsilon_0 r^2} = \frac{KQ}{r^2}$$

At $r = 5\,\text{cm}$ (on the surface):

$$E(5\,\text{cm}) = \frac{(8.9876 \times 10^9)(10 \times 10^{-6})}{(0.05)^2} = 3.6 \times 10^7\,\text{N/C}$$

At $r = 20\,\text{cm}$:

$$E(20\,\text{cm}) = \frac{(8.9876 \times 10^9)(10 \times 10^{-6})}{(0.2)^2} = 2.25 \times 10^6\,\text{N/C}$$

3. **Electric Field Lines**

- Inside the conductor ($r < R$): No field, so no lines.
- Outside the conductor ($r \geq R$): Lines radiate outward from the surface, perpendicular to the surface, since the sphere behaves as a point charge.

Solution of Exercise 02

1. The electrical resistance R of a wire is determined using the formula:

$$R = \frac{\rho L}{A}$$

Substituting the given values:

$$R_1 = \frac{(1.68 \times 10^{-8}) \times 10}{2 \times 10^{-6}} = 0.084\,\Omega$$

Thus, the initial resistance of the wire is $0.084\,\Omega$.

2. If the wire is stretched to twice its original length, its volume remains constant, implying:

$$L_1 A_1 = L_2 A_2$$

Since the final length is $L_2 = 2L_1 = 20\,\text{m}$, the new cross-sectional area is:

$$A_2 = \frac{A_1 L_1}{L_2} = \frac{(2 \times 10^{-6}) \times 10}{20} = 1 \times 10^{-6}\,\text{m}^2$$

The new resistance is given by:

$$R_2 = \frac{\rho L_2}{A_2} = \frac{(1.68 \times 10^{-8}) \times 20}{1 \times 10^{-6}} = 0.336\,\Omega$$

Thus, after stretching, the resistance of the wire increases to $0.336\,\Omega$, which is four times the original resistance.

Solution of Exercise 03

The resistance of a conductor as a function of temperature is given by the linear approximation:

$$R = R_0[1 + \alpha(T - T_0)]$$

1. Resistance at $T = 100°\text{C}$

$$R_{100} = 10[1 + (4.5 \times 10^{-3})(100 - 20)] = 13.6\,\Omega$$

Thus, the resistance of the tungsten wire at 100°C is $13.6\,\Omega$.

2. Resistance at $T = -10°\text{C}$

$$R_{-10} = 10[1 + (4.5 \times 10^{-3})(-10 - 20)] = 8.65\,\Omega$$

Thus, the resistance of the tungsten wire at −10°C is $8.65\,\Omega$.

Solution of Exercise 04

1. Capacitance and Charge Storage

(a) The capacitance C of a parallel-plate capacitor is given by:

$$C = \frac{\varepsilon_0 A}{d}$$

Substituting the values:

$$C = \frac{(8.85 \times 10^{-12}) \times (1.00)}{1.00 \times 10^{-3}} = 8.85 \text{ nF}$$

Thus, the capacitance of the capacitor is 8.85 nF.

(b) The charge stored on a capacitor is given by:

$$Q = CV$$

Substituting the values:

$$Q = (8.85 \times 10^{-9}) \times (3 \times 10^3) = 26.6\,\mu\text{C}$$

Thus, the charge stored on the plates is 26.6 μC.

2. Capacitor Design

To achieve a capacitance of $C = 1$ nF with a plate separation of $d = 1$ mm in vacuum, we use:

$$A = \frac{Cd}{\varepsilon_0} == \frac{(1 \times 10^{-9}) \times (1 \times 10^{-3})}{8.85 \times 10^{-12}} = 0.113 \text{ m}^2$$

Thus, to achieve a capacitance of 1 nF, the required plate area must be 0.113 m^2.

Solution of Exercise 05

1. The capacitance C of a system of two conductors is defined as:

$$C = \frac{Q}{V} = \frac{10 \times 10^{-6}}{10} = 1 \times 10^{-6} \text{ F} = 1\,\mu\text{F}$$

Thus, the capacitance of the system is $1\,\mu$F.

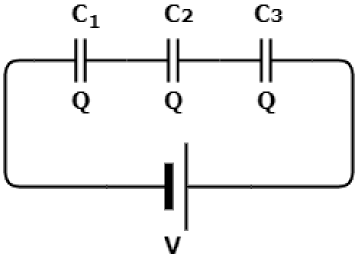
2. Since a capacitance depends only on the ***conductors' geometry and the permittivity of the medium***, it remains unchanged when the charge increases. Given the new charge $Q' = 100 \times 10^{-6}$ C, the potential difference is determined using:

$$V' = \frac{Q'}{C} = \frac{100 \times 10^{-6}}{1 \times 10^{-6}} = 100 \text{ V}$$

Thus, when the charges increase, the new potential difference between the conductors is 100 V.

Solution of Exercise 06

Case 1: Capacitors in Series



For capacitors in series, the total capacitance is given by:

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = (\frac{1}{4} + \frac{1}{6} + \frac{1}{8}) \times 10^6 = \frac{26}{48} \times 10^6$$

Thus:

$$C_{\text{eq}} = \frac{48}{26} \approx 1.85\,\mu\text{F}$$

Charge Calculation

In a series connection, all capacitors have the same charge:

$$Q = C_{\text{eq}} V = (1.85 \times 10^{-6}) \times (12) = 22.2 \mu C$$

Since the charge is the same for all capacitors:

$$Q_1 = Q_2 = Q_3 = Q = 22.2 \mu C$$

Voltage Across Each Capacitor

The voltage across each capacitor:

$$V_i = \frac{Q}{C_i}$$

- For $C_1 = 4\,\mu$F:

$$V_1 = \frac{22.2 \times 10^{-6}}{4 \times 10^{-6}} = 5.55\,V$$

- For $C_2 = 6\,\mu$F:

$$V_2 = \frac{22.2 \times 10^{-6}}{6 \times 10^{-6}} = 3.7\,V$$

- For $C_3 = 8\,\mu$F:

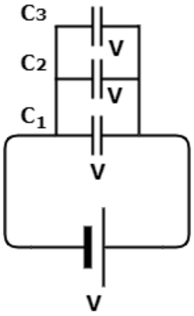
$$V_3 = \frac{22.2 \times 10^{-6}}{8 \times 10^{-6}} = 2.78\,V$$

Verification of Total Voltage

$$V_{\text{total}} = V_1 + V_2 + V_3 = 5.54 + 3.69 + 2.77 = 12V$$

Thus, the calculations are verified.

Case 2: Capacitors in Parallel



For capacitors in parallel, the total capacitance is:

$$C_{\text{eq}} = C_1 + C_2 + C_3$$

$$C_{\text{eq}} = (4 + 6 + 8) \times 10^{-6} = 18\mu F$$

Voltage Across Each Capacitor

In a parallel connection, the voltage across each capacitor is the same as the applied voltage:

$$V_1 = V_2 = V_3 = V = 12V$$

Charge on Each Capacitor

The charge on each capacitor in a parallel connection is given by:

$$Q_i = C_i V$$

- For $C_1 = 4\,\mu\text{F}$:

$$Q_1 = 4 \times 12 \times 10^{-6} = 48\,\mu\text{C}$$

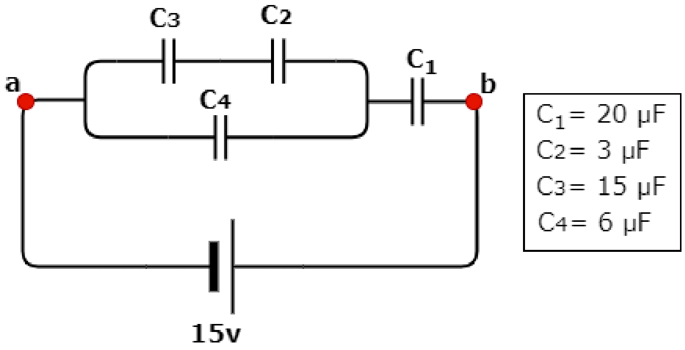
- For $C_2 = 6\,\mu\text{F}$:

$$Q_2 = 6 \times 12 \times 10^{-6} = 72\,\mu\text{C}$$

- For $C_3 = 8\,\mu\text{F}$:

$$Q_3 = 8 \times 12 \times 10^{-6} = 96\,\mu\text{C}$$

Solution of Exercise 07



1. **Equivalent Capacitance:**
   To determine the equivalent capacitance between points a and b, we analyze the network in steps:
   Step 1: Capacitors C_2 and C_3 are in series. Their equivalent capacitance C_{23} is given by:

$$\frac{1}{C_{23}} = \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{3\ \times 10^{-6}} + \frac{1}{15\ \times 10^{-6}} = \frac{2}{5} \times 10^{6}$$

   Thus:

$$C_{23} == \frac{5}{2} \times 10^{-6} = 2.5\,\mu\text{F}$$

   Step 2: Capacitors C_{23} and C_4 are in parallel. Their equivalent capacitance C_{234} is given by:

$$C_{234} = C_{23} + C_4 = 2.5\ \times 10^{-6} + 6\ \times 10^{-6} = 8.5\,\mu\text{F}$$

   Step 3: Capacitors C_1 and C_{234} are in series. The overall equivalent capacitance C_{ab} between points a and b is:

$$\frac{1}{C_{ab}} = \frac{1}{C_1} + \frac{1}{C_{234}} = \frac{1}{20 \times 10^{-6}} + \frac{1}{8.5 \times 10^{-6}} = \frac{28.5}{170} \times 10^6$$

Thus:

$$C_{ab} = \frac{170}{28.5} \times 10^{-6} \approx 5.96\,\mu\text{F}$$

Therefore, the equivalent capacitance between points a and b is approximately $5.96\,\mu$F.

2. **Charge on Each Capacitor:**

Given $V_{ab} = 15\,\text{V}$, the total charge Q stored in the network is:

$$Q = C_{ab}V_{ab} = 5.96 \times 15 = 89.4\,\mu\text{C}$$

Since C_1 is in series with the equivalent capacitance C_{234}, the charge on C_1 is the same as the total charge:

$$Q_1 = Q = 89.4\,\mu\text{C}$$

The voltage across C_{234} is:

$$V_{234} = \frac{Q}{C_{234}} = \frac{89.4 \times 10^{-6}}{8.5 \times 10^{-6}} = 10.52\,V$$

The charge on C_4 is:

$$Q_4 = C_4V_{234} = 6 \times 10^{-6} \times 10.52 = 63.12\,\mu\text{C}$$

The charge on C_2 and C_3 is the same as the charge on C_{23}:

$$Q_2 = Q_3 = Q_{23} = C_{23}V_{234} = 2.5 \times 10^{-6} \times 10.52 \approx 26.3\,\mu\text{C}$$

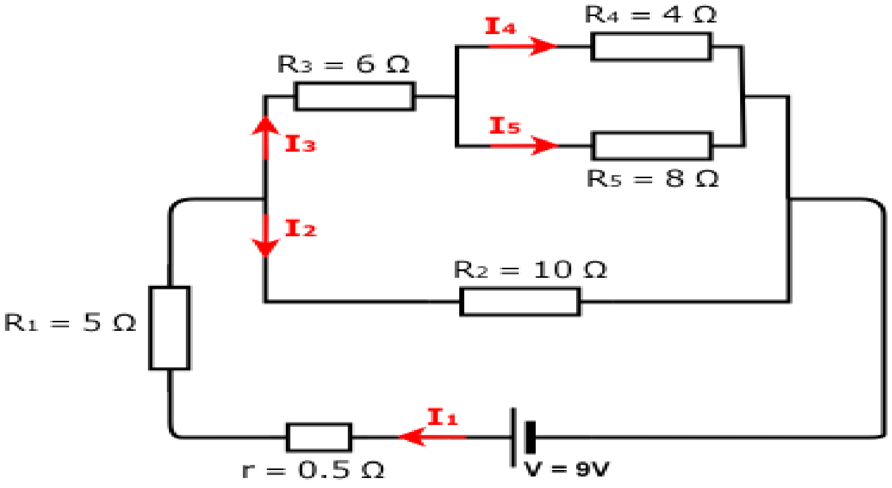
Series of Exercises 03

Exercise 01

Consider the circuit illustrated in the Figure below, determine:

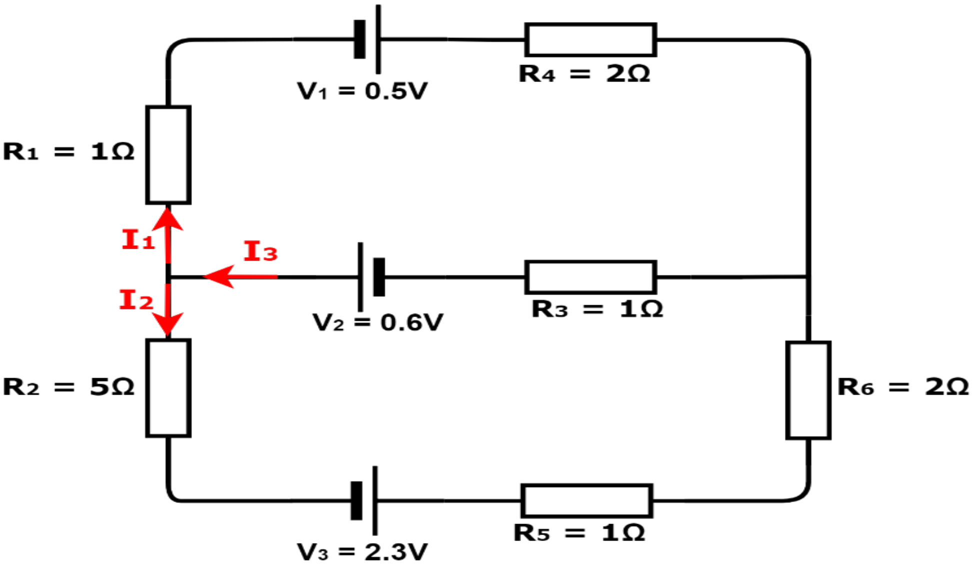
(a) The equivalent resistance and the total current from the source.

(b) The current and voltage drop across each resistor.



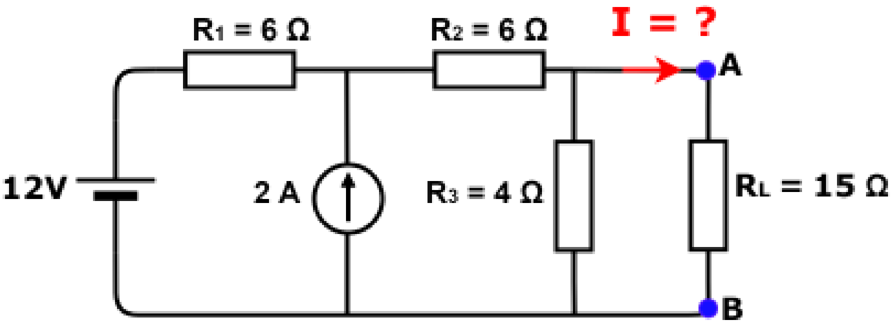
Exercise 02

Consider the circuit illustrated in the figure below. Determine the current in each branch by applying Kirchhoff's Laws and Ohm's Law.



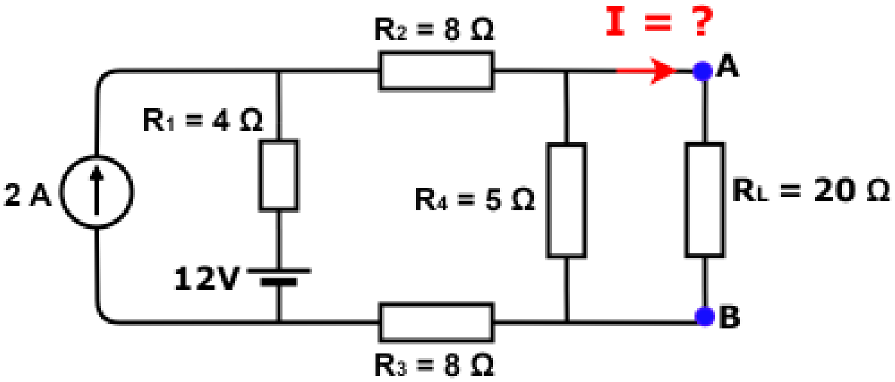
Exercise 03

Find the Thevenin equivalent circuit for the given circuit below and determine the current through the load resistor R_L.



Exercise 04

Find the Norton equivalent circuit for the given circuit below and determine the current through the load resistor R_L.



Solution of Series 03

Solution of Exercise 01

Step 1: Determine the Equivalent Resistance

Resistors R_4 and R_5 (Parallel Combination):

$$R_{45} = \left(\frac{1}{R_4} + \frac{1}{R_5}\right)^{-1} = \left(\frac{1}{4} + \frac{1}{8}\right)^{-1} = \frac{8}{3}\,\Omega \approx 2.67\,\Omega$$

Resistors R_3 and R_{45} (Series Combination):

$$R_{345} = R_3 + R_{45} = 6 + \frac{8}{3} = \frac{26}{3}\,\Omega \approx 8.67\,\Omega$$

Resistors R_{345} and R_2 (Parallel Combination):

$$R_{2345} = \left(\frac{1}{R_{345}} + \frac{1}{R_2}\right)^{-1} = \left(\frac{3}{26} + \frac{1}{10}\right)^{-1} = \frac{130}{28}\,\Omega \approx 4.64\,\Omega$$

Total Equivalent Resistance (including R_1 and r):

$$R_{\text{eq}} = R_1 + R_{2345} + r = 5 + 4.64 + 0.5 = 10.14\,\Omega$$

Step 2: Total Current from the Source

Using Ohm's Law:

$$I_1 = \frac{V}{R_{\text{eq}}} = \frac{9}{10.14} \approx 0.89\,A$$

Step 3: Current and Voltage Drop Across Each Resistor

Voltage Drop Across R_1:

$$V_1 = I_1 \cdot R_1 = 0.89 \cdot 5 = 4.45\,V$$

Voltage Drop Across Internal Resistance r:

$$V_r = I_1 \cdot r = 0.89 \cdot 0.5 = 0.45\,V$$

Voltage Drop Across R_{2345}:

$$V_{2345} = I_1 \cdot R_{2345} = 0.89 \cdot 4.64 = 4.13\,V$$

Voltage Drop Across R_2:

$$V_2 = V_{2345} = 4.13\,V$$

Current Through R_2:

$$I_2 = \frac{V_2}{R_2} = \frac{4.13}{10} = 0.41\,A$$

Current Through R_3:

$$I_3 = I_1 - I_2 = 0.89 - 0.41 = 0.48\,A$$

Voltage Drop Across R_3:

$$V_3 = I_3 \cdot R_3 = 0.48 \cdot 6 = 2.88\,V$$

Voltage Drop Across R_4 and R_5:

$$V_{45} = I_3 \cdot R_{45} = 0.48 \cdot 2.67 = 1.28\,V$$

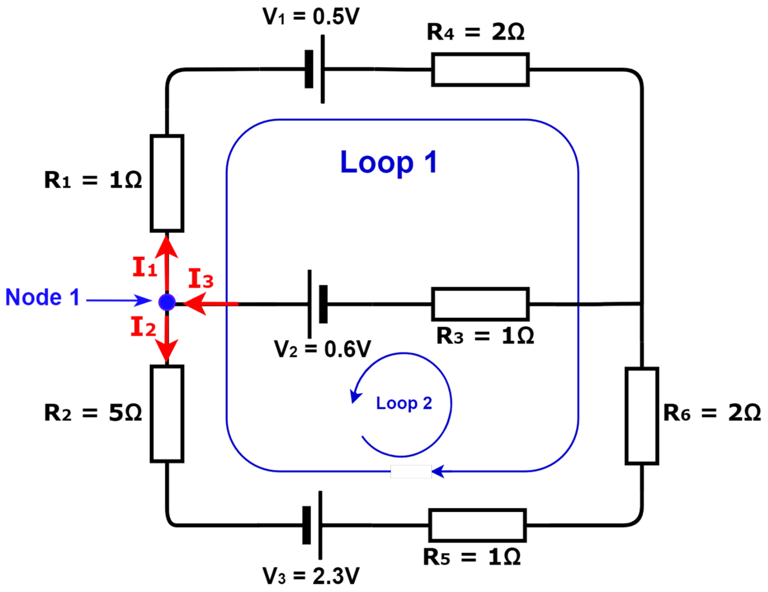
Current Through R_4:

$$I_4 = \frac{V_{45}}{R_4} = \frac{1.28}{4} = 0.32\,A$$

Current Through R_5:

$$I_5 = \frac{V_{45}}{R_5} = \frac{1.28}{8} = 0.16\,A$$

Solution of Exercise 02



Applying Kirchhoff's Current Law (KCL) at node 1:

$$I_3 = I_1 + I_2 \tag{4.2}$$

Applying Kirchhoff's Voltage Law (KVL) to loop 1:

$$V_1 - R_4 I_1 + R_6 I_2 + R_5 I_2 - V_3 + R_2 I_2 - R_1 I_1 = 0$$
$$V_1 - V_3 - I_1(R_4 + R_1) + I_2(R_2 + R_5 + R_6) = 0$$

$$-3I_1 + 8I_2 = 1.8 \tag{4.3}$$

Applying KVL to loop 2:

$$V_3 - R_5 I_2 - R_6 I_2 - R_3 I_3 - R_2 I_2 + V_2 = 0$$
$$V_2 + V_3 - I_2(R_2 + R_5 + R_6) - R_3 I_3 = 0$$

$$8I_2 + I_3 = 2.9 \tag{4.4}$$

Substituting Equation (1) into Equation (3):

$$8I_2 + (I_1 + I_2) = 2.9$$

$$I_1 = 2.9 - 9I_2 \tag{4.5}$$

Substituting Equation (4) into Equation (2):

$$-3(2.9 - 9I_2) + 8I_2 = 1.8$$

Solving for I_2:

$$I_2 = \frac{10.5}{35} = 0.3A$$

Solving for I_1 from Equation (4):

$$I_1 = 2.9 - 9(0.3) = 0.2A$$

Solving for I_3 from Equation (1):

$$I_3 = 0.2 + 0.3 = 0.5A$$

Solution of Exercise 03

Step 1: Remove the Load Resistor R_L

To determine the Thevenin equivalent circuit, we first remove R_L from the circuit, leaving two open terminals where R_L was previously connected.

Step 2: Find Thevenin Resistance (R_{th})

To find R_{th}, we deactivate all independent sources:

- Short-circuit the 12V voltage source.
- Open-circuit the 2A current source.

The resistors R_1 and R_2 are connected in series:

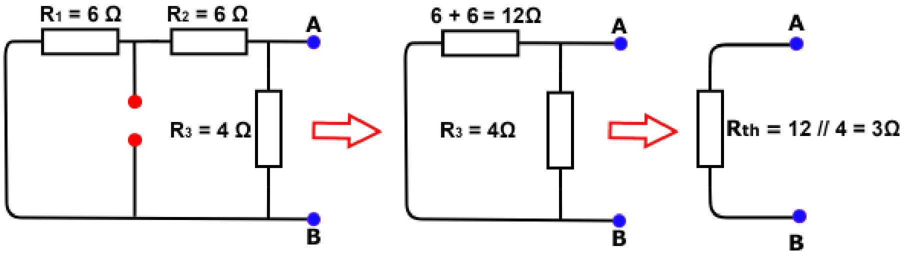
$$R_{12} = 6 + 6 = 12\,\Omega$$

The resistor R_{12} is connected in parallel with the resistor R_3 :

$$R_{123} = \frac{12 \times 4}{12 + 4} = \frac{48}{16} = 3\,\Omega$$

Thus, the Thevenin equivalent resistance is:

$$R_{\text{th}} = R_{123} = 3\,\Omega$$



Step 3: Find Thevenin Voltage (V_{th})

Applying Kirchhoff's Current Law (KCL) at node V:

$$I_3 = I_1 + I_2 \tag{4.6}$$

Using Kirchhoff's Voltage Law (KVL) in the left loop:

$$12 - 6I_1 - V = 0$$

$$I_1 = \frac{12 - V}{6} \tag{4.7}$$

Applying KVL in the right loop:

$$V - 6I_3 - 4I_3 = 0 \Rightarrow I_3 = \frac{V}{10} \tag{4.8}$$

Substituting Equations (6) and (7) into Equation (5), and given that $I_2 = 2A$, we obtain:

$$\frac{V}{10} = \frac{12 - V}{6} + 2$$

Solving for V:

$$V = 15V$$

Since V_{th} is the voltage across the 4Ω resistor:

$$V_{th} = 4 \times I_3 = 4 \times \frac{V}{10} = 4 \times \frac{15}{10} = 6V$$

Step 4: Compute the Load Current I_L

The Thevenin equivalent circuit consists of V_{th} and R_{th} in series with R_L.

$$I_L = \frac{V_{\text{th}}}{R_{\text{th}} + R_L}$$

$$I_L = \frac{6V}{3\Omega + 15\Omega} = \frac{6}{18} = 0.33A$$

Final Answer: $I_L = 0.33A$.

Solution of Exercise 04

Step 1: Remove the Load Resistor R_L

To determine the Norton equivalent circuit, we first remove R_L from the circuit, leaving two open terminals where R_L was previously connected.

Step 2: Find Norton Resistance (R_N)

To find R_N, we deactivate all independent sources:

- Short-circuit the 12V voltage source.
- Open-circuit the 2A current source.

The resistors R_1, R_2, and R_3 are connected in series:

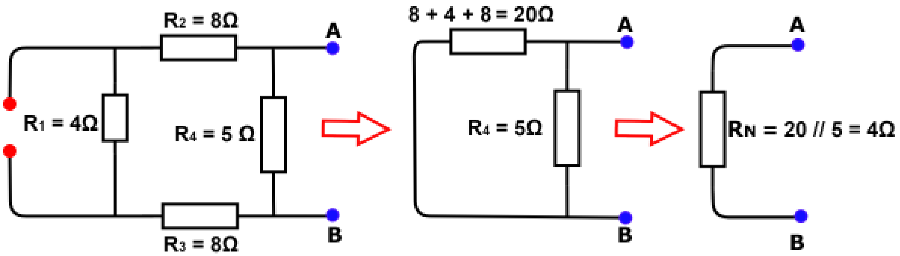
$$R_{123} = 8 + 8 + 4 = 20\Omega$$

This resistor R_{123} is connected in parallel with the resistor R_4:

$$R_{1234} = \frac{20 \times 5}{20 + 5} = \frac{100}{25} = 4\Omega$$

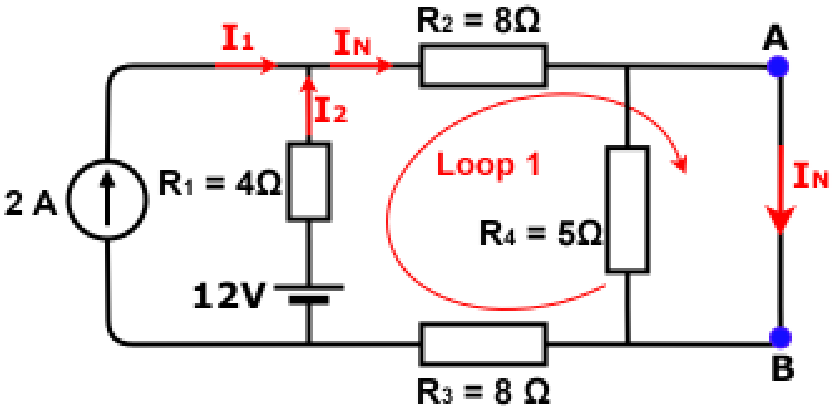
Thus, the Norton equivalent resistance is:

$$R_N = R_{1234} = 4\Omega$$



Step 3: Find Norton Current (I_N)

To determine I_N, we short-circuit the load terminals.
Important Observation: When we short-circuit the terminals, the 5Ω resistor is effectively bypassed (ignored) because all the current prefers to flow through the shorted path (zero resistance). Therefore, the 5Ω resistor does not affect I_N and is not considered in this calculation.



Applying Kirchhoff's Current Law (KCL):

$$I_N = I_1 + I_2 \tag{4.9}$$

Using Kirchhoff's Voltage Law (KVL) in the loop 1:

$$12 - 4I_2 - 8I_N - 8I_N = 0$$

$$12 - 4I_2 - 16I_N = 0 \tag{4.10}$$

From Equations (8):

$$I_2 = I_N - I_1$$

Substituting I_2 into Equation (9):

$$12 - 4(I_N - I_1) - 16I_N = 0$$

$$12 - 4I_N + 4I_1 - 16I_N = 0$$

Given that $I_1 = 2A$:

$$12 + 8 - 20I_N = 0$$

$$20I_N = 20$$

$$I_N = \frac{20}{20} = 1A$$

Step 4: Compute the Load Current I_L

The Norton equivalent circuit consists of I_N in parallel with R_N and R_L. The current I_L is given by:

$$I_L = I_N \frac{R_N}{R_N + R_L}$$

Substituting the values:

$$I_L = 1 \times \frac{4}{4+20} = \frac{4}{24} = 0.17A$$

Therefore:

$$I_L = 0.17A$$

Series of Exercises 04

Exercise 01

A charged particle with charge $q = 3.2 \times 10^{-19}$ C moves through a uniform magnetic field of magnitude $B = 1.5$ T, directed along the positive z-axis.
For each case below, determine the magnetic force $\vec{F}$ on the particle and represent the directions of the velocity $\vec{v}$, magnetic field $\vec{B}$, and magnetic force $\vec{F}$ in a diagram.

- (a) The particle moves in the positive x-direction with a velocity of 4×10^4 m/s.
- (b) The particle moves in the negative y-direction with a velocity of 5.5×10^4 m/s.
- (c) The particle moves in the positive y-direction with a velocity of 6×10^4 m/s.
- (d) The particle moves in the positive z-direction with a velocity of 3.5×10^4 m/s.
- (e) The particle is at rest.

If the charge were negative, determine the direction of $\vec{F}$ in cases (1) to (4).

Exercise 02

A straight wire of length $L = 50$ cm carries a steady current $I = 300$ mA. The wire is placed in a uniform magnetic field of magnitude $B = 2$ T. The direction of the magnetic field is along the positive y-axis.

- (a) Determine the magnitude of the magnetic force on the wire when the current flows in the:
  - i. positive x-direction
  - ii. negative y-direction
  - iii. positive z-direction
- (b) Determine the direction of the magnetic force in each case using the right-hand rule.

Exercise 03

The magnetic field B at a point P due to a straight current-carrying wire is given by:

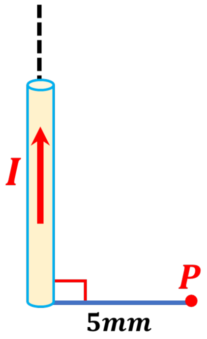
$$B = \frac{\mu_0 I}{4\pi a}(\sin\theta_2 - \sin\theta_1)$$

where:

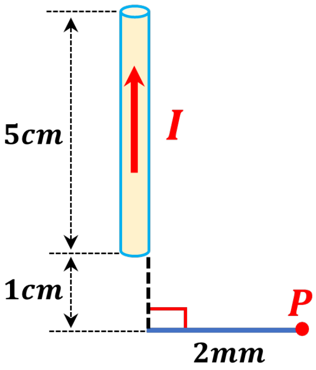
- $\mu_0 = 4\pi \times 10^{-7}$ T·m/A is the permeability of free space.
- I is the current in the wire.
- a is the perpendicular distance from the wire to the point P.
- θ_1 and θ_2 are the angles subtended at point P by the ends of the wire segment, measured with respect to the perpendicular line to the wire that passes through P.

Calculate the magnetic field B at point P for:

(a) A semi-infinite wire carrying $I = 0.5$ A. The point P is located 5 mm from the wire.

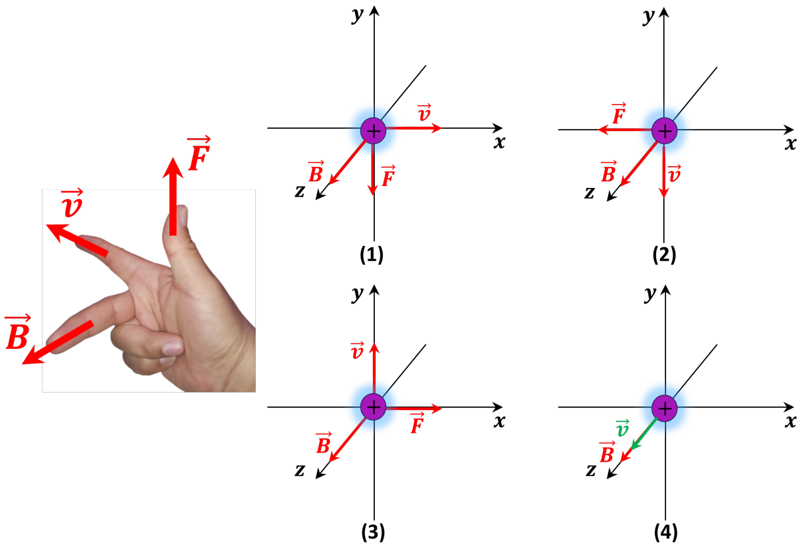


(b) A finite wire of length $L = 5$ cm, carrying $I = 1$ A. The point P is located 2 mm from the wire and 1 cm from its nearest end.



Solution of Series 04

Solution of Exercise 01



The magnitude of the magnetic force is given by:

$$F = qvB\sin\theta$$

where

$$q = 3.2 \times 10^{-19}\ \text{C}, \quad B = 1.5\ \text{T}$$

θ is the angle between the velocity and the magnetic field.

Case 1: The particle moves in the positive x-direction

$$v = 4 \times 10^{4}\ \text{m/s}, \quad \theta = 90^\circ, \quad \sin 90^\circ = 1$$

$$F = (3.2 \times 10^{-19})(4.0 \times 10^{4})(1.5)(1) = 1.92 \times 10^{-14}\ \text{N}$$

Case 2: The particle moves in the negative y-direction

$$v = 5.5 \times 10^{4}\ \text{m/s}, \quad \theta = 90^\circ, \quad \sin 90^\circ = 1$$

$$F = (3.2 \times 10^{-19})(5.5 \times 10^{4})(1.5)(1) = 2.64 \times 10^{-14}\ \text{N}$$

Case 3: The particle moves in the positive y-direction

$$v = 6 \times 10^{4}\ \text{m/s}, \quad \theta = 90^\circ, \quad \sin 90^\circ = 1$$

$$F = (3.2 \times 10^{-19})(6 \times 10^{4})(1.5)(1) = 2.88 \times 10^{-14}\ \text{N}$$

Case 4: The particle moves in the positive z-direction

$$v = 3.5 \times 10^4 \text{ m/s}, \quad \theta = 0^\circ, \quad \sin 0^\circ = 0$$

$$F = (3.2 \times 10^{-19})(3.5 \times 10^4)(1.5)(0) = 0$$

Since the velocity is parallel to the magnetic field, no force is exerted on the particle.

Case 5: The particle is at rest
Since the velocity is zero,

$$v = 0$$

$$F = (3.2 \times 10^{-19})(0)(1.5) \sin\theta = 0$$

Thus, the force is zero regardless of the magnetic field.

Case 6: The direction of $\vec{F}$ if the charge were negative
Since the force direction for a positive charge is determined using the right-hand rule, the force direction for a negative charge is simply the inverse of that shown in the diagram for each case.

Solution of Exercise 02

The magnetic force on a current-carrying wire is given by

$$\vec{F} = I\vec{L} \times \vec{B}$$

where

$$I = 3 \text{ A}, \quad L = 50 \text{ cm}, \quad B = 2 \text{ T}$$

The magnitude of the force is

$$F = ILB \sin\theta$$

where θ is the angle between the current direction and the magnetic field.

Case 1: The current flows in the positive x-direction

$$\theta = 90^\circ, \quad \sin 90^\circ = 1$$

$$F = (3)(50 \times 10^{-2})(2)(1) = 3 \text{ N}$$

Using the right-hand rule, the force is directed in the positive z-direction.

Case 2: The current flows in the negative y-direction

$$\theta = 0^\circ, \quad \sin 0^\circ = 0$$

$$F = 0$$

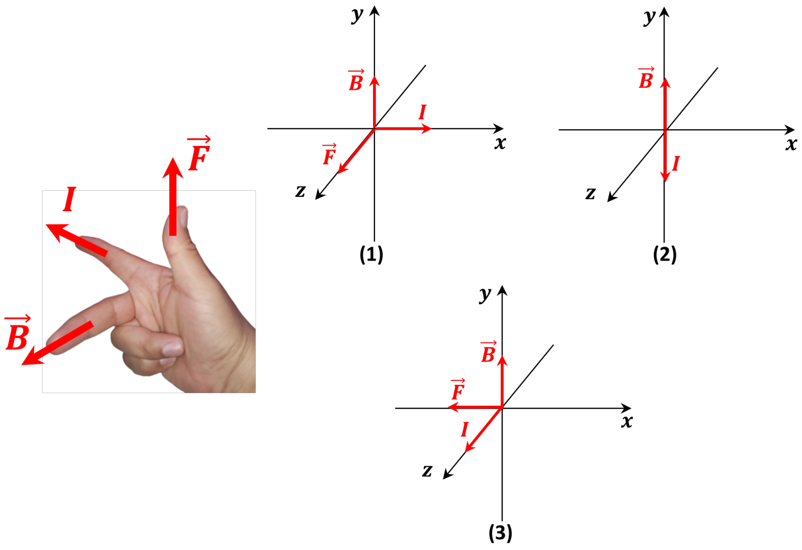
Since the current is parallel to the magnetic field, no force is exerted on the wire.

Case 3: The current flows in the positive z-direction

$$\theta = 90^\circ, \quad \sin 90^\circ = 1$$

$$F = (3)(50 \times 10^{-2})(2)(1) = 3 \text{ N}$$

Using the right-hand rule, the force is directed in the negative x-direction.



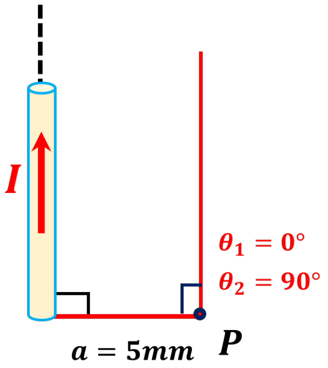
Solution of Exercise 03

Given Data

- Permeability of free space: $\mu_0 = 4\pi \times 10^{-7}$ T·m/A.
- Current: $I = 0.5$ A (Case 1), $I = 1$ A (Case 2).

Biot-Savart Law for a Straight Wire

$$B = \frac{\mu_0 I}{4\pi a}(\sin\theta_2 - \sin\theta_1)$$

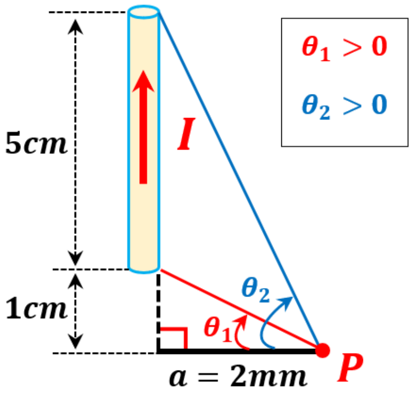


Case 1: Semi-Infinite Wire

- The wire extends infinitely in one direction, making one endpoint appear at $\theta_2 = 90°$ and the other aligned with the perpendicular at $\theta_1 = 0°$.
- Calculation:

$$B = \frac{(4\pi \times 10^{-7}) \times 0.5}{4\pi \times 5 \times 10^{-3}} = 1 \times 10^{-5}\ \text{T} = 10\,\mu T$$

Case 2: Finite Wire



- Angle calculations:

$$\sin\theta_1 = \frac{1 \times 10^{-2}}{\sqrt{(2 \times 10^{-3})^2 + (1 \times 10^{-2})^2}} \approx 0.98$$

$$\sin\theta_2 = \frac{(1+5) \times 10^{-2}}{\sqrt{(2 \times 10^{-3})^2 + ((1+5) \times 10^{-2})^2}} \approx 1$$

- Magnetic field:

$$B = \frac{(4\pi \times 10^{-7}) \times 1}{4\pi \times 2 \times 10^{-3}}(1 - 0.98) = 0.01 \times 10^{-4} = 1\,\mu T$$

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